Journal of Medical Sciences

ISSN 1682-4474

science alert

ANSInet
an open access publisher
http://ansinet.com

Research Paper

JMS (ISSN 1682-4474) is an International, peer-reviewed scientific journal that publishes original article in experimental & clinical medicine and related disciplines such as molecular biology, biochemistry, genetics, biophysics, bio-and medical technology. JMS is issued eight times per year on paper and in electronic format.

For further information about this article or if you need reprints, please contact:

Emmanuel O. Oghre
Department of Mathematics,
University of Benin,
Benin City, Nigeria

Tel: 2348033551266

J. Med. Sci., 7 (1): 141-145
1st January, 2007

Blood Viscosity and Sickle Cell Anaemia

Emmanuel O. Oghre and Daniel Okunoghae

Many factors affect the flow of blood in the arteries and other small vessels among which are the diameters of the various vessels, the viscosity of the blood and various constituents of the blood. People with the HbS type of red blood cell have their cell sickle. The present study used a one-dimension model of flow in the arteries to investigate the effect of increase viscosity on small pressure disturbance and on the arterial compliance and the sequence of shock. We deduced that increase viscosity which all sickle cell patient are prone to leads to reduced flow and hampers the distribution of oxygen to essential areas of the body. The higher the viscosity the more like the crisis will occur.

Key words: Blood viscosity, sickle cell, wall compliance, shock

ANSInet
Asian Network for Scientific Information

Department of Mathematics, University of Benin, Benin City, Nigeria

INTRODUCTION

The desire to quantitatively understand both the microscopic and macroscopic behaviour of blood when it flows through the vessels has supplied the motivation for appropriate investigations. Young summarized the essence of this desire as understanding the physical events that take place in normal animal and thereby make contribution to physiology. A secondary aim is to make contribution to medicine by analyzing particular abnormal or diseased states in the hope of improving diagnosis or treatment (Pedley, 1980) .

Within the blood vessels are red blood cells, which contain a protein called haemoglobin whose function is to carry oxygen from the lungs to all parts of the body. Most people have normal haemoglobin or HbA; their blood cells are doughnut shaped and flexible, allowing them to pass easily through even the smallest vessels. Those with sickle cell anaemia have a different type of haemoglobin, HbS. Their blood cells are sickle shaped and rigid and therefore cannot pass through small blood vessels. This results in blockage and a lack of oxygen reaching parts of the body causing pain or damage to vital organs.

There are three primary factors that determine resistance to blood flow within a single vessel: diameter R (radius), length of vessel L and viscosity of blood η (Klahunde, 2005). The relationship is given by:

$$R = \frac{\eta L}{r^4} \tag{1}$$

The relationship between flow F, the Pressure gradient ΔP and Resistance R is given by the well-known Poiseuille equation as

$$F \alpha \frac{\Delta P}{R} = \frac{\Delta P r^4}{\eta L} \tag{2}$$

This implies that flow is inversely proportional to blood viscosity. The viscosity (thickness) of blood or any liquid, for that matter, is controlled by both the chemical components and temperature (Bedham, 2004). Temperature induced changes are attributable to the way molecules in the liquid react to a gain or loss of thermal energy. Just as the Poiseuille equation suggests viscous liquids resist internal flow; viscous blood would resist flow through blood vessels and would consequently not be as effective at distributing heat to either areas of the body that are in need of it or away from places that have excess of it.

It is well known that the viscosity of a sickle cell patient is greater than that of a normal patient. Mazzoni *et al.* (2002) found that as blood viscosity decreases, cardiac output increases. This implies also that increase in blood viscosity decreases cardiac output. Chien's measurement (Chien, 1970) of effective viscosity as a function of particle shape in dilute suspension found that thin discs have higher viscosity. Chien's results also suggest that continuous surface deformations in response to flow conditions may further reduce their contribution to blood viscosity. This implies that in a sickle cell patient where the cell are sickle and resist deformation the viscosity will be higher than normal. In this study we investigate the effect of increase viscosity on small pressure disturbance and the arterial compliance. We also determine the effect of this increase on the time of shock formation i.e., the onset of sickle cell anaemia patient crisis.

