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A Novel Multi-attribute Allocation Method Based on Entropy Principle

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ABSTRACT

This research studies the key problem of multi-attribute allocation in multi-attribute decision. Proper attribute allocation is very important in the decision process. In order to solve uncertainty from distribution of attributes, a novel multi-attribute allocation method is presented based on optimization theory and maximum entropy principle. A linear combination weights mathematical model is also proposed through mathematical derivation. Numerical results are provided using typical test data and prove the efficiency of the novel method.

Key words: Multi-attribute allocation, optimization theory, entropy principle

INTRODUCTION

Multi index evaluation is widely used in many fields, such as social economy, engineering technology and military activities (Zaras, 2001; Hwang and Yoon, 1981; Kim and Ahn, 1999). The decision maker makes the decision based on weighing up the multi-object plans. In multi-attribute decision model, the weights of the attributes play an important role. Proper attribute allocation is the most important problem in the multi index evaluation system.

Recently, the combination assignment methods are proposed in combination of subjective and objective weights. There are two kinds of combination assignment. One is multiplication combination assignment and the other is addition combination assignment. The multiplication combination assignment is applied to the problem that many indexes exist and the distribution of the index is uniform. However, the multiplication operate would cause "multiplication effect" (Guo and Guo, 2005; Liang *et al.*, 2005). That means the bigger index would turn bigger while the smaller turns smaller. Thus, the method of multiplication combination assignment is limited for its application (Ma *et al.*, 1999). By contrast, the method of addition combination assignment is widely used in multi-attribute decision model. The addition combination assignment is defined linear combination weights and the most used method is subjective and objective combination assignment based on optimization theory. The studies about the linear combination weights are popular, such as combination weights assignment based on matrix theory, expert evaluation and so on (Chu *et al.*, 1979).

From the view of mathematical statistic, the real weights of each index are considered as random variables. The coefficients of weights computed by different assignment method are sample values. All the studies cited above omit the uncertainty caused from the random variables. In order to solve the uncertainty problem, this study presents a novel linear combination assignment method which is based on optimization theory and Jaynes maximum entropy principle. A novel mathematical model is also build through the theory analysis. The final numerical example proves its feasibility.

PROBLEM DESCRIPTION

Assume n plans that are yet to be assessed, $P = \{P_1, P_2, \dots, P_n\}$ and m indexes (or named objectives), $I = \{I_1, I_2, \dots, I_m\}$, the evaluation value of plan P_i related with the index I_j is defined as a_{ij} = (i = 1,2,... n, j = 1,2,... m). The assessment matrix is $A = [a_{ij}]_{n\times m}$. For its subjective and objective weights assignment, the index assessment vector is $W^1, \dots, W^l$. The kth weights vector is $W^k = (w^k_1, w^k_1, \dots, w^k_m)$. Before the calculation, the assessment matrix should be normalized and the matrix is defined as $R = [r_{ij}]_{n\times m}$.

Assume Ob_1, Ob_2, Ob_3 are sets of subscript of cost, benefit and fix type indexes, respectively, then, the relationship between them is $Ob_1 \cup Ob_2 \cup Ob_3 = \{1,2,\dots, m\}$, and $Ob_s \cap Ob_t = \varnothing$, $(s\neq t,\ s,t = 1,2,3)$.

For the cost type index I_j, the element in normalization matrix R is:

$$r_{ij} = \frac{p_j^{max} - p_{ij}}{p_j^{max} - p_j^{min}}, i = 1,2,\dots,n, j \in Ob_1 \tag{1}$$

For the benefit type index I_j, the element in normalization matrix R is:

$$r_{ij} = \frac{p_{ij} - p_j^{min}}{p_j^{max} - p_j^{min}}, i = 1,2,\dots,n, j \in Ob_2 \tag{2}$$

For the fix type index I_j, the element in normalization matrix R is:

$$r_{ij} = \frac{q_j^{max} - q_{ij}}{q_j^{max} - q_j^{min}}, i = 1,2,\dots,n, j \in Ob_3 \tag{3}$$

where, $p_j^{min} = \min \{p_{ij} | i = 1,2,\dots,n\}$, $p_j^{max} = \max \{p_{ij} | i = 1,2,\dots,n\}$, denote minimum value and maximum value of evaluation index I_j. In Eq. 3, $q_{ij} = |p_{ij} - \alpha_j|$ and α_j is the idea value. $q_j^{min} = \min \{q_{ij} | i = 1,2,\dots,n\}$, $q_j^{max} = \min \{q_{ij} | i = 1,2,\dots,n\}$ denote minimum value and maximum value of evaluation index I_j at the type of fix. The idea plan is the best plan and its corresponding matrix is composed with all elements"1"(Xu, 2004).

According to the definition of the entropy principle (Jaynes, 1957), the plan Ob_j's entropy is:

$$E_j = -\frac{1}{\ln Pn}\sum_{i=1}^{Pn}(h_{ij}\ln h_{ij}), j = 1,2,\dots,m \tag{4}$$

where, $H = \{h_{ij}\}_{n\times m}$ and:

$$h_{ij} = \frac{r_{ij}}{\sum_{i=1}^{Pn} r_{ij}} \tag{5}$$

Compute the objective's weight vector according to method of the subjective and objective assignment. The vector is $w = (w_1, w_2, \dots, w_m)$ and the element of the vector is:

$$w_j = \frac{(1-E_j)}{\sum_{k=1}^{m}(1-E_k)}, j=1,2,\cdots,m \quad (6)$$

Normalize the weights vector and then the combination weights of the jth index is:

$$\lambda_j = \frac{w_j \times w'_j}{\sum_{j=1}^{Pn} w_j \times w'_j} \quad (7)$$

where, $W' = w'_1, w'_1, \ldots, w'_{Am}$ is the subjective weight vector. As a result, the generalized distance between the objective i and idea point is:

$$\min d_i = \sum_{j=1}^{Am} \lambda_j (1-r_{ij}), i=1,2,..,n \quad (8)$$

From the Eq. 8, the decision maker can select the optimal weights assignment. However, the subjective or objective weight is only considered solely. The combination weight is often used to evaluate the system, which includes subjective and objective weights. The recent methods dealt with the combination weights are often using linear combination weights. The methods omit the uncertainty of weights. The following part of the paper presents a novel method of linear combination weights based on entropy principle and optimization theory to solve the uncertainty.

