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Implicit Algorithm of Hybrid Hardening Elastic-Plastic Constitutive Relation and its Application in Autofrettage Residual Stress Analysis

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ABSTRACT
Based on the ABAQUS/Standard platform and the characteristics of loading and unloading for autofrettage, the User-defined Material Mechanical Behavior (UMAT) subroutine of hybrid hardening constitutive relationship was developed. In this implicit algorithm, parameter of back stress, loading parameters and isotropic hardening coefficient were introduced, algorithm for stress compensation updating and consistent stiffness matrix updating were used. After the reliability verification, this UMAT was used for autofrettage residual stress analysis of notch ring specimen's model. Experimental and Comparison results showed that analysis results of residual stress field based on UMAT were accurate. The effect of isotropic hardening coefficient on maximum residual equivalent stress, maximum compressive residual stress and depth of residual compressive stress zone were examined. The results showed that under the same autofrettage process condition, the residual compressive stress would increases with isotropic hardening coefficient but the depth of residual compressive stress zone decreases with the increase of isotropic hardening coefficient. The findings have significant influence on the improvement of autofrettage technology.

Key words: Residual stress, constitutive relation, hybrid hardening, autofrettage, FEM

INTRODUCTION
Ultrahigh pressure thick wall vessel has been widely used in petroleum, chemical, machinery, food processing and other industries. Under the effect of ultrahigh pressure, the incoherent position of inner structure of the thick wall vessel forms stress concentration easily. The maximum value of stress is even higher than the material's yield strength. Therefore, under the constant-amplitude loading, the thick wall vessel would cease to be effective because of cracking and fatigue. As an effective means of improving the stress concentration of inner structure and enhancing the fatigue life of the vessels, the autofrettage has been widely concerned by researchers who focus on the thick walled vessel under ultrahigh pressure (Brunnet *et al*., 2014). In order to precisely calculate the distribution of residual stress field about autofrettage with the finite element method, an accurate model of the material's constitutive relation needs to be established.

In the process of plastic loading and unloading, metal materials exhibits complicated strain hardening behaviors and Bauschinger effect. In order to simplify the calculation, the Bilinear Kinematic Hardening model is often adopted to analyze the residual stress about autofrettage (Salami *et al*., 2014; Qian *et al*., 2011). Although, this model considers the plastic strain hardening behavior and the Bauschinger effect, the actual hardening behavior of metal materials is more complicated than the Linear hardening (Badr *et al.*, 2000). To precisely describe the hardening properties of metal materials, some scholars (Xu and Liu, 1995) put forward the rule of hybrid hardening which describes the hardening behavior of materials as the combination of Isotropic hardening and Kinematic hardening. Hybrid hardening rule and actual hardening behavior is closer to the metal materials and suitable for use in a simulation with reverse loading and cyclic loading conditions.

Due to the complexity of hybrid hardening elastic-plastic constitutive relation, the specific types of analysis and load characteristics need to be considered to choose suitable stress updating algorithm and write material constitutive relation interface programs that can be used in the finite element analysis. ABAQUS is the large international general finite element software, providing users with two user material subroutine UMAT and VUMAT interfaces and it is convenient for users to define the material model which is not available in the material database (Hibbitt, Karlsson and Sorensen Inc., 2002). In consideration of the slow ladder loading and unloading of hydraulic autofrettage in ultrahigh pressure thick wall vessels, this study introduced the back stress, loading parameters and isotropic hardening coefficient to establish a stress compensation updating algorithm and a consistent stiffness matrix updating algorithm based on the platform of ABAQUS/Standard and then developed an hybrid hardening elastic-plastic constitutive relation with UMAT. After reliable verification, it has been applied to analyze the autofrettage residual stress of the notch ring model.

MATERIALS AND METHODS

Hybrid hardening elastic-plastic constitutive relation: As the metal material hybrid hardens, the size, location and shape of yield surface change with the development of plastic. Hybrid hardening could be simplified as the weighted sum of isotropic hardening and kinematic hardening (Xu and Liu, 1995), so isotropic hardening coefficient c could be introduced as weight in the classic elastic-plastic constitutive relation and then the yield surface of the Mises yield criterion f (σ, σ_F, b) = 0 is changed to the following equation:

$$\begin{cases} \frac{1}{2}(\sigma' - b') : (\sigma' - b') - \frac{1}{3}\sigma_F^2 = 0 \\ \sigma_F(\bar{\varepsilon}^P) = (1-c)\sigma_{S0} + cY(\bar{\varepsilon}^P) \end{cases} \tag{1}$$

where, σ' is the deviator of stress tensor σ, b' is the deviator of back stress tensor b, σ_F is the Von Mises principal stress which refers to the size of yield surface, $\bar{\varepsilon}^P$ is the equivalent plastic strain, σ_{s0} is the initial yield stress, Y ($\bar{\varepsilon}^P$) is the stress and plastic strain curve that is obtained from the test. When weight is equal to one, the yield criterion degenerates to isotropic hardening yield criterion. When weight is equal to zero, the yield criterion is kinematic hardening yield criterion.