MATHEMATICAL FORMULATION

Due to the large wavelength of the arterial pulse waves compared to the vessel diameters, one-dimensional models permits an efficient simulation of the wave propagation in the large arteries which allows us to study the effects of local changes on the global system. The governing equations of the one dimensional model are obtained by applying conservation of mass and momentum to a one dimensional impermeable tubular control volume of inviscid, incompressible and Newtonian fluid and by considering a tube law that relates changes in pressure to changes in cross-sectional area. The continuity equation is one of the equations governing the fluid dynamics of the blood and states the conservation of matter i.e., what goes into a section must either come out or be stored. Under the assumption of one dimensionality the continuity equation is

$$\frac{\partial A}{\partial t} + \frac{\partial (uA)}{\partial x} = 0 \tag{3}$$

where A denotes the cross sectional area of the artery, u the axial blood velocity (average over the cross-section), x the axial coordinate and t is the time. The first term is the rate of storage of blood within the arterial segment; the second is the difference between the blood flowing into and out of the segment in the axial direction.

The second fluid dynamics relation is the momentum equation, which balances inertia, friction and pressure forces acting on the blood. In one dimension it is given as

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{\partial P}{\partial x} + \tau = 0 \tag{4}$$

where P is the pressure difference across the tube and p is the fluid density. The first term is the acceleration of the blood; the second is the convective acceleration due to axial rate of change of velocity, the third the pressure gradient and the fourth term represents the friction at walls of the arteries. It is assumed that the lumen area is related instantaneously to the pressure (Vander Werff, 1974) and there is no phase lag; thus the one dimension state equation is given as

$$A = A(x,p) \tag{5}$$

Although the shear stress in pulsating flows is not directly related to the instantaneous mean velocity, it is assumed to be given by the laminar Poiseuille relation

$$\tau = -\frac{8\pi\mu u}{A} \tag{6}$$

where μ is the blood viscosity.
Equation 4 then becomes

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} = -\frac{8\pi\mu u}{\rho A} \tag{7}$$

METHOD OF SOLUTION

If we assume that the wave amplitude is smaller than the wavelength, then the cross-sectional area can be assumed to be constant along the elastic vessel.

$$\therefore \frac{\partial A}{\partial x} << 1 \tag{8}$$

and Eq. 3 reduces to

$$\frac{1}{A}\frac{\partial A}{\partial t} + \frac{\partial u}{\partial x} = 0 \tag{9}$$

A special example of a pressure area relationship which has been empirically formulated to model the pulmonary artery is

$$A = \pi\left(R_0 + \alpha P\right)^2 \tag{10}$$

where R_0 is the vessel radius at zero pressure and α is the compliance constant of the vessel wall.

We can thus use Eq. 7, 9 and 10 to determine the speed of propagation of a small disturbance in an elastic vessel. This may allow us quantify the compliance and the effect of increase in viscosity on it. If we substitute Eq. 10 into 9 we have

$$\frac{2\alpha}{R_0 + \alpha P}\frac{\partial P}{\partial t} + \frac{\partial u}{\partial x} = 0 \tag{11}$$

If the initial fluid velocity is zero, then again because the wave amplitude is small the convective acceleration term in Eq. 7 can be neglected. Thus we have

$$\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{-8\pi\mu u}{\rho A} \tag{12}$$

Differentiating Eq. 11 with respect to t and Eq. 12 with respect to x, subtracting the resulting equation and neglecting the second order terms we obtain

$$\frac{\partial^2 P}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} = 0 \tag{13}$$

Equation 12 is the wave equation where the quantity c is the wave speed and is given as

$$c^2 = \frac{R_0 + \alpha P}{2\rho\alpha} \tag{14}$$

We can the determine the vessel compliance α as

$$\alpha = \frac{R_0}{2\rho c^2 - P} \tag{15}$$

Therefore we can determine the vessel compliance if we know the other measurable parameters.