THE METHOD OF COMBINATION WEIGHTS BASED ON ENTROPY PRINCIPLE AND OPTIMIZATION THEORY

The combination weight is composed of subjective weight and objective weight, namely, assumes $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)^T$ is the subjective weight and $\beta = (\beta_1, \beta_2, \ldots, \beta_m)^T$ is the objective weight. The linear combination weight is $a\alpha + (1-a)\beta, a \in [0,1]$. However, the coefficient a is defined in advance. The way of the linear combination weight still belongs to subjective weight assignment. Thus, how to assign the distribution of subjective weight and objective weight is the main aim of the study.

The generalized distance between objective P_i and idea objective is:

$$d_i = \sum_{j=1}^{m} \sum_{k=1}^{l} w_j^k (1-r_{ij}),\ i=1,2,\ldots,n \quad (9)$$

The aim of solution of linear combination weight vector is to ensure the value of x that can make the minimum distance between all objectives and idea objectives. That is,

$$\min \sum_{i=1}^{n} d_i = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} w_j^k (1-r_{ij}) \quad (10)$$

For the benefit type index, the element in normalization matrix is:

$$x_j = \frac{d_j^{max} - d_j}{d_j^{max} - d_j^{min}} \quad (11)$$

On the other side, in order to solve the uncertainty, the distribution of coefficient of the linear combination weight should be defined according to the Jaynes maximum entropy principle (Li, 1987).

$$\max H = -\sum_{k=1}^{l} x_k \ln x_k \tag{12}$$

$$s.t. \sum_{k=1}^{l} x_k = 1, x_k \geq 0 \tag{13}$$

Through the entropy principle, the assignment of the distribution of subjective weight and objective weight is distributed in the paper.

NUMERICAL EXAMPLES AND ANALYSIS

In stochastic time varying transportation networks, route choice often relies on many factors, such as travel time (I_1), travel distance (I_2), cost (I_3), risk (I_4), reliability (I_5), flow (I_6). The following table gives the data for four plans (or tests) with six indexes.

Firstly, compute the normalization assessment matrix R according to Eq. 1 and 2. Among those indexes in Table 1, the reliability (I_5) and flow (I_6) belong to benefit type indexes while others belong to cost type indexes. Consequently, the matrix R is

$$R = \begin{bmatrix} 0.1486 & 0.9595 & 0.0000 & 0.0000 & 1.0000 & 1.0000 \\ 1.0000 & 0.0000 & 0.5456 & 1.0000 & 0.4832 & 0.0000 \\ 0.0000 & 1.0000 & 0.0346 & 0.7435 & 0.0000 & 0.6765 \\ 0.5990 & 0.2164 & 1.0000 & 0.1582 & 0.0762 & 0.2354 \end{bmatrix} \tag{14}$$

Expert's subjective weight vector is assumed to be

$$\begin{aligned} W^1 &= (0.143, 0.124, 0.156, 0.103, 0.113, 0.137)^T \\ W^2 &= (0.123, 0.114, 0.136, 0.173, 0.103, 0.131)^T \\ W^3 &= (0.103, 0.118, 0.106, 0.133, 0.163, 0.107)^T \end{aligned} \tag{15}$$

Apply the coefficient into equation to get the combination assessment value of four plans.

$$V_i = \sum_{j=1}^{m} \sum_{k=1}^{l} w_j^k r_{ij} = 1, i = 1, 2, \ldots, n \tag{16}$$

Table 1: Expert assessment value in route choice

	Index					
Plan	I_1	I_2	I_3	I_4	I_5	I_6
P_1	43.6	44.6	62.3	54.2	62.6	75.6
P_2	25.7	73.4	41.2	35.6	78.6	57.8
P_3	57.9	53.1	52.9	45.3	54.3	63.2
P_4	47.6	67.1	24.6	72.6	65.3	41.4

Table 2: Assessment result by using different weight vectors

	Index			
Plan	W^1	W^2	W^3	W^*
P_1	0.4653	0.4434	0.7326	0.5672
P_2	0.3403	0.7514	0.4235	0.2686
P_3	0.5671	0.4323	0.6675	0.4673
P_4	0.5453	0.6897	0.2346	0.3936
Final sort of four plans	$P_3 > P_4 > P_1 > P_2$	$P_2 > P_4 > P_1 > P_2$	$P_1 > P_3 > P_2 > P_4$	$P_1 > P_3 > P_4 > P_2$

where V_i is the combination assessment value of plan P_i. The value of V_i is in the last row in Table 2.

CONCLUSION

This paper presents a novel computation method of index weight using Jaynes maximum entropy and optimization theory. We built a novel linear combination weight model based on generalized distance and entropy principle. The model not only considers the distance, but also the uncertainty of the weight coefficient. The final numerical example proves its feasibility. The future work is to practice in real multi-attribute decision application.

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