According the classical elastic-plastic theory of the normal rule, Prager's assumption and Ziegler's assumption etc. (Brovko, 2013; Yang *et al.*, 2012; Qi *et al.*, 2013) the update equation of the deviator of back stress tensor is obtained as follows:

$$b'_{t+\Delta t} = b'_t + \frac{2}{3}(1-c)\alpha E_P \frac{1}{2\mu(1+E_P/3\mu)} \times \left(\left((\sigma'_{t+\Delta t} - b'_t) : (\sigma'_{t+\Delta t} - b'_t) \right)^{1/2} - \sqrt{\frac{2}{3}}\sigma_F \right) \times \left(\sqrt{\frac{3}{2}} \frac{\sigma' - b'}{\sigma_F} \right) \tag{2}$$

where, α is loading parameter. It is used to distinguish the hardening material state of loading from unloading. When $\alpha = 1$, the hardening material is in loading situation; when $\alpha = 0$, it is in unloading situation. The E_p is the function of plastic accumulative deformation $\bar{\varepsilon}^P$ which could be obtained by curve in the uniaxial tension:

$$E_p = \frac{dY(\bar{\varepsilon}^p)}{d\bar{\varepsilon}^p}$$

$$\mu = \frac{E}{2(1+\nu)}$$

where, E is Young modulus and v is Poisson ratio.

The Eq. 1 and 2 constitute the hybrid hardening elastic-plastic constitutive relation.

Prediction of initial elasticity: The stress σ_n, the strain ε_n and strain increment $\Delta\varepsilon_{n+1}$ in the step N are already known. Firstly, it is assumed that all the strain increments $\Delta\varepsilon_{n+1}$ are elastic strains. So, the elastic stress prediction according to linear-elastic theory can be calculated:

$$\sigma_{n+1}^{trail} = \sigma_n + D : \Delta\varepsilon_{n+1} \tag{3}$$

where, D is elastic stiffness matrix.

Elastic-plastic state determination: The identification of elastic-plastic state has a great influence on the accuracy of the calculation results. Before starting increment or iteration steps, stress σ, back stress b, equivalent plastic strain $\bar{\varepsilon}^P$ and strain increment $\Delta\varepsilon$ are already known and then predict the elastic-plastic state by using the elastic stress σ_{n+1}^{trail}. The procedures that judge the elastic-plastic state are as follows:

- Calculating equivalent principal stress $[3J_2(\sigma\text{-}b)]^{1/2}$
- Utilizing plastic accumulative deformation and plastic stress-strain curve to calculate Y ($\bar{\varepsilon}^P$)
- Judging whether yield happens, according to the Eq. 1 that shows the condition of yield

Plastic correction and updating stress: When the material yields, the stress compensation updating algorithm should be applied based on elastic tensor. The strain increment of incremental step is including elastic part and plastic part:

$$\Delta\varepsilon = \Delta\varepsilon^e + \Delta\varepsilon^p$$

On the basis of trial stress, subtract the redundant stress increment corresponded to plastic strain increment and update stress. The expression is:

$$\sigma_{t+\Delta t} = \sigma_t - 2\mu\Delta\varepsilon^p \tag{4}$$

At the same time, the back stress is taken as the initial value of the next increment step or iterative sub-step.

Updating algorithm for the stiffness matrix of hybrid hardening elastic-plastic constitutive relation model: The abrupt transformation of metal materials from elasticity into plasticity due to yield may lead to pseudo-loading and pseudo-unloading. So, implicit algorithm adopts a system linearization algorithm modulus that is the consistent tangent stiffness matrix based on constitutive integration (Simo and Taylor, 1985).

