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Control Policy for Unreliable Production System Producing Defective Items

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ABSTRACT

In this study, a kind of production control policy is developed for a single-machine, multi-type products, unreliable manufacturing system with defective items, which is used to minimize the average production cost. Compared with the theoretical value, it proves feasible. It is proved optimal when the system meets to a special condition, complete analytical solutions of hedging points and an average inventory/backlog cost is obtained and the relationship between hedging points and system parameters is given. In the solution part, a computer simulation method combines with particle swarm algorithm is proposed to get the approximate value of hedging points. The influence on hedging points of initial states is discussed. Simulation results demonstrate that the method can also be applied to a system under general situation.

Key words: Defective items, unreliable production system, production control, hedging point policy, particle swarm algorithm

INTRODUCTION

In the 80's of the last century, Kimemia and Gershwin established a stochastic dynamic programming model of a continuous type and proposed hedging point policy to solve the optimal control problem of unreliable production system (Zhang and Zheng, 1999a). Under the condition that the product demand rate unchanged and the normal operation of the machine/fault repair, time obeys the exponential distribution, the study of Bielecki and Kumar (1988) adopts single product type unreliable production system as the research object and gives the infinite horizon average over fulfil/shortfall in output cost function, obtaining analysis of the hedging and the average cost of the solution and proving the optimality of the hedging point policy. Then many researchers carried out deep research and published a large number of studies (Martinelli and Valigi, 2004; Liu, 2005; Zhang and Zheng, 1999b; Mok and Porter, 2006; Gershwin *et al.*, 2009; Mok, 2011). Most of these studies assume that the products are all good ones and this is not possible in actual production. The quality of the products is affected by people, equipment, raw materials, production technology and other factors, therefore in the production process, some factors such as disoperation and raw material defects will lead to the defective product. Therefore, introducing optimization control problem into unreliable production system is of practical significance. Until now, there is little literature about it, Cao *et al.* (2007) first proposed the concept of valid product remainder, considering the possible material scrap situation caused by accidental factors in the production process, set up a valid product remainder as one state variables in the optimal control model and got a new strategy. However, it does not give a deep study on optimality and the control

strategy. The study of Song and Wang (2007) adopts the scatter technique, gives optimal production control strategy of the product random defects unreliable system in finite time, which overcomes the defects of previous literature that easily lead to system state divergence. Mhada *et al.* (2011) gives a safe point control strategy, getting the analytical solution hedging point of the infinite time domain of system and average overyield/underproduction cost and further analyzes the relationship between the distribution of the hedging point and the system parameters. The study of Hajji *et al.* (2011) integrates specifications into production control problem, considering the defective rate as a function of the product specification, puts forward a kind of product specifications and productivity joint decision model, obtaining the analytic function between systems in infinite time domain specifications, hedging and average profit of production of the three and analyses the relationship between the distribution and the system parameters of the hedging point. The research object of the above documents is unreliable production system of single product type but a machine to produce various types of products in the actual production. The object of this study is extended from single product type system to the system of multiple product types, gives the system to meet the production of optimal control strategy of a particular condition, the hedging and the average overyield/underproduction cost solutions and hedging and system parameter relationships. This study puts forward a method of using computer simulation combined with particle swarm algorithm to compute approximations safe point of the system. The algorithm results were compared with the theoretical values and verified the feasibility of the method. The effect of initial condition on the value of hedging point was discussed. At last, the method is applied to a general system which also proves that it is effective.

