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## **Aeroelastic Dynamic Response and Control of an Airfoil Section with Hysteresis Nonlinearity**

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### **ABSTRACT**

A state feedback suboptimal control law based on the state-dependent Riccati equation is derived for aeroelastic response and flutter suppression of a two-dimensional airfoil section with hysteresis nonlinearity in pitch. An observer was constructed to estimate the unavailable state variables of the system. With the control law designed, nonlinear effect of time delay between the control input and actuator are investigated by numerical approach. The closed-loop system including the observer and nonlinear controller is asymptotically stable. The simulation results show that the observer can give precise estimations for the plunge displacement and the velocities in pitch and plunge and that the controller is effective for flutter suppression. The time delay between the control input and actuator may jeopardize control performance and cause high-frequency vibration.

**Key words:** Aeroelasticity, flutter, airfoil, nonlinear control system, state estimation, time delay

### **INTRODUCTION**

Under the condition of a certain flight, aeroelastic systems exhibit a variety of phenomena including instability, limit cycle and even chaotic vibration (Fung, 1995; Dowell *et al.*, 1995; Zhao and Yang, 1990). Flutter instability can jeopardize aircraft structure and its performance. A number of investigators have considered control problems for such systems and designed controllers for flutter suppression. Kurdila and Akella (2001) gave a good summary of nonlinear control methods for high-energy limit-cycle oscillations. Mukhopadhyay (2003) presented a historical perspective on analysis and flutter control of aeroelastic systems. In recent years, a large number of control strategies have been developed for the flutter suppression (Behal *et al.*, 2006; Lee and Singh, 2007; Tadi, 2003; Bhoir and Singh, 2005) such as nonlinear adaptive control (Behal *et al.*, 2006) and global robust control using output feedback (Lee and Singh, 2007). According to Behal *et al.* (2006) and Lin and Chin (2006), an output feedback and an adaptive decoupled fuzzy sliding-mode control laws have been implemented for suppressing flutter and reducing the vibrational level in subcritical flight speed range. The State-Dependent Riccati Equation (SDRE) approach provides an extremely effective algorithm for synthesizing nonlinear feedback controls by allowing nonlinearities in the system states while additionally offering great design flexibility through design matrices (Cimen, 2010). Elhami and Narab (2012) made a comparison of SDRE and Sliding Mode Control (SMC) approaches for flutter suppression in a nonlinear wing section. The SDRE control technique was developed to design suboptimal control laws for nonlinear aeroelastic systems (Tadi, 2003; Bhoir and Singh, 2005).

The active feedback control involves many technical problems, one of which is the unavoidable presence of a time delay between the controller and actuators (Hu *et al.*, 1998). Time delay feedback control has received much attention in recent years (Qian and Tang, 2008). The flutter instability of actively controlled airfoils involving a time-delayed feedback control related to the aeroelasticity of 2-D lifting surfaces is considered via Pontryagin's approach in conjunction with Stépán's theorems (Librescu *et al.*, 2005). As indicated by Marzocca *et al.* (2005), the actuators may input energy at the moment when the controlled system does not need it. The time delay is very detrimental, because redundant energy may be inputted into the controlled system which can lead to a reduction of the control performance and even cause instability of the dynamical system. Zhao (2009, 2011) presented a systematic study on aeroelastic stability of a two dimensional airfoil with a single or multiple time delays in the feedback control loops and investigates the effects of time delay on the flutter instability of an actively controlled airfoil. Huang *et al.* (2012) revealed the effect of input time delay on the stability of a controlled high-dimensional aeroelastic system and present a new optimal control law to suppress the flutter.

In previous research, the flutter control was studied without considering nonlinearity such as hysteresis in the airfoil (Behal *et al.*, 2006; Lee and Singh, 2007; Tadi, 2003; Bhoir and Singh, 2005; Lin and Chin, 2006; Cimen, 2010; Elhami and Narab, 2012). In this study, hysteresis nonlinearity in pitch has been considered in the design of a control law for the flutter control of nonlinear aeroelastic systems by state variable feedback. The model represents a prototypical aeroelastic wing section which has been traditionally used for the theoretical and experimental study of two-dimensional aeroelastic behavior. The purpose of the study is to investigate the effect of hysteresis nonlinearity on the dynamic response and flutter suppression with the control law designed. In addition, the effect of time delay between the actuator control input and the control surface action is also investigated.

## MATERIALS AND METHODS

**Aeroelastic model and control problem:** The prototypical aeroelastic wing section is shown in Fig. 1. The governing equations of motion are provided by Ko *et al.* (1997, 1998):

$$\begin{bmatrix} m & mx_a b \\ mx_a b & I_a \end{bmatrix} \begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_a \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_a(\alpha) \end{bmatrix} \begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \quad (1)$$

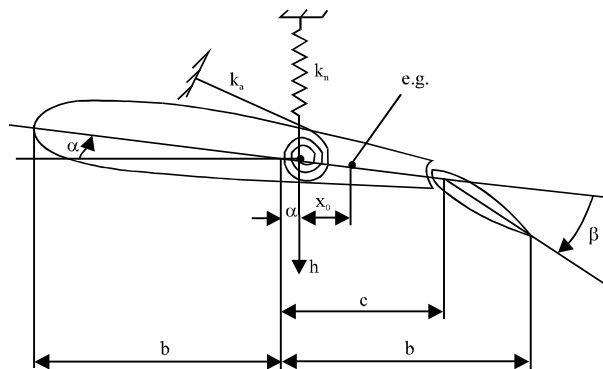


Fig. 1: Schematic of airfoil section with a control surface

where,  $h$  is the plunge displacement and  $\alpha$  is the pitch angle. The parameter  $m$  is the mass of the wing,  $I_a$  is the moment of inertia;  $b$  is the semichord of the wing;  $x_a$  is the nondimensionalized distance of the center of mass from the elastic axis;  $c_a$  and  $c_h$  are the pitch and plunge damping coefficients, respectively. The parameters  $M$  and  $L$  are the aerodynamic lift and moment. Assuming a quasi-steady aerodynamic model, the aerodynamic lift and moment are given by:

$$L = \rho U^2 b c_{l\alpha} [\alpha + (\dot{h}/U) + (\frac{1}{2} - \alpha) b (\dot{\alpha}/U)] + \rho U^2 b c_{l\beta} \beta \tag{2}$$

$$M = \rho U^2 b^2 c_{m\alpha} [\alpha + (\dot{h}/U) + (\frac{1}{2} - \alpha) b (\dot{\alpha}/U)] + \rho U^2 b^2 c_{m\beta} \beta \tag{3}$$

where,  $\alpha$  is the nondimensionalized distance from the midchord to the elastic axis,  $c_{l\alpha}$  and  $c_{m\alpha}$  are the lift and moment coefficients per angle of attack and  $c_{l\beta}$  and  $c_{m\beta}$  are lift and moment coefficients per control surface deflection  $\beta$ ,  $k_a(\alpha)$  and  $k_h$  are the pitch and plunge stiffness coefficients, respectively. The structural nonlinearities are represented by the nonlinear functions  $M(\alpha)$ . In this study, system Eq. 1 was investigated for a hysteresis model in pitch, where  $M(\alpha)$  is illustrated in Fig. 2 and given by Liu *et al.* (2002):

$$M(\alpha) = \begin{cases} \alpha - \alpha_f + M_0, & \alpha < \alpha_f \uparrow, \\ \alpha + \alpha_f - M_0, & \alpha < -\alpha_f \downarrow, \\ M_0, & \alpha_f \leq \alpha \leq \alpha_f + \delta \uparrow, \\ -M_0, & -\alpha_f - \delta \leq \alpha \leq -\alpha_f \downarrow, \\ \alpha - \alpha_f - \delta + M_0, & \alpha > \alpha_f + \delta \uparrow, \\ \alpha + \alpha_f + \delta - M_0, & \alpha < -\alpha_f - \delta \downarrow, \end{cases} \tag{4}$$

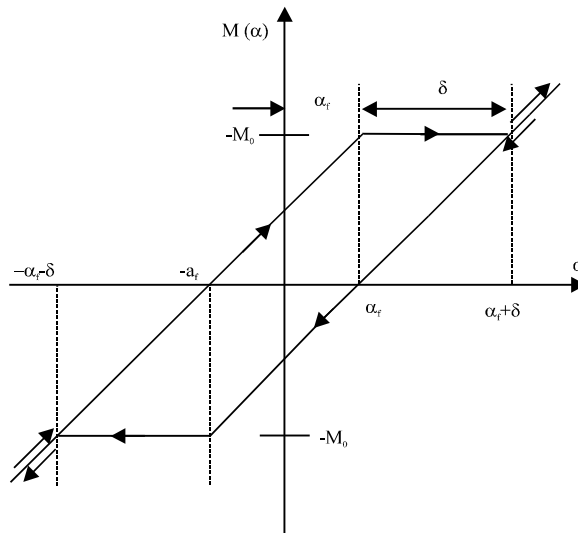


Fig. 2: General sketch of a hysteresis stiffness

where,  $\uparrow$  and  $\downarrow$  represent the motion in the increasing and decreasing  $\alpha$  direction, respectively.  $M_0$ ,  $\delta$  and  $\alpha_f$  are constants.

Let the vector  $x \in \mathbb{R}^4$  be given by  $x = [h, \alpha, \dot{h}, \dot{\alpha}]$ ; then the preceding equations can be written in a state-space form given by:

$$\dot{x} = A(x)x + B\beta(t) \quad (5)$$

where,  $\beta$  is the command input. The definition of matrix  $A(x)$  and  $B$  is given by:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_1 & -k_2 U^2 - p(x_2) & -c_1 & -c_2 \\ -k_3 & -k_4 U^2 - q(x_2) & -c_3 & -c_4 \end{bmatrix} \quad (6)$$

$$B = [0 \quad 0 \quad b_3 U^2 \quad b_4 U^2]^T$$

where the parameters are given as follows:

$$d = m (I_a - m x_a^2 b^2)$$

$$k_1 = I_a k_h / d, \quad k_2 = (I_a \rho b c_{l\alpha} + m x_b^3 \rho c_{m\alpha}) / d$$

$$k^3 = -m x_a b k_h / d, \quad k_4 = -(m x_b^2 \rho c_{l\alpha} + m b^2 \rho c_{m\alpha}) / d$$

$$p(x_2) = -m x_a b k_\alpha(x_2) / d, \quad q(x_2) = m k_\alpha(x_2) / d$$

$$c_1 = [I_a (c_h + \rho U b c_{l\alpha}) + m x_a \rho U b^3 c_{m\alpha}] / d$$

$$c_2 = [I_a \rho U b^2 c_{l\alpha} (\frac{1}{2} - a) - m x_a b c_a + m x_a \rho U b^4 c_{m\alpha} (\frac{1}{2} - a)] / d$$

$$c_3 = -m (x_a b c_h + x_a \rho U b^2 c_{l\alpha} + \rho U b^2 c_{l\alpha}) / d$$

$$c_4 = m [c_a - x_a \rho U b^3 c_{l\alpha} (\frac{1}{2} - a) - \rho U b^3 c_{m\alpha} (\frac{1}{2} - a)] / d$$