In the classic elastic-plastic theory, the expression of stress is:

$$\begin{aligned}\dot{\sigma}_{ij} &= \dot{\eta}_{ij}\sigma_s + \eta_{ij}\dot{\sigma}_s + \frac{1}{3}\delta_{ij}\dot{\sigma}^{pr}_{kk} \\ &= (K - \frac{2\mu\sigma_s}{3\bar{\sigma}^{pr}})\delta_{ij}\dot{\varepsilon}_{mm} + \frac{2\mu\sigma_s}{\bar{\sigma}^{pr}}\dot{\varepsilon}_{ij} + (\frac{h}{h/3\mu+1} - \frac{3\mu\sigma_s}{\bar{\sigma}^{pr}})\eta_{ij}\eta_{kl}\dot{\varepsilon}_{kl}\end{aligned} \quad (5)$$

Where:

$$K = \frac{E}{3(1\text{-}2\nu)},\ h = \frac{d\sigma_s}{d\bar{\varepsilon}^{pl}}$$

if:

$$G^* = \frac{\mu\sigma_s}{\bar{\sigma}^{pr}},\ \lambda^* = K - \frac{2}{3}G^*$$

then the Eq. 5 could be simplified as follows:

$$\dot{\sigma}_{ij} = \lambda^*\delta_{ij}\dot{\varepsilon}_{mm} + 2G^*\dot{\varepsilon}_{ij} + (\frac{h}{h/3\mu+1} - 3G^*)\eta_{ij}\eta_{kl}\dot{\varepsilon}_{kl} \quad (6)$$

Through the above equation of derivation, the consistent tangent stiffness matrix can be obtained. The procedures that update consistent tangent stiffness matrix are as follows:

- Calculating the equivalent stress $\bar{\sigma}^{pr}$ and the direction of plastic flow (h) according to the elastic stress prediction σ^{trail}_{n+1} and back stress b
- Utilizing plastic accumulative deformation and plastic stress-strain curve to calculate yield stress σ_s
- Updating consistent tangent stiffness matrix according to the Eq. 6

Calculation steps and the flow chart for hybrid hardening elastic-plastic constitutive relation implicit algorithms in ABAQUS: The UMAT is a FORTRAN program interface which is provided for the users who define their material properties in ABAQUS (Hibbitt, Karlsson and Sorensen Inc., 2002). According to their research needs, users could program the material model which is not available in the material database.

In ABAQUS/Standard, UMAT offers the strain increment of every incremental step, the stress state of material at the beginning of incremental step and the state variables that are used to save back stress and equivalent plastic strain.

In ABAQUS/Standard, the increment of every incremental step, the stress state of material at the beginning of incremental step and the state variables that are used to save back stress and equivalent plastic strain are offered to UMAT as input. Steps to achieve the implicit algorithm of hybrid hardening constitutive relations are as follows:

- Getting the stress σ_n, back stress b_n and equivalent plastic strain $\bar{\varepsilon}^P$ at the beginning of incremental step N and current strain increment $\Delta\varepsilon_{n+1}$ of incremental step N from the UMAT interface
- Calculating the elastic stress for prediction $\sigma_{n+1}^{trail} = \sigma_n + D : \Delta\varepsilon_{n+1}$ according to the generalized Hooke law
- Calling the subroutine of plastic stress-strain and calculating Y ($\bar{\varepsilon}^P$)
- Calculating the equivalent stress $\bar{\sigma}^{pr}$ and the direction of plastic flow h
- Substituting the elastic stress for prediction into yield criterion with hybrid hardening and judging whether yield happens
- If it dose not yield, $\sigma_{n+1} = \sigma_n$, stiffness matrix is consistent with elastic stiffness matrix
- If it yields, calculate the current increment plastic strain of incremental step $\Delta\varepsilon = \Delta\gamma n$, update stress, back stress and equivalent plastic strain. Calculate consistent tangent stiffness matrix and update according to the Eq. 6
- Ending and going back to the main program

According to the above calculation procedures, this study developed the UMAT of elastic-plastic constitutive relation with hybrid hardening. The flow chart is depicted in Fig. 1.