Recall Eq. 3, 5 and 7

$$\frac{\partial A}{\partial t} + \frac{\partial(uA)}{\partial x} = 0 \tag{3}$$

$$A = A(x, P) \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} = -\frac{8\pi\mu u}{\rho A} \tag{7}$$

Let us introduce the wave speed in the form

$$c^2 = \frac{A}{\rho}\frac{\partial P}{\partial A} \tag{16}$$

Note that this form is conforms with Eq. 14
Using (5) and (16) Eq. 3 becomes

$$\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x} + \rho c^2\frac{\partial u}{\partial x} + \rho c^2\frac{u}{A}\frac{\partial A}{\partial x} = 0 \tag{17}$$

if we consider a uniform arterial segment as in Eq. 8, we have

$$\frac{\partial P}{\partial t} + u\frac{\partial P}{\partial x} + \rho c^2 \frac{\partial u}{\partial x} = 0 \tag{18}$$

Now let us consider the flow as a perturbation of the undisturbed state $u = 0$, $P = P_0$= constant, $A = A_0$= constant (Hoogstraten and Smit, 1978; Akinrelere and Ayeni, 1983; Ayeni and Akinrelere, 1984). We also assume that

$$\frac{\partial A}{\partial P}(P_0) > 0$$

for $x \geq 0$ i.e., the tube wall react instantaneously on pressure change in the blood. Thus if we use the subscript 0 to denote the initial value then we have

$$P(0, t) = P_0 + rt + 0(t^2) \qquad r > 0 \tag{19}$$

Let

$$\tau = t - \frac{x}{c_0} \qquad c_0 = c(P_0) \tag{20}$$

Let also the wave expansion for P, u, T be of the form.

$$P = P_0 + \tau P_1(t) + \frac{\tau^2}{2} P_2(t) + \ldots \quad P_1(0) = r \tag{21}$$

$$u = \tau u_1(t) + \frac{\tau^2}{2} u_2(t) + \ldots \tag{22}$$

Thus P_1, u_1 are a measure of the jump in the normal derivatives of P, u, respectively. We now transform Eq. (7) and (18) to ordinary differential equations by noting that

$$\frac{\partial}{\partial t} = \frac{d}{dt} + \frac{d}{d\tau} \text{ and } \frac{\partial}{\partial x} = -\frac{1}{c_0}\frac{d}{d\tau} \tag{23}$$

This leads us to have

$$P_1 = \rho c_0 u_i \tag{24}$$

$$\frac{dP_1}{dt} + P_2 - \frac{u_1 P_1}{c_0} - \rho c_0 u_2 - 2\rho \left.\frac{dc}{dP}\right|_{P_1} P_1 u_1 = 0 \tag{25}$$

$$\frac{du_1}{dt} + u_2 - \frac{u_1^2}{c_0} - \frac{P_2}{\rho c_0} - \frac{8\pi}{\rho A_0}\mu u_1 = 0 \tag{26}$$

Using Eq. (24) to eliminate u_1 from Eq. (25) and (26) we have

$$\frac{dP_1}{dt} - \left(\frac{1}{\rho c_0^2} + \frac{1}{c_0}\left.\frac{dc}{dP}\right|_{P_1}\right) P_1^2 + \frac{4\pi\mu P_1}{\rho A_0} = 0 \tag{27}$$

If we let

$$a = \left(\frac{1}{\rho c_0^2} + \frac{1}{c_0}\left.\frac{dc}{dP}\right|_{P_1}\right) \text{ and } b = \frac{4\pi\mu}{\rho A_0}$$