$$b_3 = -(I_a \rho b c_{l\beta} + m x_a \rho b_3 c_{m\beta}) / d$$

$$b_4 = -(m x_a b^2 \rho c_{l\beta} + m \rho b^3 c_{m\beta}) / d$$

If a time delay  $\tau$  exists between the control input and actuator, Eq. 5 becomes:

$$\dot{x} = A(x)x + B\beta(t - \tau) \quad (7)$$

**State variable feedback control law:** A nonlinear flutter control law based on the state-dependent Riccati equation method (Tadi, 2003; Bhoir and Singh, 2005; Cimen, 2010) is designed here. Consider an optimal control (infinite-horizon regulator) problem in which for the nonlinear system expressed by Eq. 5, the performance index of the form:

$$J = \frac{1}{2} \int_0^{\infty} (x^T Q(x)x + R\beta^2) dt \quad (8)$$

where,  $Q(x)$  is a positive definite symmetric matrix and  $R > 0$  for  $x \in \mathbb{R}^4$ .

For the system modeled in Eq. 5, the controllability matrix is given by:

$$D(x) = [B, A(x)B, A^2(x)B, A^3(x)B] \quad (9)$$

A hysteresis model consists of six linear subsystems governing six regions in its state space, so long as the controllability matrix  $D(x)$  is nonsingular in the six regions to ensure that the system is controllable.

In order to obtain the suboptimal solution of the preceding problem using the SDRE method, one solves the state-dependent Riccati equation given by:

$$A^T(x)P(x) + P(x)A(x) - P(x)BR^{-1}B^T P(x) + Q(x) = 0 \quad (10)$$

for the matrix  $P(x) > 0$ .

The feedback control law is then given by:

$$\beta(t) = -R^{-1}B^T P(x)x \quad (11)$$

Substituting the control law Eq. 11 in Eq. 5 gives the closed-loop system.

$$\dot{x} = [A(x) - BR^{-1}B^T P(x)]x \doteq \bar{A}(x)x \quad (12)$$

where,  $\bar{A}(x)$  is the closed-loop system matrix.

The performance of the closed-loop system depends on the matrix  $A(x)$  and the weighting matrices  $Q(x)$  and  $R$ .

**State estimator:** In general, not all of the states are available online and the feedback law must be based on an estimate of the states. Assuming that only the pitch angle  $\alpha$  as a function of time can be directly measured, the output equation is given by:

$$y = \alpha = [0, 1, 0, 0] x = Cx \quad (13)$$

To design an observer based on SDRE, it is required that the system be observable. For the pointwise observability, it is required that the following matrix has a full rank for all times:

$$E(x) = [C^T (CA(x))^T (CA^2(x))^T (CA^3(x))^T]^T \quad (14)$$

The determinant of the matrix  $E(x)$ :

$$\det(E(x)) = -k_3^2 + c_3c_1k_3 - c_3^2k_1 \quad (15)$$

As long as the value of the determinant is not zero, it means the matrix is nonsingular, the observability of the system is only related to the system parameters. If  $\hat{x}(t)$  is the estimate of the state then the observer dynamics is given by:

$$\dot{\hat{x}} = A(\hat{x})\hat{x} + B\beta(t) + K(y - C\hat{x}) \quad (16)$$

Where:

$$K = -P_0C^TV^{-1} \quad (17)$$

The matrix  $P_0$  is the positive definite solution of the algebraic Riccati equation:

$$A(\hat{x})P_0 + P_0A^T(\hat{x}) - P_0C^TV^{-1}CP_0 + Q_0 = 0 \quad (18)$$

where,  $V^{-1}$  and  $Q_0$  are constants.

Substituting  $\hat{x}$  for  $x$  in Eq. 11, the complete closed-loop system was obtained which is composed of controlled object, observer and state feedback.

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A(x) & -BR^{-1}B^TP \\ P_0C^TV^{-1}C & A(x) - P_0C^TV^{-1}C - BR^{-1}B^TP \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \quad (19)$$

## RESULTS AND DISCUSSION

In this section, numerical results for the control of the aeroelastic system are obtained. The values for the system parameters are taken from Singh and Yim (2003) and listed as follows:  $b = 0.135$  m,  $m = 12.387$  kg,  $I_a = 0.065$ ,  $x_a = 0.3267$ ,  $k_h = 2844.4$  N m<sup>-1</sup>,  $ch = 27.43$  N sec m<sup>-1</sup>,  $c_a = 0.036$  N sec,  $\rho = 1.225$  kg m<sup>-3</sup>,  $\alpha = -0.68$ ,  $c_{1\alpha} = 6.28$ ,  $c_{1\beta} = 3.358$ ,  $c_{ma} = -1.1304$ ,  $c_{mp}$ . The nonlinear parameter in pitch stiffness is  $m_0 = 0.25^\circ$ ,  $\delta = 0.5^\circ$ ,  $a_f = 0.25^\circ$ . For the specific data given in above, the determinant of the observability matrix  $E(x)$  is equal to  $-3,487,750.89(\neq 0)$ . The matrix has a full rank for all times and therefore, it is possible to design an on-line observer. The performance of the estimator is shown as Fig. 3. It is thus obvious that the estimator can quickly make an accurate estimate of non-direct measurement variables.