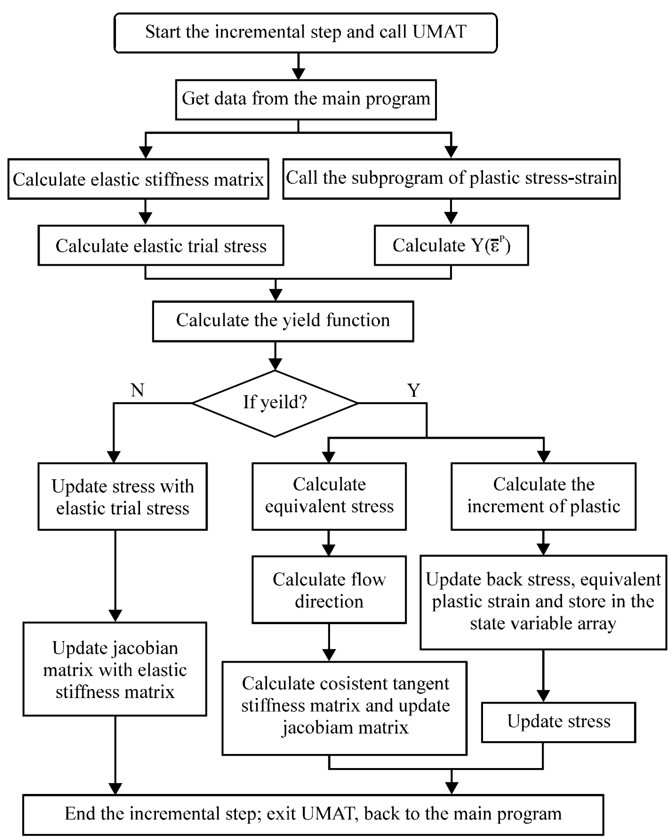

Fig. 1: Flow chart of the hybrid hardening elastic-plastic constitutive relation UMAT

RESULTS AND DISCUSSION

To examine the calculation precision of the implicit algorithm developed for hybrid hardening elastic-plastic constitutive relationship, the relevant UMAT was applied to a specific model for calculating the autofrettage residual stress. On the same model and conditions, the UMAT of this study was compared with other published constitutive models and algorithms in simulation results. Even more, an experiment was designed to validate the accuracy of all simulation results. In addition, the influence of isotropic hardening coefficient on residual stress was discussed by applying the UMAT.

Three-dimensional finite element model: In order to simulate and analyze the condition of local stress concentration of lumen in an ultrahigh pressure vessel with complex cavity, the notch ring specimens was designed. The material is $25Cr_2Ni_4MoV$. The wall cylinder was thick, with external diameter of the ring 85 mm, inner diameter 50 mm, diameter ratio 1.7. With the thickness of the ring 25 mm, the notch was a semicircle. Figure 2 shows the notch ring.

Processing of material performance parameters: Through uniaxial tensile test, the Young modulus and Poisson ratio of notch ring specimens material was obtained:

$$E = 210 \text{ GPa}, \ v = 0.3, \ \sigma_s = 956 \text{ MPa}$$

The curve data of true stress and plastic strain were shown in Fig. 3. Short cylindrical test bar was firstly stretched to a certain amount of plastic deformation and then compressed into submission on the tension and compression experimental machine. The isotropic hardening coefficient c value 0.6 was obtained from the experiment.

