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Research Article

An Intersection Model of RCC-5 for Spatial Relationships and its Application

[1]Jiang Baode and [1,2]Wei Dong-Qi

[1]National Engineering Research Center for Geographic Information System, China University of Geosciences, 430074 Wuhan, China
[2]Xi'an Center of Geological Survey, CGS, 710054 Xi'an, China

Abstract

Background: Intersection model and RCC model are two types of typical models of spatial relations in GIS, some of them can be translated to each other but for RCC-5, there is no corresponding intersection model. **Materials and Methods:** This study presents an extended 4-intersection model (E4-IM) for representing topological relations as RCC-5 do. Learning from the idea of RCC-5, which is insensitive to the region's boundary, it combines the boundary with the exterior of a region in the 4-intersection matrix and gets an extended 4-intersection matrix model of RCC-5. Then, the property of this new model is given and it is proved to be Jointly Exhaustive and Pairwise Disjoint (JEPD). At last, an application is presented. **Results:** The result shows that this new model can be extended further to describe the topological relations of a simple region and a region with a hole. **Conclusion:** It has the same expression ability as RCC-5 and can be easy to be extended further to represent topological relations of complex regions.

Key words: Topological relations, topology representation, extended 4-intersection model, spatial relation, GIS

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Corresponding Author: Wei Dong-Qi, Xi'an Center of Geological Survey, CGS, 710054 Xi'an, China

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INTRODUCTION

Models of spatial relations are a key component of Geographical Information System (GIS). Many efforts have been made to formally define spatial relations[1-3]. Because topological properties are the most fundamental, compared to Euclidean, metric and vector spaces[4], so for a long time, topological relations between spatial objects are one of the most important spatial relationships in GIS. They have been widely studied by the GIS and spatial database communities for more than two decades and are widely applied in spatial query, spatial reasoning, spatial analysis, content-based image retrieval and robot path navigation etc[5-8].

Over the past two decades, study has been conducted on how to apply fundamental mathematical theories for modeling and describing topological relations. There are mainly two classical types of topological relations representing models[9,10]. One is based on the point-set topology theory, such as 4-intersection model (4-IM) and 9-intersection model (9-IM)[11,12]. Another is based on logical method of Region Connection Calculus (RCC)[13], such as RCC-5 and RCC-8. These two type's models have their advantages and disadvantages, respectively. The RCC model is a qualitative description model, it has been popularly adopted by the qualitative spatial reasoning, but it is lack of formal description and hard to calculate. Instead, the intersection model is easy to formalize and calculate and it is widely used in spatial topological relation query. If they can translate to each other, their advantages can be got together to use. In fact, both 4-IM and RCC-8 can represent 8 types of topological relations of two simple spatial regions and they can be translated to each other. But for RCC-5, there is no corresponding intersection model.

In view of the above-mentioned fact, this study presents an extended 4-intersection model (E4-IM) for representing topological relations as RCC-5 do. After recalling the basic concepts of RCC model and intersection model, the extended 4-intersection model of RCC-5 is introduced, following with its property and proof. Then as an application, the idea of this new model is used to describe the topological relations of a simple region and a region with a hole.

MATERIALS AND METHODS

RCC model: In 1992, Randell *et al.*[13] gave the Region Connection Calculus (RCC) theory, based on Clarke's spatial calculus logical axiom[14]. Now RCC theory has been further studied and obtained improvement, such as RCC-5 relations and RCC-8 relations. The RCC-8 can distinguish 8 types of topological relations between two simple spatial regions, including: Disconnected (DC), Externally Connected (EC), Partially Overlapping (PO), Tangential Proper Part (TPP), Non-Tangential Proper Part (NTPP), equal (EQ), Tangential Proper Part Inverse (TPPI) and Non-Tangential Proper Part Inverse (NTPPI). The RCC-5 ignores the influence of boundaries of regions and can distinguish 5 types of topological relations of two simple spatial regions. The relationship between RCC-8 and RCC-5 is showed in Fig. 1, that's DC and EC are united to DR, TPP and TPP are united to PP, TPPI and NTPPI are united to PPI.