**Case 0:** Simulation is performed for the open-loop system ( $\beta = 0$ ) with the initial conditions  $\alpha(0) = 0.1$  rad,  $h(0) = 0.01$  m,  $\hat{a}(0) = 0$  rad sec<sup>-1</sup> and  $h(0) = 0$  and for the values of  $U = 13$  m sec<sup>-1</sup>. The open-loop eigenvalues of the system are in the right half plane at  $2.9046 + j18.3698$  and  $2.9046 - j18.3698$  and the remaining eigenvalues are in the left half plane. Thus the origin

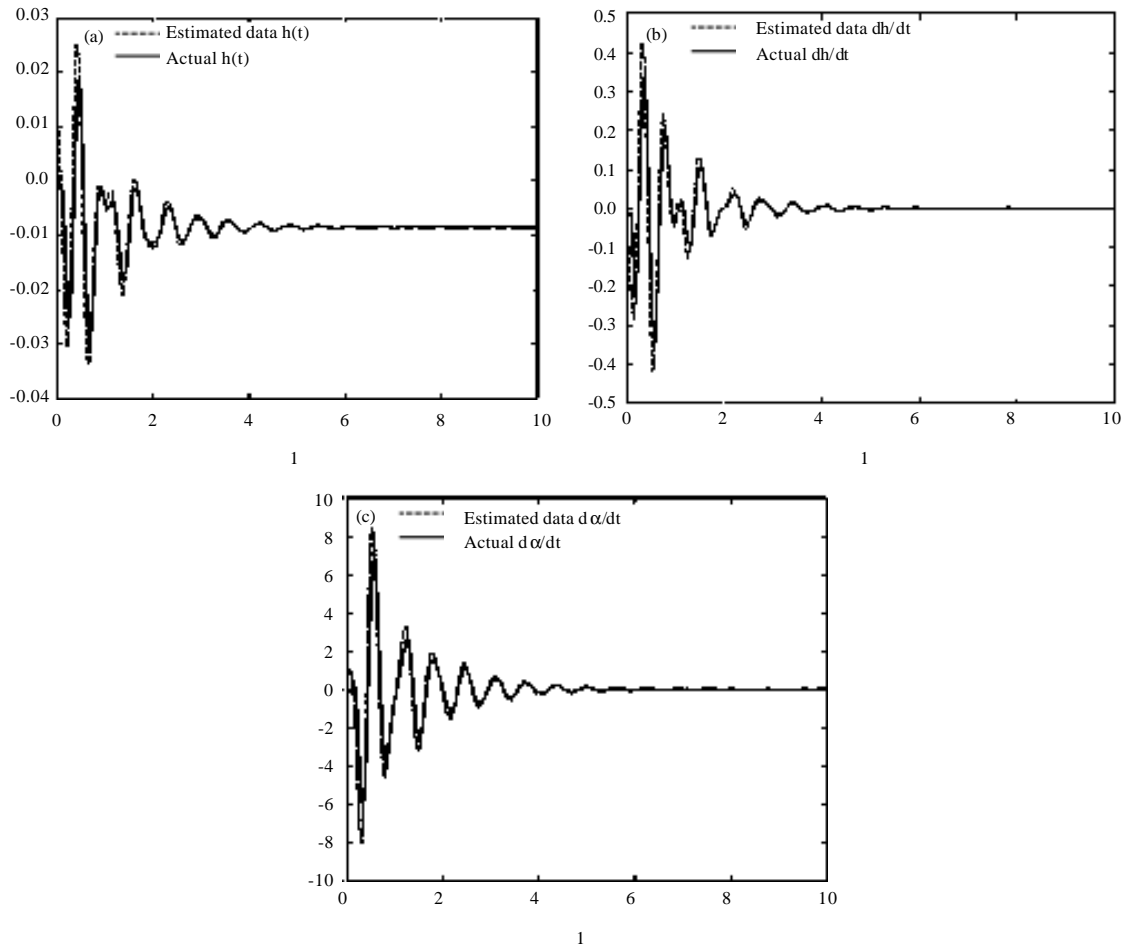


Fig. 3(a-c): Performance of the estimator as a function of time, (a)  $h(t)$ , (b)  $dh/dt$  and (c)  $d\alpha/dt$

$x = 0$  is locally unstable. From Fig. 4, it is seen that for the chosen initial condition, after an initial transient, the pitch angle and the plunge displacement trajectories converge to limit cycles.

**Case 1:** Now the closed-loop system (Eq. 19) including the nonlinear control law (Eq. 11) is simulated. The parameter  $U$  and the initial conditions of case 0 are retained. The weighting matrix and the scalar function in the performance index are selected as  $Q = \text{diag}(1, 10, 1, 10)$  and  $R = 1000$ , respectively. From the results shown in Fig. 5, it can be observed that the control law designed in the closed-loop system makes the plunge displacement and the pitch angle converge. The maximum control magnitude for stabilization is 0.17 rad. The settling time for the stabilization of both the plunge displacement and the pitch angle is of the order of 1.6 sec which is fast. The peaks in the plunge displacement and pitch angle do not exceed their initial values of 0.01 m and 0.1 rad, respectively. If the weighting parameters  $R = 1000$  and  $R = 100$  changes, the shape of the response characteristics will also change. As shown in Fig. 6, when the parameter  $R = 1000$  changes to  $R = 100$ , the settling time for the stabilization of the response is of the order of 1.3 sec. Moreover, the maximum control magnitude for stabilization is 0.34 rad. Figure 7 shows the response at different parameter  $Q$ . It can be seen the settling time for the stabilization is of the