then equation can be written in the form

$$\frac{dP_1}{dt} - aP_1^2 + bP_1 = 0 \tag{28}$$

We consider first the case when $\mu = 0$ i.e., $b = 0$ and we have

$$\frac{dP_1}{dt} - aP_1^2 = 0 \tag{29}$$

This gives us

$$P_1 = \frac{r}{1 - art}, \qquad P_1(0) = r \tag{30}$$

P_1 becomes infinite when

$$t = t_s = \frac{1}{ar}, \qquad \text{if } a > 0 \tag{31}$$

We now consider when the viscosity $\mu \neq 0$ by solving the equation

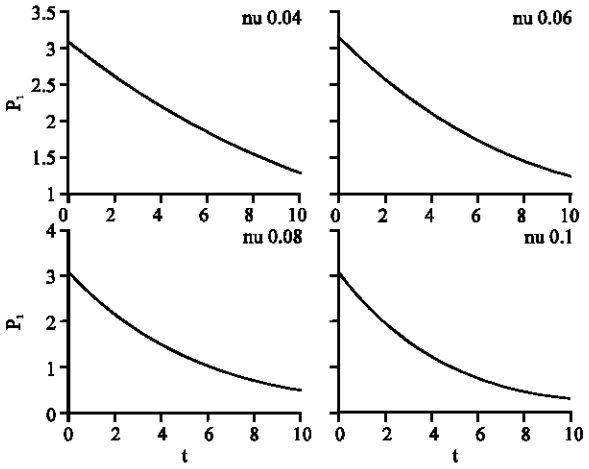
$$\frac{dP_1}{dt} - aP_1^2 + bP_1 = 0 \tag{32}$$

This gives us

$$\frac{1}{P_1(t)} = \frac{1}{r}\exp(bt) - \frac{a}{b}\left[\exp(bt) - 1\right] \tag{33}$$

$$= \frac{1}{r}\exp\left(\frac{4\pi\mu t}{\rho A_0}\right) - \frac{\rho a A_0}{4\pi\mu}\left[\exp\left(\frac{4\pi\mu t}{\rho A_0}\right) - 1\right] \tag{34}$$

We plot below the graph of $P_1(t)$ against t for various values of μ



$P_1(t)$ and invariably $v_1(t)$ become infinite at time

$$t_s = -\frac{\rho A_0}{4\pi\mu}\log_e\left(1-\frac{4\pi\mu}{\rho A_0 ar}\right) \quad e\left(\frac{4\pi\mu}{\rho A_0 ar}<1\right) \qquad (35)$$

This is the time when shock occurs.

DISCUSSION

The circulation time of blood is given by the ratio of volume to flow. A minimum volume will result in a reduced transformation time of information through the circulation delivered by hormones, oxygen carbon dioxide and other blood solute. Chien (1970) has shown that continuous surface deformation in response to flow conditions reduces their contribution to blood viscosity. For a sickle cell anaemia patient the sickle nature which causes rigidity prevents this thus making the blood density higher than would have been reduced. From Eq. (14) we see that the wave speed is inversely proportional to the to the density of the fluid. Therefore as the density and invariably the viscosity increases, the wave-speed decreases. The wall compliance is also inversely proportional to the density and as it increases the compliance decreases and the vessel response is decreased. Therefore there is a blockage and a lack of oxygen reaching parts of the body causing pain or damage to vital organs.

The occurrence of shock is synonymous with the breakdown of flow or what is termed crisis in the patient. Equation (35) clearly shows when this is likely to occur and we can also see that as μ increases t decreases i.e., the existence of shock becomes prominent as the viscosity increases.

REFERENCES

Akinrelere, E.A. and R.O. Ayeni, 1983. Shock waves in arterial blood flow. J. Nig. Math. Soc., 2: 91-98,.

Ayeni, R.O. and E.A. Akinrelere, 1984. On temperature factor and the effect of viscosity on blood temperature in the arteries. J. Theor. Biol., 109: 479-487.

Bedham, S.J., 2004. Body Temperature Regulation in Ask A Scientist. Ar gone National Laboratories, Department of Energy, University of Chicago, Newton BBS.

Chien, S., 1970. Shear dependence of effective cell volume as a determinant of blood viscosity. Science, 168: 977-979.

Hoogstraten, H.W. and C.H. Smit, 1978. A mathematical theory of shock waves formation in arterial blood flow. ACTA. Mecha., 30: 145-155.

Klahunde, R.E., 2005. Determinants of Resistance to Flow (Poiseuille Equation). Cardiovascular Physiological Concepts. Http://www.cvphysiology.com/Hemodynamics/H003.htm.

Mazzoni, M.C., T.G. Amy and I. Marcos, 2002. Blood and plasma viscosity and microvascular function in hemodilution. Eur. Sur. Res., 34: 101-105.

Pedley, T.J., 1980. The Fluid Dynamics of Large Blood vessels. Cambridge University Press.

Vander-Werff, T.J., 1974. Significant parameters in arterial pressure and velocity development. J. Biomechan., 7: 437-447.