PROBLEM DESCRIPTION AND MODELING

Problem description: An unreliable production system includes a machine which can produce $N(N>1)$ types of products and $N(N>1)$ detection device. There is a buffer zone between each detection device and the machine. Each testing device adopts total inspection way for the products with detection rate and demand rate consistent. There are some hypothesis of aspects for this kind of system:

- Raw material is sufficient
- Normal operation/product processing time is far less than the machine fault time
- To adjust the time when switching between different products of the machine is negligible
- The maximum productivity of various types of products is the same
- Rate and the repair rate are exponentially distributed machine fault
- Product demand rate is constant in the infinite time domain
- Defective occurrence obeys Bernoulli distribution, the defective rate is known
- Once leave the buffer zone, defective products would not consider cost
- Detection device does not have any fault, without considering the detection error
- The material flow can be regarded as a continuous process

Dynamic characteristic and mathematical model: The dynamic characteristics of the system are:

$$x_{i1}(t) = u_i(t)(1-\beta_i) - d_i \quad i = 1, 2, \cdots, N$$

$$x_{i2}(t) = \begin{cases} 0 & x_{i1}(t) < 0 \\ \beta_i x_{i1}(t)/(1-\beta_i) & x_{i1}(t) \geq 0 \end{cases}$$

Constraint conditions are:

$$\sum_{i=1}^{N} \tau_i u_i(t) \leq \alpha(t) \tag{1}$$

$$\sum_{i=1}^{N} d_i \tau_i /(1-\beta_i) < \frac{r}{r+p} \tag{2}$$

Equation 1 represents constraints of machine capacity and Eq. 2 represents the necessary conditions that demands can be met.

According to the dynamic characteristics of the system and the preceding assumptions, averaging over production/shortfall in output cost function for a system can be obtained by:

$$J = \frac{1}{T} E \int_0^T \left[\sum_{i=1}^{N} g_i^+ x_{i1}^+(t) + \sum_{i=1}^{N} g_i^- x_{i1}^-(t) + \sum_{i=1}^{N} h_i x_{i2}(t) \right] dt \tag{3}$$

Definitions of some related parameters and variables are as follows:

J : Average over fulfill/shortfall cost

$x_i(t)$: Inventory of the ith product at t time, $x_i(t) = x_{i1}(t) + x_{i2}(t)$

$x_{i1}(t)$: Inventory of qualified products of the ith product at t time, $x_{i1}^+(t) = \max(x_{i1}, 0)$ represents over fulfill number, $x_{i1}^-(t) = \max(-x_{i1}, 0)$ represents shortfall number

$x_{i2}(t)$: Inventory of the defective products of the ith product at t time

g_i^+ : Ith product of over production cost coefficient

h_i : Ith product of defective cost coefficient. The defective products will not only make the system and produce the storage costs but also will lead to additional costs such as waste machine production capacity, so the inferior cost coefficient is larger than over production cost coefficient (Hajji *et al.*, 2011), that is $h_i > g_i^+$

g_i^- : Ith product of shortfall in output cost coefficient. Usually a bad crop will have negative effects on the production credit of the enterprise, which will bring great losses to the enterprise, so the cost of production shortfall in output coefficient is greater than the cost coefficient and the inferior cost coefficient. that is $g_i^- > h_i > g_i^+$

$u_i(t)$: Productivity of the ith product in t time, $0 \leq u_i(t) \leq u_{i\max}$

τ_i : Reverse of the most productivity u_{max}, $\tau = 1/u_{max}$

$\alpha(t)$: Condition of machine in t time. When machine runs normally $\alpha(t)$ is 1, else 0

d : Demand rate of product, $d = (d_1, d_2, \ldots, d_N)$

p : Failure rate of the machine

r : Repair rate of machine

β_i : Product defective rate of the ith product

THEORETICAL ANALYSIS

For multiple types of products of an unreliable production system, Sethi and Zhang (1999) gave the optimal control strategy and obtained analytic solution of the hedging point of infinite horizon and average cost and production/shortfall in output without considering the quality of products. In theoretical analysis of product random defects, the optimal control strategy and hedging system are given in special conditions and obtained the analytical solution of hedging point and the average cost of production/shortfall in output.