4-intersection model: Pullar and Egenhofer[15] originally described a formal model based on point-set topology for classifying topological relationships between one-dimensional intervals of IR^1. Egenhofer and Franzosa[12] adopted the same method for classifying topological relationships between area features in IR^2. The results of such formalization are the so-called 4-intersection model (4-IM)[12]. In the 4-IM, the topological relations between two entities A and B are defined in terms of the intersections of A's interior and boundary with B's interior and boundary. It can be represented by a matrix of values:

$$\begin{pmatrix} A^o \cap B^o & A^o \cap \partial B \\ \partial A \cap B^o & \partial A \cap \partial B \end{pmatrix}$$

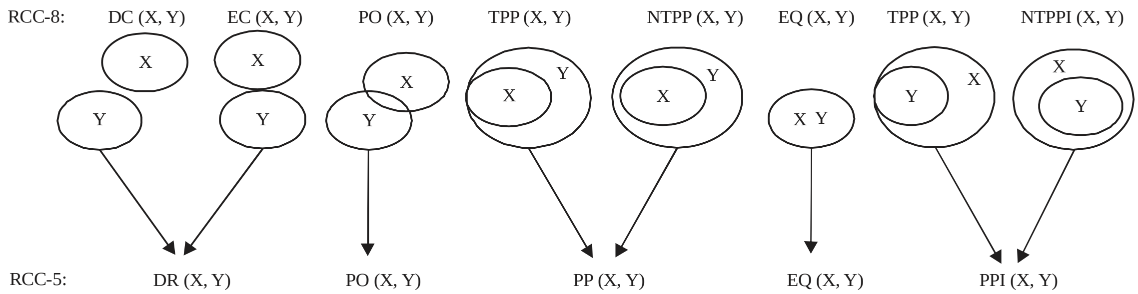

Fig. 1: Relationship between RCC-8 and RCC-5

Table 1: 4-intersection matrix representation of RCC-8 relations

RCC-8	DC (A, B)	EC (A, B)	PO (A, B)	TPP (A, B)	NTPP (A, B)	EQ (A, B)	TPP (A, B)	NTPPI (A, B)
4-IM	$\begin{pmatrix}0&0\\0&0\end{pmatrix}$	$\begin{pmatrix}0&0\\0&1\end{pmatrix}$	$\begin{pmatrix}1&1\\1&1\end{pmatrix}$	$\begin{pmatrix}1&0\\1&1\end{pmatrix}$	$\begin{pmatrix}1&0\\1&0\end{pmatrix}$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}1&1\\0&1\end{pmatrix}$	$\begin{pmatrix}1&1\\0&0\end{pmatrix}$

Table 2: Extended 4-intersection matrix representation of RCC-5 relations

RCC-5	DR (A, B)	PO (A, B)	PP (A, B)	EQ (A, B)	PPI (A, B)
Extended 4-IM	$\begin{pmatrix}0&1\\1&1\end{pmatrix}$	$\begin{pmatrix}1&1\\1&1\end{pmatrix}$	$\begin{pmatrix}1&0\\1&1\end{pmatrix}$	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$	$\begin{pmatrix}1&1\\0&1\end{pmatrix}$

Here, A^{o} means the interior of A and ∂A means the boundary of A. each intersection may be empty ($\varnothing$) or nonempty ($\neg\varnothing$), resulting in a total of $2^4 = 16$ combinations. But when considering the objectives in real world, only 8 types of topological relations corresponding to RCC-8 relations can be obtained (Table 1).

Extended 4-intersection model of RCC-5: Both 4-IM and RCC-8 can represent 8 types of topological relations of two simple spatial regions and they can be translated to each other. But for RCC-5, there is no corresponding intersection model. This section will introduce an extended intersection model for representing topological relations as RCC-5 do.

Definition of extended 4-intersection model: For two simple regions by not considering the boundaries like RCC-5, IR^2-$\{\partial A \cup \partial B\}$ can be decomposed to 4 parts $A^o \cap B^o$, $A^o \cap (B^C)^o$, $(A^C)^o \cap B^o$ and $(A^C)^o \cap (B^C)^o$, here A^C means the complement of A (Fig. 2). Similarly, the RCC-5 relations can be described as the following 4-intersection matrix:

$$\begin{pmatrix} A^o \cap B^o & A^o \cap (B^C)^o \\ (A^C)^o \cap B^o & (A^C)^o \cap (B^C)^o \end{pmatrix}$$

It is called extended 4-intersection model (E4-IM). Each intersection may be empty ($\varnothing$) or nonempty ($\neg\varnothing$), resulting in a total of $2^4 = 16$ combinations. But when considering the objectives in real world, only 5 types of topological relations corresponding to RCC-5 relations can be obtained (Table 2), the RCC-5 relations set is called Θ.