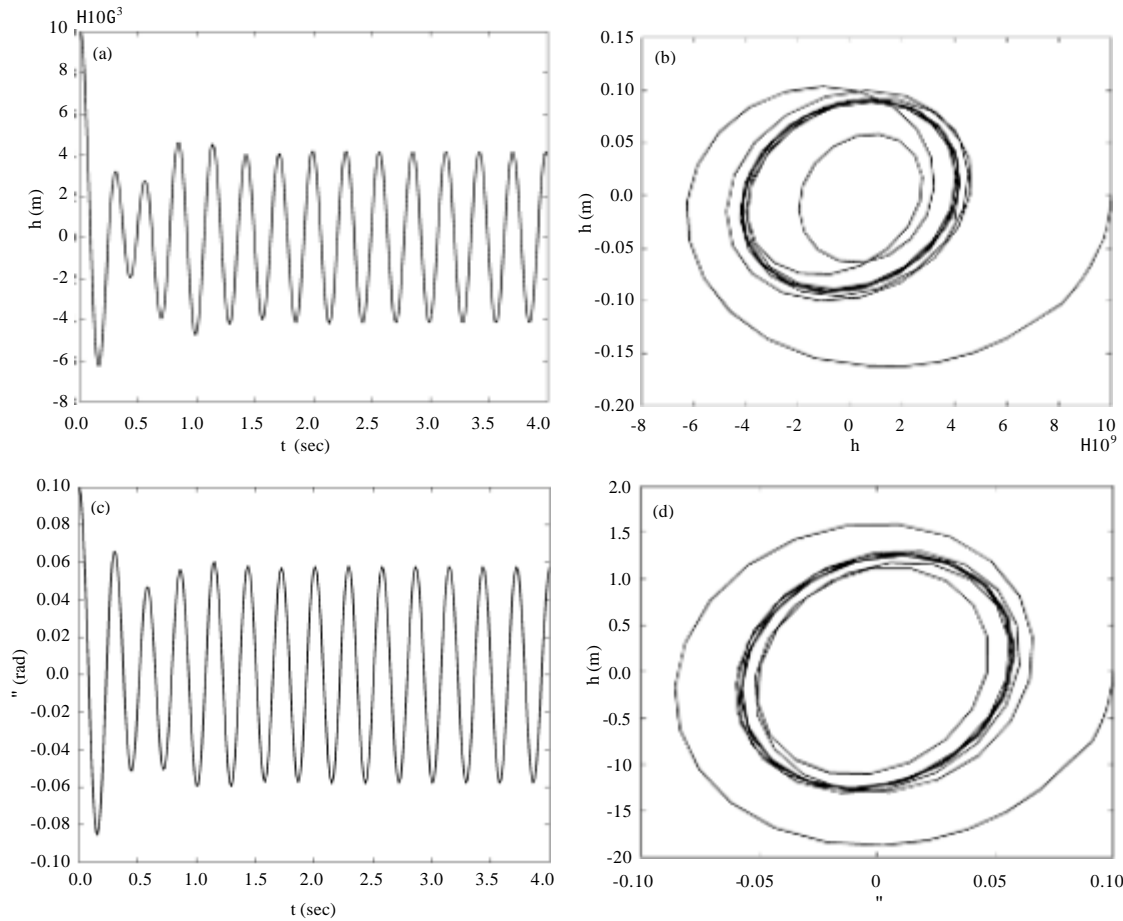


Fig. 4(a-d): Open-loop responses at  $U = 13 \text{ m sec}^{-1}$  (a) Plunge, (b) Phase plane plot  $h$ - $\dot{h}$ , (c) Pitch and (d) Phase plane plot  $\alpha$ - $\dot{\alpha}$

order of 1.4 sec and the maximum control magnitude for stabilization is 0.26 rad. It is found that the choice of larger  $Q$  and  $R$  reduces the peaks in the state variables and gives faster convergence but requires larger control input. Hence, the effects of the weighting parameters  $Q$  and  $R$  on the response characteristics should be seriously considered.

**Case 2:** In the above analysis, time delays in control loops are ignored. In this section, the effect of time delay on an aeroelastic system is investigated. First a comparison was made for the control input time histories of the closed-loop system at  $U = 10 \text{ m sec}^{-1}$  with and without time delay. From the results shown in Fig. 8, it can be observed that the control law designed in the previous section makes the system response converge without any time delay. If a time delay  $\tau$  between the control input and actuator occurred at time  $t$ , the control input  $\beta(t-\tau)$  would be derived from the previous state  $x(t-\tau)$  at time  $\tau$  before the present state. The control input  $\beta(t-\tau)$  would drive the system to produce a deflection angle which would cause oscillations of the system state and control input. We observe that this vibration was convergent when  $\tau \geq 0.006$  sec. With the time delay increasing, a high-frequency vibration of small amplitude arose in the control input  $\beta(t)$  but the vibration is still