Definition 1: If the product demand rate remained unchanged in the infinite horizon, normal operation/machine fault repair time obeys the exponential distribution and the product cost coefficient and the defective rate are the same, such as Eq. 4, the special conditions are said to meet the system.

$$\begin{cases} g_1^+ = g_2^+ = \cdots = g_N^+ = g^+ \\ g_1^- = g_2^- = \cdots = g_N^- = g^- \\ h_1 = h_2 = \cdots = h_N = h \\ \beta_1 = \beta_2 = \cdots = \beta_N = \beta \end{cases} \tag{4}$$

Theorem 1: The optimal control strategy of product random defects of single product type under unreliable production system in the special conditions is:

$$u_i(t) = \begin{cases} 0 & i \in Q'(\boldsymbol{x}) \text{ or } \alpha(t) = 0 \\ d_i' & i \in K(\boldsymbol{x}) \text{ and } \alpha(t) = 1 \\ \left(1 - \sum_{j \in K(\boldsymbol{x})} \tau_j d_j'\right) d_i' \bigg/ \left(\tau_i \sum_{l \in L(\boldsymbol{x})} d_l'\right) & i \in L(\boldsymbol{x}) \text{ and } \alpha(t) = 1 \end{cases} \tag{5}$$

where, $x = (x_1, x_2, \ldots, x_N)$, $Q'(x) = \{q: x_{q1} > z_q\}$, $K(x) = \{j: x_{j1} = z_j\}$, $L(x) = \{l: x_{l1} < z_l\}$ and $d' = d/(1-\beta)$.
Hedging point z_i is:

- When $D < u_{max}(1-\beta)\dfrac{r}{p+r}$, $F \leq 0$, $z_i = 0$

- When $D < u_{max}(1-\beta)\dfrac{r}{p+r}$, $F > 0$, $z_i = \dfrac{d_i\left[u_{max}(1-\beta)-D\right]}{ru_{max}(1-\beta)-(r+p)D} \ln \dfrac{pu_{max}(1-\beta)(Q_1+Q_2)}{Q_1\left[u_{max}(1-\beta)-D\right](r+p)}$

- When $D \geq u_{max}(1-\beta)\dfrac{r}{p+r}$, $z_i = \infty$

The average cost producing/shortfall in output is:

- When $z_i = 0$, $J = \dfrac{Q_2 pu_{max}(1-\beta)D}{(r+p)\left[ru_{max}(1-\beta)-(r+p)D\right]}$

- When $z_i > 0$, $J = \dfrac{Q_1 D}{r+p} + \dfrac{Q_1 D\left[u_{max}\left(1-\beta\right)-D\right]}{r u_{max}\left(1-\beta\right)-\left(r+p\right)D}\ln\dfrac{p\left(Q_1+Q_2\right)u_{max}\left(1-\beta\right)}{Q_1\left[u_{max}\left(1-\beta\right)-D\right]\left(r+p\right)}$

where, $D = \displaystyle\sum_{i=1}^{N} d_i$, $Q_1 = g^+ + h\dfrac{\beta}{1-\beta}$, $Q_2 = g^-$, $F = \dfrac{u_{max}\left(1-\beta\right)\left(Q_1+Q_2\right)p}{Q_1\left[u_{max}\left(1-\beta\right)-D\right]\left(r+p\right)}-1$

From Eq. 3 we can get:

$$
\begin{aligned}
J &= \frac{1}{T}E\int_0^T\left[\sum_{i=1}^{N}g_i^+ x_{i1}^+(t)+\sum_{i=1}^{N}g_i^- x_{i1}^-(t)+\sum_{i=1}^{N}h_i x_{i2}(t)\right]dt\\
&= \frac{1}{T}E\int_0^T\left[\sum_{i=1}^{N}g_i^+ x_{i1}^+(t)+\sum_{i=1}^{N}g_i^- x_{i1}^-(t)+\sum_{i=1}^{N}\frac{h_i\beta_i}{1-\beta_i}x_{i1}^+(t)\right]dt\\
&= \frac{1}{T}E\int_0^T\left[\sum_{i=1}^{N}\left(g_i^+ + \frac{h_i\beta_i}{1-\beta_i}\right)x_{i1}^+(t)+\sum_{i=1}^{N}g_i^- x_{i1}^-(t)\right]dt
\end{aligned}
$$