Property of extended 4-intersection model: The property of the extended 4-intersection model is given.

Theorem 1: The topological relations of two simple regions given by extended 4-intersection model are exclusive and complete.

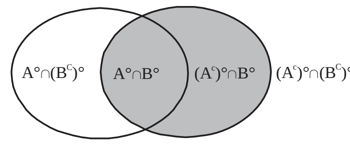

Fig. 2: Partition of two simple regions intersection

Proof: Given any two simple regions A and B. Since the 4 sets in the extended 4-intersection matrix make certain empty or nonempty, there is a 0/1 matrix given by the extended 4-intersection matrix corresponding to two simple regions A and B. Notice that a set is either empty or nonempty and these 4 intersections are disjoint in pairs, so there is a unique topological relation given by the extended 4-intersection matrix corresponding to the two simple regions A and B. This means the topological relations of two simple regions given by the extended 4-intersection model are complete. On the other hand, if there are two 0/1 matrices M and N given by the extended 4-intersection matrix corresponding to the regions A and B, then there exist an element in the matrix that is empty and nonempty simultaneously, this is a contradiction. Therefore, the topological relations of two simple regions given by the extended 4-intersection model are exclusive.

An application: The RCC-5 is difficult to be extended and it is not applicable to be used to represent topological relations of complex regions, but the E4-IM is relatively easy. As an application, an extended 6-intersection matrix model for a simple region and a region with a hole based on the E4-IM was investigated. First, the definition of 6-intersection matrix is given and then, three constraints of 6-intersection matrix are described to calculate the realizable topological relations, at last the comparison of 6-intersection with E4-IM is given.

Definition of 6-intersection matrix: Following the idea of extended 4-intersection model, the topological relations of a simple region and a region with a hole can be described. Suppose A is a simple region, B is a region with a hole and B^h

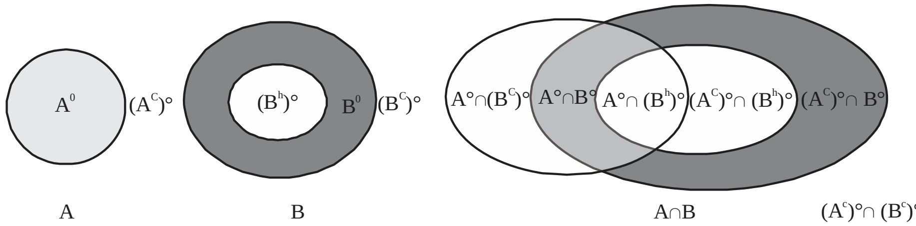

Fig. 3: Different partitions of IR^2 by A, B and A∩B

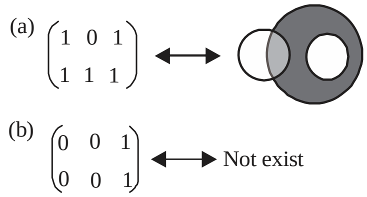

Fig. 4(a-b): Two examples of 6-intersection matrixes and their corresponding topological relations implementations, (a) A kind of 6-intersection matrix that can be implemented and (b) A kind of 6-intersection matrix that cannot be implemented

is the hole of B. By not considering the boundaries like RCC-5, IR^2-∂A is decomposed to two parts A^o and $(A^C)^o$, IR^2-{$\partial B \cup \partial B^h$} is decomposed to three parts B^o, $(B^h)^o$ and $(B^C)^o$ and A∩B is decomposed by IR^2-{$\partial A \cup \partial B \cup \partial B^h$} to 6 parts $A^o \cap (B^C)^o$, $A^o \cap B^o$, $A^o \cap (B^h)^o$, $(A^C)^o \cap (B^h)^o$, $(A^C)^o \cap B^o$ and $(A^C)^o \cap (B^C)^o$ (Fig. 3).

Being similar to the extended 4-intersection model, A∩B could be represented by a 6-intersection matrix of values:

$$\begin{pmatrix} A^o \cap B^o & A^o \cap (B^h)^o & A^o \cap (B^C)^o \\ (A^C)^o \cap B^o & (A^C)^o \cap (B^h)^o & (A^C)^o \cap (B^C)^o \end{pmatrix}$$

Each intersection may be empty (∅) or nonempty (¬∅), resulting in a total of $2^6 = 64$ combinations theoretically. But when consider the objectives in real world, there are same topological relations that do not exist. For example, a 6-intersection matrix given in Fig. 4a can find a realizable topological relation, while a 6-intersection matrix given in Fig. 4b does not exist.