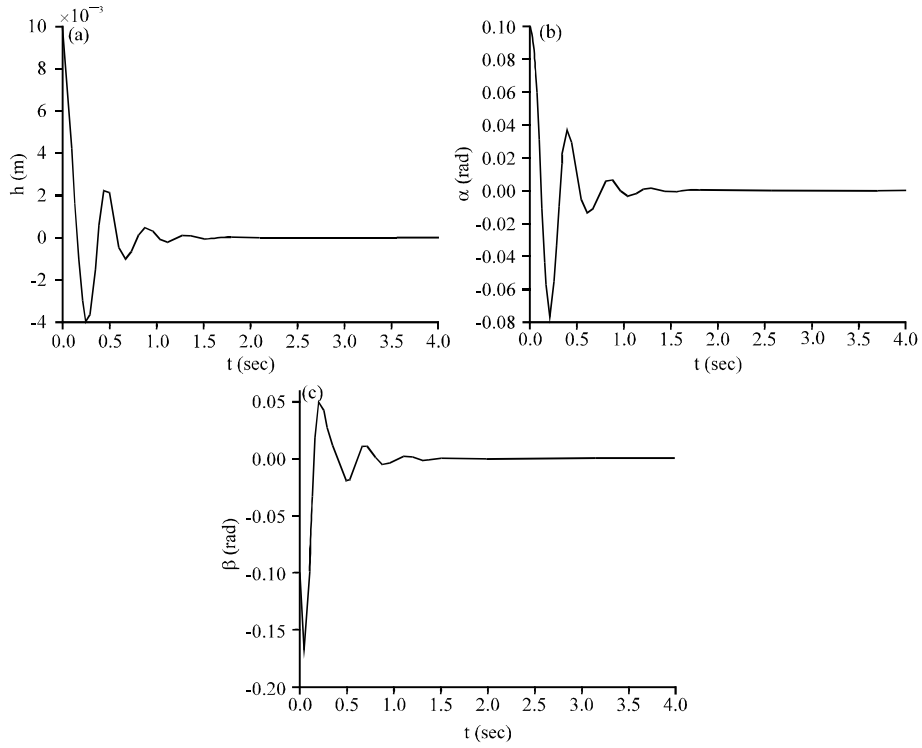


Fig. 5(a-c): Nonlinear flutter control:  $U = 10 \text{ m sec}^{-1}$ ,  $R = 1000$ ,  $Q = \text{diag}(1, 10, 1, 10)$  (a) Plunge, (b) Pitch and (c) Control input

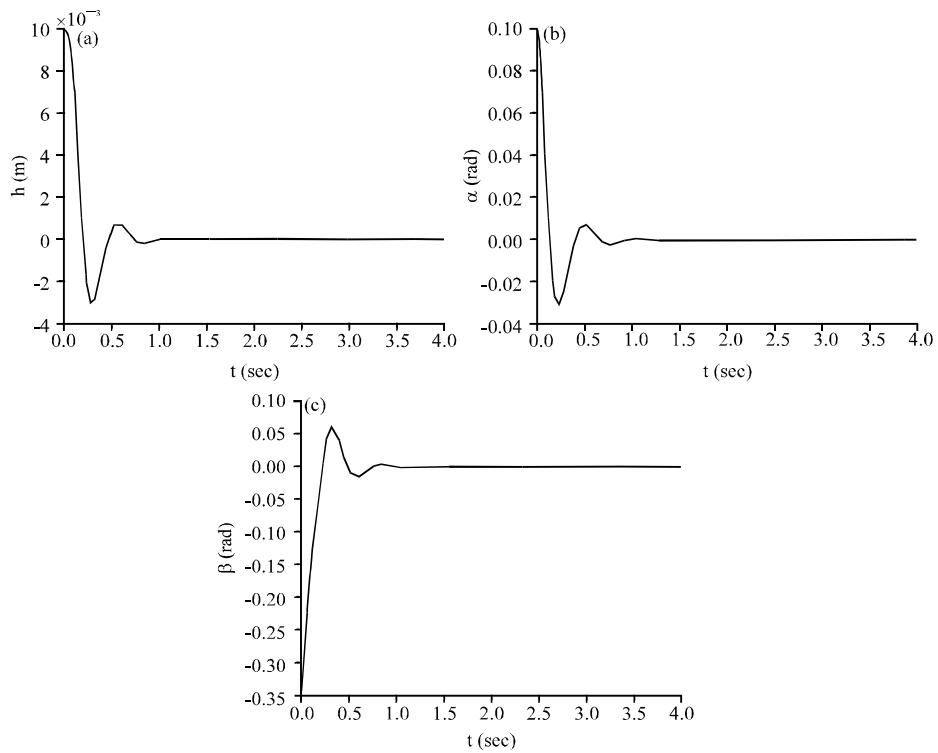


Fig. 6(a-c): Nonlinear flutter control:  $u = 10 \text{ m sec}^{-1}$ ,  $R = 100$ ,  $Q = \text{diag}(1, 10, 1, 10)$  (a) Plunge, (b) Pitch and (c) Control input