When system meets Eq. 4, based on study of Sethi and Zhang (1999), the above equation can be further represented by:

$$
\begin{aligned}
J &= \lim_{T\to\infty}\frac{1}{T}E\int_0^T\left[\sum_{i=1}^{N}\left(g^+ + \frac{h\beta}{1-\beta}\right)x_{i1}^+(t)+\sum_{i=1}^{N}g^- x_{i1}^-(t)\right]dt\\
&= \lim_{T\to\infty}\frac{1}{T}E\int_0^T\left[\sum_{i=1}^{N}Q_1 x_{i1}^+(t)+\sum_{i=1}^{N}Q_2 x_{i1}^-(t)\right]dt\\
&= \lim_{T\to\infty}\frac{1}{T}E\int_0^T\left[Q_1\sum_{i=1}^{N}x_{i1}^+(t)+Q_2\sum_{i=1}^{N}x_{i1}^-(t)\right]dt
\end{aligned}
\tag{6}
$$

Compare Eq. 6 with the average over production/shortfall in output cost function in Sethi and Zhang (1999), we can find that the two are completely same in form. Therefore, the hedging point strategy of the system is:

$$
u_i(t) = \begin{cases}
0 & i\in Q'(\boldsymbol{x}) \text{ or } \alpha(t)=0\\
d_i' & i\in K(\boldsymbol{x}) \text{ and } \alpha(t)=1\\
\left(1-\displaystyle\sum_{j\in K(\boldsymbol{x})}\tau_j d_j'\right)d_i'\Big/\left(\tau_i\displaystyle\sum_{l\in L(\boldsymbol{x})}d_l'\right) & i\in L(\boldsymbol{x}) \text{ and } \alpha(t)=1
\end{cases}
$$

where, $x = (x_1, x_2, \ldots, x_N)$, $Q'(x) = \{q:x_{q1}>z_q\}$, $K(x) = \{j:x_{j1}=z_j\}$, $L(x) = \{l:x_{l1}<z_l\}$, $d' = d/(1-\beta)$.

Before solving Eq. 6, we first discuss Eq. 7:

$$
J' = \lim_{T\to\infty}\frac{1}{T}E\int_0^T\left[Q_1\eta^+(t)+Q_2\eta^-(t)\right]dt
\tag{7}
$$

Where:

$$\eta = \sum_{i=1}^{N} x_{i1}$$

the system represented by the above equation can be expressed as when a certain type of product is out of stock, other types of products can be used to replace them, therefore:

$$Q_1 \eta^+(t) + Q_2 \eta^-(t) \le Q_1 \sum_{i=1}^{N} x_{i1}^+(t) + Q_2 \sum_{i=1}^{N} x_{i1}^-(t)$$

so, $J' \le J$. In fact, the system represented by Eq. 7 is like a single product type unreliable production system whose demand rate is:

$$D = \sum_{i=1}^{N} d_i$$

According to Bielecki and Kumar (1988), its optimal control strategy is:

$$u(t) = \begin{cases} 0 & \eta(t) > z^* \text{ or } \alpha(t) = 0 \\ D/(1-\beta) & \eta(t) = z^* \text{ and } \alpha(t) = 1 \\ u_{max} & \eta(t) < z^* \text{ and } \alpha(t) = 1 \end{cases}$$

Hedging point z^* is:

- When $D < u_{max}(1-\beta)\dfrac{r}{p+r}, F \le 0, z^* = 0$

- When $D < u_{max}(1-\beta)\dfrac{r}{p+r}, F > 0, z^* = \dfrac{D[u_{max}(1-\beta)-D]}{r u_{max}(1-\beta)-(r+p)D} \ln \dfrac{p u_{max}(1-\beta)(Q_1+Q_2)}{Q_1[u_{max}(1-\beta)-D](r+p)}$