In order to find the realizable topological relations, three constraints of the 6-intersection matrix are given in the following.

Constraints of 6-intersection matrix: Notice that B^o is the internal part of B, B^h is the hole of B and $(B^h)^o$ is the internal part of B^h. By not considering the boundaries like RCC-5,

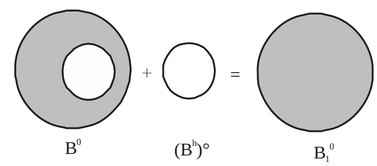

Fig. 5: Result of combining B^o and $(B^h)^o$

$B^o \cup (B^h)^o$ is the internal part of a simple region without a hole in it as shown in Fig. 5.

Then doing a union computation with the first two columns of the 6-intersection matrix above, it can be got:

$$A^o \cap B^o \cup \left(A^o \cap \left(B^h\right)^o\right) = A^o \cap \left(B^o \cup \left(B^h\right)^o\right) = A^o \cap B_1^o$$
$$\left(\left(A^C\right)^o \cap B^o\right) \cup \left(\left(A^C\right)^o \cap \left(B^h\right)^o\right) = \left(A^C\right)^o \cap \left(B^o \cup \left(B^h\right)^o\right) = \left(A^C\right)^o \cap B_1^o$$

Because B^h is the hole of B, by not considering the boundaries, $(B^h)^o$ and B only have one type topological relationship, that is 'by contain'. So if the topological relationships between the two simple regions A and B_1 represented by the E4-IM can be realized, the topological relationships between A and B (B has a hole in it) can also be realized. Thus, the problem that whether the topological relationships of a simple region and a region with a hole can be realized presented by 6-intersection matrix can be changed to the problem that whether the topological relationships of two simple regions presented by E4-IM can be realized. Because a realizable extended 4-intersection matrix is corresponding to a RCC-5 relations set (Table 2), so there is a constraint as following.

Constraint 1: If a 6-intersection matrix corresponds to a realizable topological relation, then, after doing a union computation with the first two columns of the 6-intersection matrix, it can be got an extended 4-intersection matrix that satisfies the RCC-5 relations set. That is:

$$\begin{pmatrix} (A^o \cap B^o) \cup (A^o \cap (B^h)^o) & A^o \cap (B^C)^o \\ ((A^C)^o \cap B^o) \cup (A^C)^o \cap (B^h)^o) & (A^C)^o \cap (B^C)^o \end{pmatrix} \in \Theta$$

In other words, the result of doing a '∪' computation with the first two columns of the 6-intersection matrix is

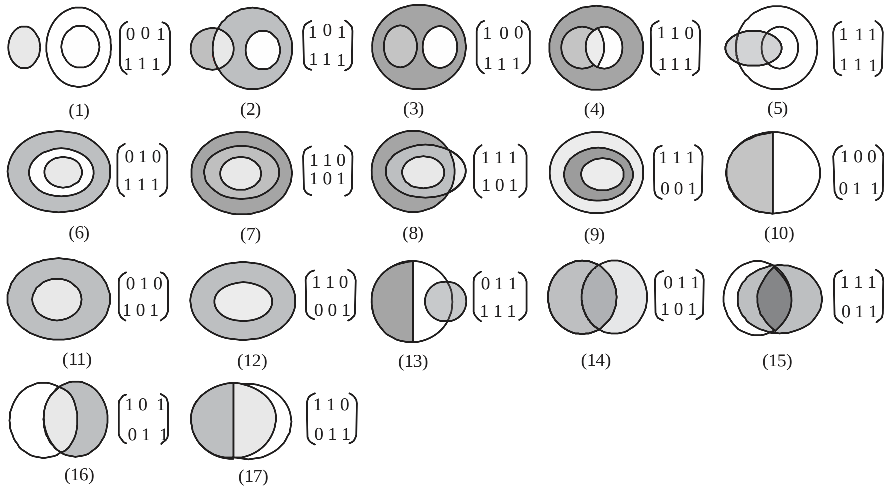

Fig. 6: Seventeen kinds of topological relations between a simple region and a region with a hole

corresponding to an extended 4-intersection model of two simple regions. So, if the result of the first two columns of any 6-intersection matrix doing a '$\cup$' computation is of RCC-5 relations set Θ, thus 6-intersection matrix is a realizable topological relation.