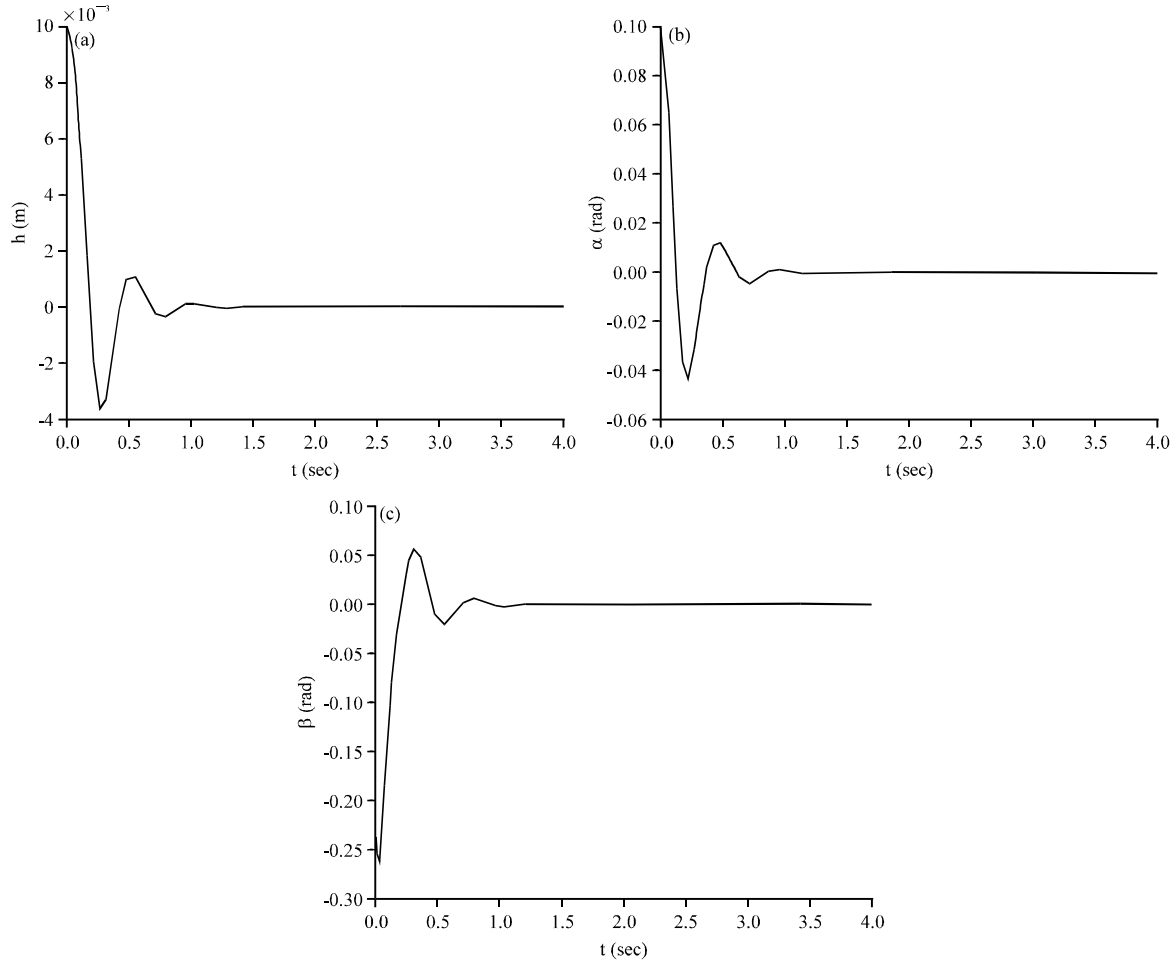


Fig. 7(a-c): Nonlinear flutter control:  $U = 10 \text{ m sec}^{-1}$ ,  $R = 1000$ ,  $Q = \text{diag}(10, 100, 10, 100)$ ,  
 (a) Plung, (b) Pitch and (c) Control input

convergent. However, the vibration becomes divergent when  $\tau = 0.029 \text{ sec}$ . From the analysis results, it can be seen that the time delay will produce an additional motion in the system responses. At the flow velocity  $U = \text{m sec}^{-1}$ , the time histories of pitch and plunge responses are shown in Fig. 9. If a time delay was set between the control input and actuator, the system response behaved differently. It is obvious that the pitch and plunge responses does not converge when the time delay  $\tau = 0.029 \text{ sec}$ . The results indicate that the time delay between the control input and actuator may impair the performance of a designed control law and cause instability of the system. Bifurcation diagrams of closed-loop system response as function of time delay are given in Fig. 10. It can be see that the amplitudes of plunge and pitch responses are quite small until the time delay is higher than 0.025 sec which means that a quite small time delay may lead to high-frequency vibration of the system but the control law designed is still effective to suppress the flutter. Increasing the time delay further, closed-loop response amplitudes become large rapidly. The plunge and pitch amplitudes are almost the same with open-loop response amplitudes when  $\tau = 0.029 \text{ sec}$ . And now the designed controller has no effect for flutter suppression.

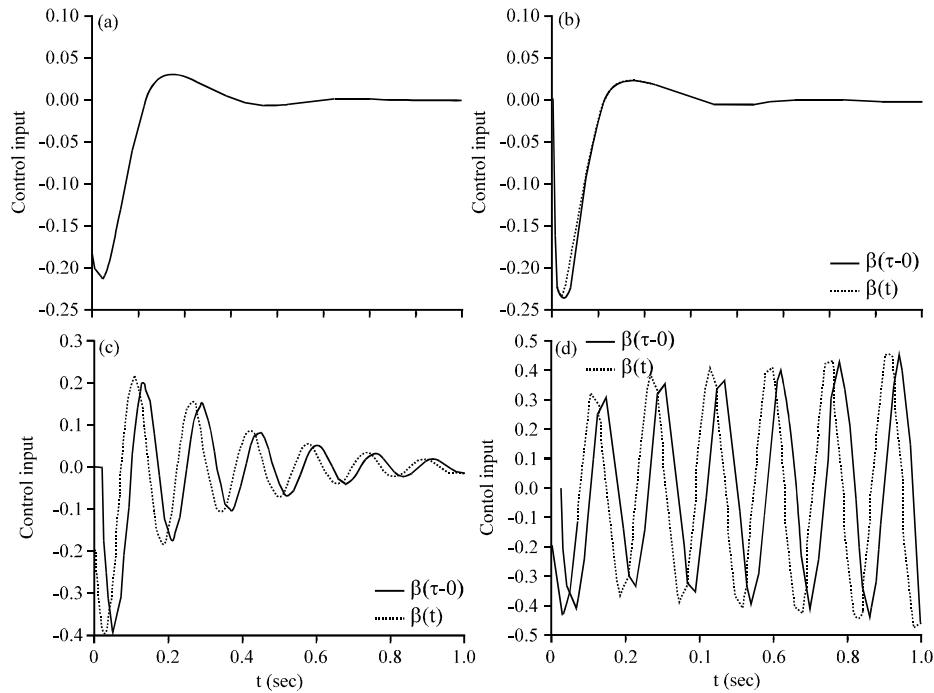


Fig. 8(a-d): Time histories of control input at  $U = 10 \text{ m sec}^{-1}$  and (a)  $\tau = 0$  sec, (b)  $\tau = 0.001$  sec, (c)  $\tau = 0.005$  sec and (d)  $\tau = 0.006$  sec