- When $D \ge u_{max}(1-\beta)\dfrac{r}{p+r}, z^* = \infty$

The average cost of production/shortfall in output J' is:

- When $z^* = 0, J' = \dfrac{Q_2 p u_{max}(1-\beta)D}{(r+p)[r u_{max}(1-\beta)-(r+p)D]}$

- When $z^* > 0, J' = \dfrac{Q_1 D}{r+p} + \dfrac{Q_1 D[u_{max}(1-\beta)-D]}{r u_{max}(1-\beta)-(r+p)D} \ln \dfrac{p(Q_1+Q_2)u_{max}(1-\beta)}{Q_1[u_{max}(1-\beta)-D](r+p)}$

According to Sethi and Zhang (1999) theorem 4.1, hedging point:

$$z_i = \frac{d_i}{D} z^*$$

is the average cost of production/shortfall in output $J = J'$ and Eq. 5 is the optimal production control strategy.

Corollary 1: When $D < u_{max}(1-\beta)\frac{r}{p+r}$, F>0, z_i is monotone decreasing function about g^+ and h and is monotone increasing function about g^-.

Proof:

$$z_i = \frac{d_i \left[u_{max}(1-\beta) - D \right]}{r u_{max}(1-\beta) - (r+p)D} \ln \frac{p u_{max}(1-\beta)(Q_1 + Q_2)}{Q_1 \left[u_{max}(1-\beta) - D \right](r+p)}$$
$$= A \ln \left[F(Q_1) + 1 \right] = A \ln \left[F(Q_2) + 1 \right]$$

Where:

$$A = \frac{d_i \left[u_{max}(1-\beta) - D \right]}{r u_{max}(1-\beta) - (r+p)D}, \ F(Q_1) = F(Q_2) = F$$

Because:

$$D < u_{max}(1-\beta)\frac{r}{p+r}, \ F > o$$

therefore, A>0, F+1>1.

- $\forall Q_1' > Q''$

$$F(Q_1') - F(Q_1'') = \frac{p u_{max}(1-\beta)(Q_1' + Q_2)}{Q_1' \left[u_{max}(1-\beta) - D \right](r+p)} - 1 - \frac{p u_{max}(1-\beta)(Q_1'' + Q_2)}{Q_1'' \left[u_{max}(1-\beta) - D \right](r+p)} + 1$$
$$= \frac{p u_{max}(1-\beta)Q_2}{Q_1' \left[u_{max}(1-\beta) - D \right](r+p)} - \frac{p u_{max}(1-\beta)Q_2}{Q_1'' \left[u_{max}(1-\beta) - D \right](r+p)} < 0$$

We can find that F is monotone decreasing function about Q_1. Because:

$$Q_1 = g^+ + h\frac{\beta}{1-\beta}$$

it is obvious that Q_1 about g^+ and h are both monotone decreasing, therefore, $F(Q_1)$ about g^+ and h are both monotone decreasing, so z_i about g^+ and h are both monotone decreasing.

• $\forall Q'_2 > Q''_2$

$$F\left(Q'_2\right) - F\left(Q''_2\right) = \frac{pu_{max}\left(1-\beta\right)\left(Q_1 + Q'_2\right)}{Q_1\left[u_{max}\left(1-\beta\right)-D\right]\left(r+p\right)} - 1 - \frac{pu_{max}\left(1-\beta\right)\left(Q_1 + Q''_2\right)}{Q_1\left[u_{max}\left(1-\beta\right)-D\right]\left(r+p\right)} + 1$$

$$= \frac{pu_{max}\left(1-\beta\right)Q'_2}{Q_1\left[u_{max}\left(1-\beta\right)-D\right]\left(r+p\right)} - \frac{pu_{max}\left(1-\beta\right)Q''_2}{Q_1\left[u_{max}\left(1-\beta\right)-D\right]\left(r+p\right)} > 0$$

We can find that F is monotone decreasing function about Q_2. Because $Q_2 = g^-$, so F is monotone increasing function about g^-, so z_i is monotone increasing function about g^-.