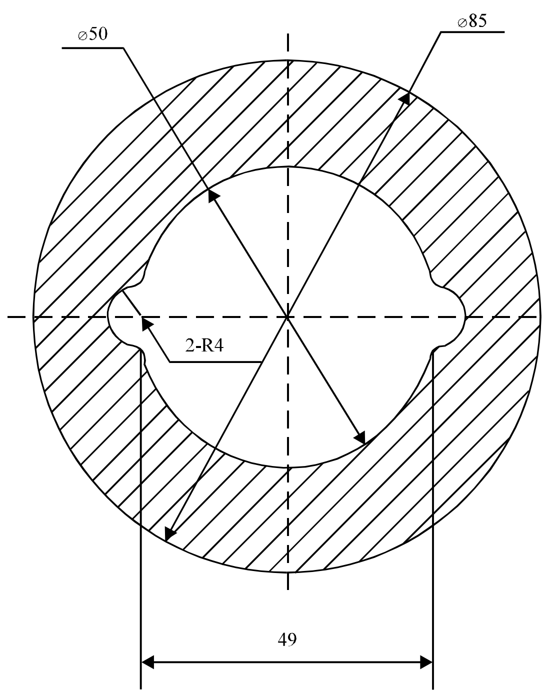

Fig. 2: Model of notch ring

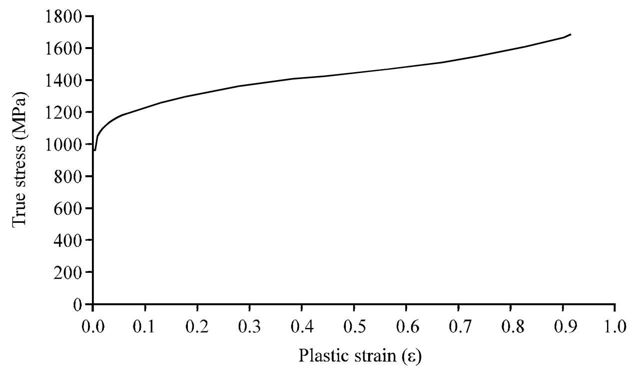

Fig. 3: True stress-plastic strain curve of material

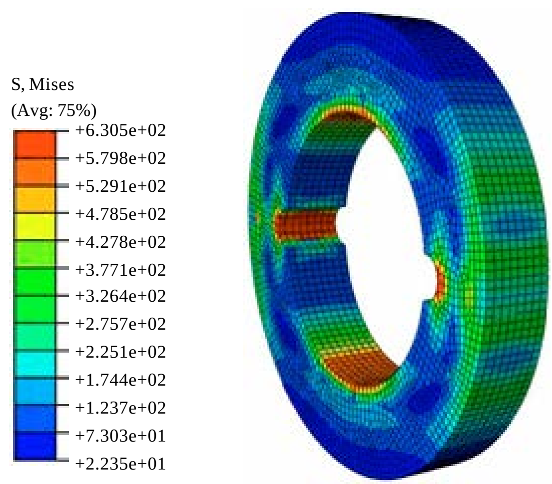

Fig. 4: Distribution of equivalent stress after unloading from P_z

Analysis of autofrettage residual stress based on hybrid hardening model: To produce residual stress in notch ring specimens, the pressure was slowly put on internal torus of the block to swell the ring. When stress concentration in the semicircle notch experienced a certain state of plastic deformation, pressure on internal torus of the block reduced to zero. At this time, due to elasticity shrinkage outside the ring surface, residual compressive stress field was produced in the semicircular notch.

In order to analyze the elastic-plastic finite element of loading and unloading process, two load steps were applied by ABAQUS/Standard: P_z = 300 MPa pressure was put on the internal torus of the notch ring specimens in the first analysis step and P_z = 0 MPa pressure was put on the internal torus of the notch ring specimens in the second analysis step. Element C3D8R was chosen to divide grid for the notch ring specimens and "enhance" option was selected in the property “HOURGLASS STIFFNESS”. After the completion of the elastic-plastic finite element analysis, the maximum equivalent stress point appeared at the ring notch. The maximum principal stress of the point was tensile stress and the minimum principal stress was compressive stress. As shown in Fig. 4, the stress concentrated at the notch significantly after unloading, with the maximum equivalent stress at 630.5 MPa. Figure 5 shows the residual compressive stress of the notch remained after unloading and the maximum compressive stress was -678 MPa.

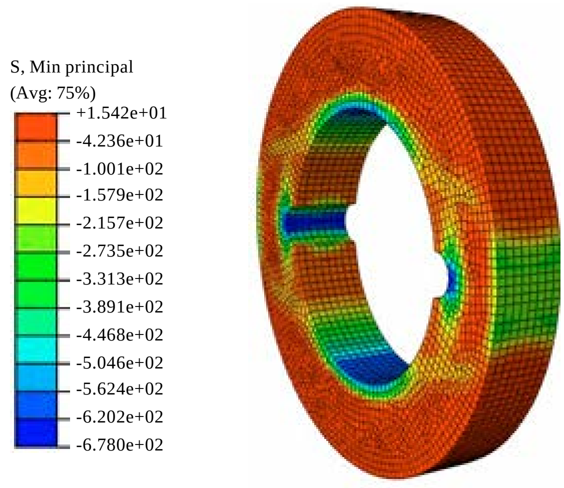

Fig. 5: Distribution of residual stress after unloading from P_z

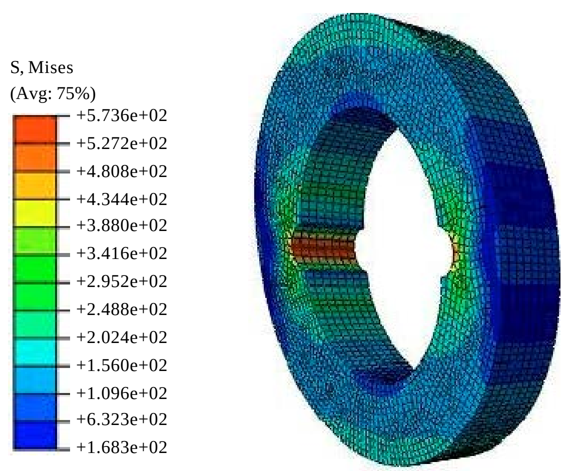

Fig. 6: Distribution of equivalent stress in the ring without autofrettage treatment