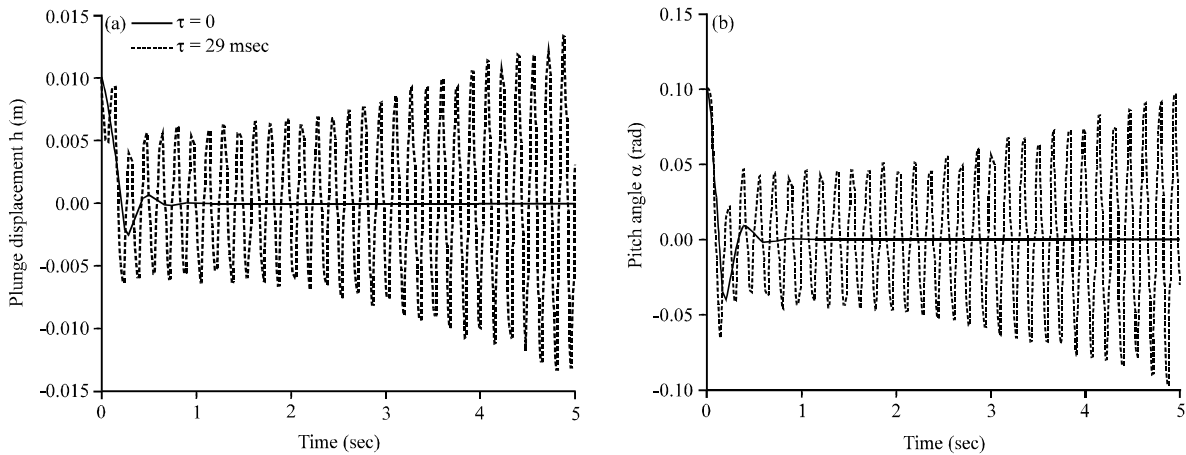


Fig. 9(a-b): Time histories (a) Plunge and (b) Pitch at  $U = 20 \text{ m sec}^{-1}$ ,  $\tau = 0.029$  sec

As mentioned in the introduction, some very similar analysis to that presented here were also undertaken by Tadi (2003), Bhoir and Singh (2005), Singh and Yim (2003) and Elhami and Narab (2012). They investigated the dynamic response of an airfoil with a freeplay and cubic nonlinearities. However, in previous research, the flutter control was studied without considering nonlinearity such as hysteresis in the airfoil. Further, they found that the effect of the weighting parameter  $Q$  on the effect of the shape of the response characteristics but they did not find the parameter  $R$ . In the current study, hysteresis nonlinearity has been considered in the design of a

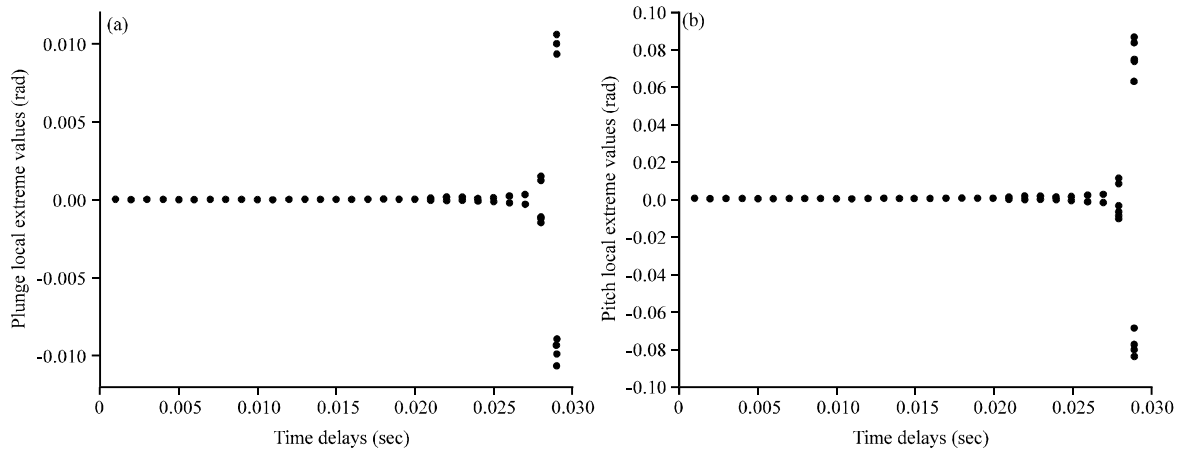


Fig. 10(a-b): Bifurcation diagrams of (a) Plunge and (b) Pitch response as function of time delay at  $U = 20 \text{ m sec}^{-1}$

control law for the flutter control of nonlinear aeroelastic systems. The effect of the weighting parameter  $R$  on the effect of the shape of the response characteristics was investigated. In addition, the effect of time delay between the actuator control input and the control surface action is also investigated.

## CONCLUSION

Based on the state space model of a two-dimensional airfoil section with hysteresis nonlinearity, a feedback control law based on the estimated states was designed by using the state-dependent Riccati equation method and applied for dynamic response suppression in this study. An observer was constructed to estimate the unavailable state variables of the system. The effects of time delay between the control input and actuator on the aeroelastic responses has been investigated.

The nonlinear control law accomplishes asymptotic regulation of the pitch and plunge motion to the system equilibrium at zero deflections. Simulation results show that the observer can give precise estimations for the plunge displacement and the velocities in pitch and plunge and in the closed-loop system, the designed controller is effective in flutter suppression. The system response is sensitive to the time delay between the control input and actuator. The bifurcation diagram of system response as function of time delay indicates that a small time delay may lead to high-frequency vibration. And with the time delay increasing, the system responses become divergent in the end.

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