PARTICLE SWARM ALGORITHM

Individual coding: In the particle swarm algorithm, the particle position and velocity are expressed in real number field, this field is easy to obtain hedging point. Therefore, particle can be directly encoded as $(z_1, z_2, \ldots, z_N)$ where, N is number of types of product.

Fitness function: Use computer language to simulate the whole production process simulation system and the repeated simulation results of mean value calculation results as each particle corresponds to the target value is:

$$J'' = \frac{1}{w}\sum_{\delta=1}^{w} J_\delta$$

where, J_δ is the calculation result of the δth production process, its function form like Eq. 8, w is simulation times, taken as 100. Fitness function is:

$$\Phi\left(z\right) = \frac{\lambda}{J''}$$

where, λ is a constant which is positive.

$$J_\delta = \frac{1}{T}\int_0^T \left[\sum_{i=1}^{N} g_i^+ x_{i1}^+\left(t\right) + \sum_{i=1}^{N} g_i^- x_{i1}^-\left(t\right) + \sum_{i=1}^{N} h_i x_{i2}\left(t\right)\right]dt \tag{8}$$

EXPERIMENTAL RESULTS

     Assume the unreliable production system with a single machine and two types of products, system parameters (Table 1). Particle swarm algorithm parameters are with particle swarm size 40, the inertia weight is 0.5, learning factor is taken as 2 and the iteration number as the 100 generation. In the memory for 4G Intel Core (TM) i5-2400 CPU 3.10 GHZ computer using Compaq Visual Fortran programming operation, the calculated results in the Origin Lab 8 drawn.

Algorithm results vs. theoretical value: From Theorem 1, we can see that when T→∞, the value of hedging point is not related with the initial state, while the initial in finite time state will

influence the hedging of the system. For comparison, the initial state assumptions of products are (0,0). The ratios of different types of products obtained by algorithm (Table 2) are always close to the theoretical value and with T increasing, hedging points obtained by algorithm are gradually close to theoretical value, so the algorithm used in this study is feasible. In addition, we can find that the method used in this study can get safe point of the system in finite time, it is convenient for it to be applied to the demand of the products only in a short period of time to maintain constant production system. What we need to point out is the initial state of the value refers to the inventory of qualified products in initial time system.

Influence of initial states: Assume that the demand rate is (1,1), the simulation time is 200. The initial state is different (Table 3), different safety values and the initial state value hedging point of the product is relatively small. For the shortfall in output of products, in order to meet the demand, hedging point of such products is relatively large for producing many products, in order to avoid over production causing too much production cost, safety of such products is relatively small.

General case: Another advantage of adopting the combined method of simulation with particle swarm algorithm to obtain hedging point is that it has less restrictive conditions which can be used in the general system.

Table 1: System parameters

u_{max}	p	r	g+	g-	h	β
5	0.01	0.1	(1 1)	(15 15)	(3 3)	(0.1 0.1)

Table 2: Algorithm results and theoretical value

	d = (1 1)		d = (1 1.5)		d = (2 1)	
T	z_1	z_2	z_1	z_2	z_1	z_2
80	1.19	1.17	2.51	3.91	6.57	3.33
100	1.76	1.81	3.10	4.60	7.66	3.78
500	6.11	6.09	7.67	11.53	24.94	12.23
1000	6.24	6.24	9.14	13.50	25.95	12.99
3000	7.58	7.47	10.03	15.32	30.50	15.05
∞	7.56	7.56	10.50	15.75	30.16	15.08