Under the influence of working load P_0 with value 90 MPa, the ring without autofrettage treatment experienced obvious stress concentration at the notch (Fig. 6) and the maximum equivalent stress occurred at the center of the notch, with its value 573.6 MPa. The notch ring no longer experienced stress concentration at the notch after given autofrettage treatment (Fig. 7) under the working load P_0 = 90 MPa. The maximum equivalent stress occurred inside the ring which was far from notch center and the equivalent stress value of the notch center was about 160 MPa. The comparison showed that the autofrettage treatment decreased the equivalent stress of the notch sharply and distributed the stress in the notch more evenly.

Experimental verification: To test the accuracy of hybrid hardening constitutive model UMAT in the elastic-plastic finite element analysis, several groups of notch ring specimens, as well as fixtures applying internal pressure, were studied. The cone angle is 45° as shown in Fig. 8, apply

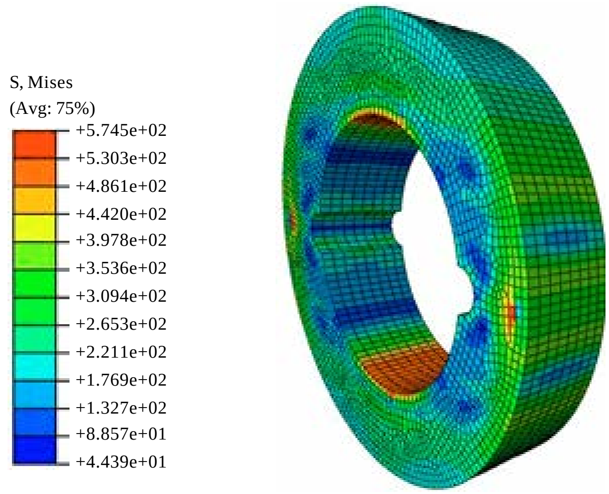

Fig. 7: Distribution of equivalent stress in the ring with autofrettage treatment

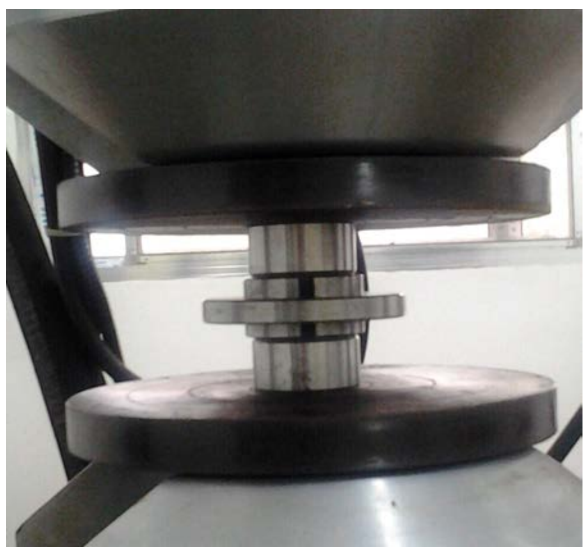

Fig. 8: Notched ring pressurized equipment and auxiliary device

autofrettage pressure load needed inside the ring by increasing pressure load in the cone. The pressure loaded on the cone was obtained by converting the ring's internal surface pressure.

The ring was tested in the stepped compression experiment on 500 t of hydraulic press. The applied pressure rose slowly from zero to P_1. Just as the stress-concentrated notch got to the yield state, give stepped pressure in accordance with an increase of 0.05 P_1 after every 3 min until the pressure loaded into P_2, thus the plastic strain reaching a certain degree. Finally, gradually unload to zero, generating residual compressive stress field at the ring notch. This experiment was repeated 3 times.

In this experiment, circumferential stress of the two points was measured through distributing two points in the center of two ring notches, as shown in the position of point A in Fig. 9. After unloading pressure completely, residual compressive stress existed in two notches. The average residual compressive stress measured by X-ray method on the surface of the u-shaped notch center was 653.4 MPa.