Table 3: Hedging points under different initial states

Initial state	z_1	z_2
(-20 -20)	4.95	4.92
(-20 -10)	4.58	2.69
(-20 -5)	4.62	1.16
(-10 -10)	4.76	4.73
(-10 0)	4.69	3.13
(-10 10)	4.30	4.26
(0 0)	4.51	4.47
(0 10)	4.58	4.50
(0 20)	4.21	4.35
(10 20)	4.42	4.27
(20 20)	4.40	4.40

Table 4: System parameters

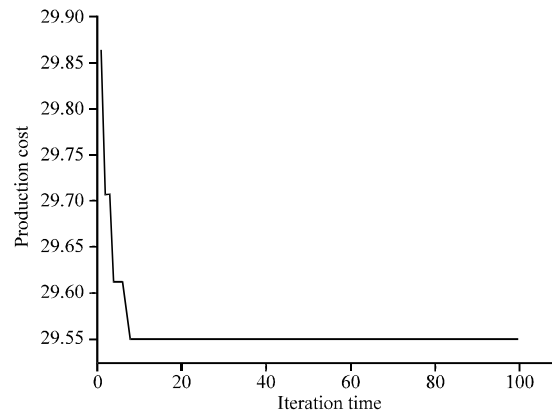| u_{max}	g⁺	g[−]	h	β
5	(2 1)	(10 15)	(3 4)	(0.1 0.15)

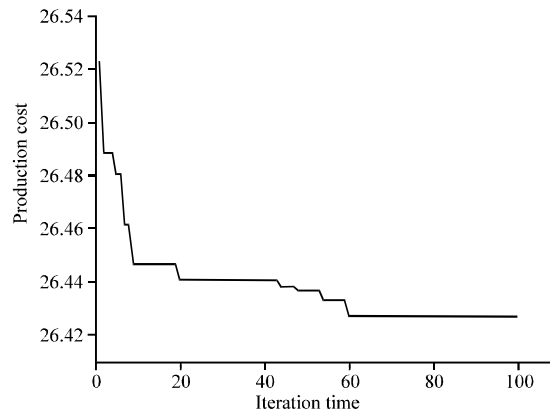Fig. 1: Simulation result of the lognormal distribution case

Fig. 2: Simulation result when parameters are different

Non exponential distribution: Actual operation/production machine fault time not only obeys the exponential distribution but also is likely to be subjected to the other distributions such as the logarithmic distribution, gamma distribution and Will Bull distribution etc. Suppose, the machine failure rate follows a mean, standard deviation for the logarithmic distribution 100, repair rate follows a mean value, standard deviation was 10 log distribution. Other parameters are shown in Table 1, the simulation taken time is 200, product demand rate (1,1) for the initial state (0,0). The program runs 187 and the algorithm runs 8 generation reach convergence (Fig. 1).

All products with different parameters: In order to facilitate the theoretical analysis, assume that parameters of the products are the same. In the actual production, parameters of each product are usually different (Table 4). Assume that the demand rate is (1,1) for the initial state (0,0), failure rate and repair rate obeys the lognormal distribution with the same, the simulation time is 200. The program runs in 190 sec and the algorithm runs the 60 generation of convergence (Fig. 2).

For the two cases above, after the program runs in the finite time, the objective function can be stable in a certain scope, therefore, method for solving the hedging point of the system generally is feasible.

CONCLUSIONS

This study took into account the existence of defects in the actual production and studied the production control problem of single product type of unreliable production system. First, it analyzed the dynamic characteristics of the system and established the mathematical model of the problem. Second, it gave the optimal production control strategy of the system to meet some special condition in infinite horizon and got the hedging point and average analytical solution of shortfall in output cost over production and finally, it put forward the method of computer simulation combined with particle swarm algorithm hedging point approximation algorithms, the results were compared with the theoretical values to verify the feasibility of the method. It discussed the effect of initial condition on hedging point, the method was applied to the system in a general case and the example showed that the method is also effective. Research on production control problem of unreliable production system product random defects multi-machine multi-product type will be the next study.

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