We discuss the implicit algorithm of hybrid hardening elastic-plastic constitutive relationship in comparison to previously published material constitutive model for residual stress calculation. Based on the notch ring model and applying the same load condition, the commonly used material models such as the Isotropic Hardening Plasticity model (Xu and Liu, 1995; Yang *et al.*, 2012), the Bilinear Kinematics Hardening Plasticity model (Qian *et al.*, 2011; Badr *et al.*, 2000), VUMAT for explicit algorithm of Hybrid Hardening Plasticity model (Xu and Liu, 1995; Salami *et al.*, 2014) were used to calculate the residual stress. All these results are compared with the results of simulations and experiments mentioned above.

As shown in Table 1, the VUMAT for display algorithm of Hybrid Hardening Plasticity is not suitable for hydraulic autofrettage process simulation because this VUMAT is generally applicable to solving the dynamic problems related to time, such as collision, impact and explosive However, hydraulic pressure autofrettage is a slow loading process which belongs to static problem.

Contrasted with experimental results of residual compressive stress, results of simulation calculated by Isotropic Hardening Plasticity model was 12.9% higher; results of simulation calculated by Bilinear Kinematics Hardening Plasticity model was 8.4% lower; simulation results that used UMAT put forward in this study was 3.76% higher. Obviously, the UMAT method produces the most accurate simulation results, a deviation of 3.76% thus acceptable in engineering

Table 1: Comparison of different material models used in residual stress calculation for notch ring

Comparison	Result of simulation (MPa)	Result of experiment (MPa)	Difference (%)
Isotropic hardening plasticity	-737.6	-653.4	+12.90
Bilinear kinematics hardening plasticity	-598.5	-653.4	-8.40
VUMAT for explicit algorithm of hybrid hardening plasticity	-	-653.4	-
UMAT (Put forward in this study)	-678.0	-653.4	+3.76

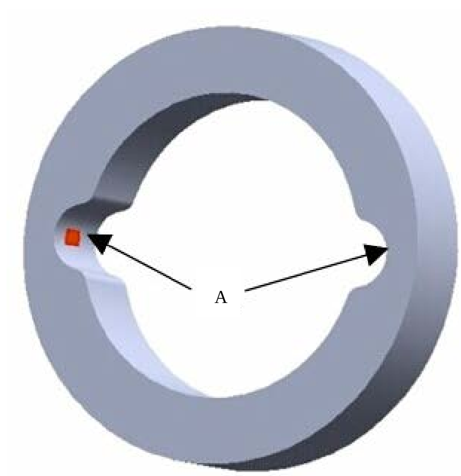

Fig. 9: Circumferential stress measuring points of the notch

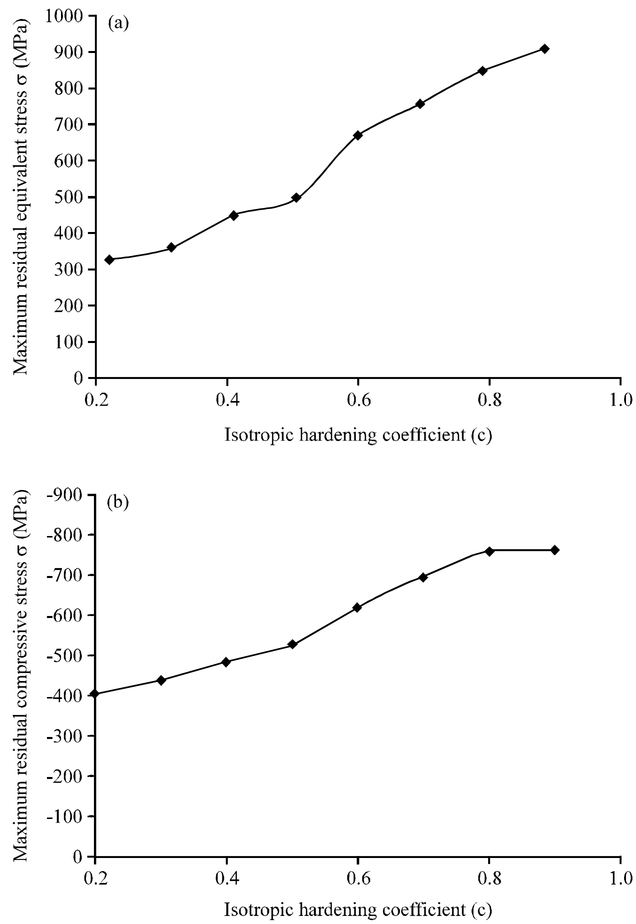

Fig. 10(a-b): Influence of isotropic hardening coefficient on the maximum residual (a) Equivalent and (b) Compressive stress

applications. The comparison shows that the implicit algorithm of hybrid hardening elastic-plastic constitutive relation has high calculation accuracy in residual stress simulation.

Influence of isotropic hardening coefficient on residual stress: By taking different values of isotropic hardening coefficient into the model of hybrid hardening elastic-plastic constitutive relation, the simulation of autofrettage residual compressive stress field of the notched ring investigated the influence of material isotropic hardening coefficient c on maximum residual equivalent stress, the maximum residual compressive stress and the depth of residual compressive stress zone depth of the ring notch. According to the characteristics of Ultrahigh pressure vessel materials, this part discusses the variation of these three state variables as the isotropic hardening coefficient is 0.2, 0.3, 0.4, 0.5, 0.6, 0.8 and 0.9, respectively.

Figure 10 and 11 show that the maximum residual equivalent stress and maximum residual compressive stress of the ring notch increase as the isotropic hardening coefficient c increases; the residual compressive stress zone depth of the ring notch decreases with the increase of the isotropic hardening coefficient c.

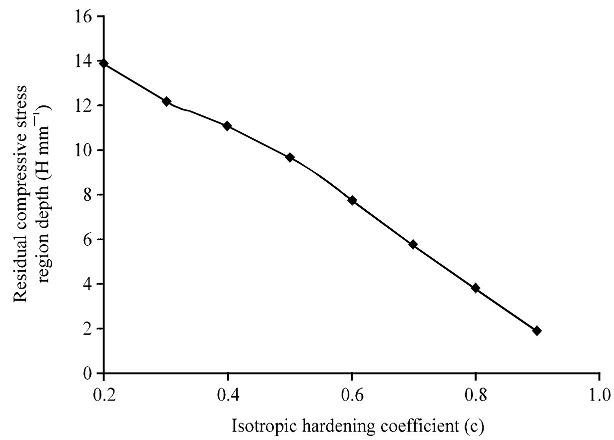

Fig. 11: Influence of isotropic hardening coefficient on the residual compressive stress region depth

The isotropic hardening coefficient characterizes the isotropic strength when the materials harden. According to the analysis of the notch ring model, the influence law of materials isotropic hardening coefficient c is obtained as follows:

- When the isotropic hardening coefficient c increases, the materials tend to be isotropic, the hardening degree of reverse loading is lowered at the notch, the reverse yield limit and the peak value of residual compressive stress rise under the same condition
- When the isotropic hardening coefficient c increases, the plastic deformation area reduces. After reverse loading, the distribution area of residual stress field becomes smaller and the stress gradient of residual compressive stress is greater

With regard to the Ultrahigh pressure thick wall vessel that has stress concentration in the inner cavity. The greater isotropic hardening coefficient is, the higher the autofrettage residual compressive stress is under the same condition of autofrettage process and the longer fatigue life is under the cycle working load action. If the depth of residual stress field is increased, the isotropic hardening coefficient should be reduced moderately.

CONCLUSION

- The hybrid hardening rule is close to the real hardening behavior of metal materials; therefore, it is suitable for simulating the autofrettage residual stress analysis with reverse loading or cyclic loading. In order to obtain the distribution of autofrettage residual stress field accurately, it is necessary to develop the elastic-plastic constitutive relation with hybrid hardening subroutine UMAT
- The stress compensation updating algorithm and the consistent stiffness matrix updating algorithm were applied to the development of the elastic-plastic constitutive relation with hybrid hardening subroutine UMAT. The reliability verification, comparison results and experimental results show that the UMAT on numerical simulation analysis of the hydraulic autofrettage residual stress field yield more accurate results

- In order to simulate and analyze the condition of local stress concentration of lumen in an ultrahigh pressure vessel with complex cavity, the notch ring specimens were designed. Through the analysis of autofrettage residual stress on the notch ring specimens, the paper focuses on the effect of isotropic hardening coefficient on the maximum residual equivalent stress, the maximum residual compressive stress and the depth of residual compressive stress zone at the ring notch. The results provide guidance in the selection of the yield ratio of ultrahigh pressure thick wall vessel materials and heat treatment process which has theoretical significance to the autofrettage treatment technology

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