In addition, the union for the corresponding elements of upper and down row of the 6-intersection matrix can get:

$$\left((A^o \cap B^o) \cup (A^C)^o \cap B^o) \ (A^o \cap (B^h)^o) \cup ((A^C)^o \cap (B^h)^o) \ (A^o \cap (B^C)^o) \cup ((A^C)^o \cap (B^C)^o)\right)$$
$$= \left((A^o \cup (A^C)^o) \cap B^o \ (A^o \cup (A^C)^o) \cap (B^h)^o \ (A^o \cup (A^C)^o) \cap (B^C)^o\right)$$

Because, by not considering the boundaries, $A^o \cup (A^C)^o$ is the IR^2, the intersection of any part of IR^2 with IR^2 is 1, so there is another constraint.

Constraint 2: If a 6-intersection matrix corresponds to a realizable topological relation, the result of a union for the corresponding elements of upper and down row of the 6-intersection matrix is a proper subset. That is:

$$(A^o \cap B^o) \cup ((A^C)^o \cap B^o) = 1 \ (A^o \cap (B^h)^o) \cup ((A^C)^o \cap (B^h)^o) =$$
$$1 \ ((A^o \cap (B^C)^o) \cup ((A^C)^o \cap (B^C)^o)) = 1$$

At last, for any two simple regions, their external parts must be of intersection, which can be obtained:

Constraint 3: For two simple regions A and B, $(A^C)^o \cap (B^C)^o = 1$. According to the constraints above, the all realizable topological relations of a simple region and a region with a hole represented by the 6-intersection matrix can be calculated out. The following is the algorithm implementation.

Algorithm implementation: The main idea of algorithm is: (1) Each 6-intersection matrix is given in the form of row vector [a_1 a_2 ... a_6]. Then there are 2^6 types of 0/1 matrixes in theory, that is a matrix M of 2^6 row vectors. (2) Scan the matrix M row by row and sign all of the row vectors of M satisfying all the constraints. (3) Save all of the row vectors of M satisfying the constraints to the matrix N and output the results.

Description of the algorithmic pseudo code that calculates the all topological relations for a simple region and a region with a hole is following:

Topological Relation Gen (NULL, TR)
Input: NULL
Output: Topological relations that satisfy all the constraints:

```
TRAll←2^6 basic topological relations
TR←NULL
for each t in TRAll
if t satisfies constraint 1
if t satisfies constraint 2
if t satisfies constraint 3
TR ←t//Put topological relation into TR
end if
end if
end if
end for
return TR
```

By program experiments, 17 kinds of 6-intersection matrixes are calculated out. The corresponding topological relations are shown in Fig. 6.

Table 3: Relationship between 6-intersection model and E4-IM

6-intersection model	E4-IM
1, 6, 11, 13, 14	DR
2, 4, 5, 7, 8	PO
3	PP
10	EQ
9, 12, 15, 16, 17	PPI

RESULTS

By contrast, the expression ability of 6-intersection model and E4-IM is given in the following. In fact, both E4-IM and 6-intersection model can be used to describe the topological relationships of a simple region and a region with a hole. But based on the E4-IM, only 5 types of topological relations can be distinguished, which is corresponded to RCC-5. While based on the 6-intersection model, 17 kinds of topological relationships can be distinguished. The relationship between the two extended intersections models are shown in Table 3.

As seen in the table, the 'DR' relation in E4-IM is divided into 5 kinds of topological relationships in 6-intersection model. It is corresponded to the number 1, 6, 11, 13 and 14 in Fig. 6. So as the 'PO' and 'PPI' relations, they are also be divided into 5 kinds of topological relationships in the 6-intersection model respectively, the 'PO' is corresponded to the number 2, 4, 5, 7 and 8 and the 'PPI' is corresponded to the number 9, 12, 15, 16 and 17 in Fig. 6. So, 6-intersection model can distinguish more types of topological relationships than E4-IM for modeling the topological relationships of a simple region and a region with a hole and its expression ability is stronger than E4-IM.

DISCUSSION

Compared with the existing models, the E4-IM has the following advantages. From the perspective of intersection model, whether the basic intersection models of 4-IM and 9-IM or the later extended models such as V9-IM and DE-9IM etc., they model the topological relations of regions either considering the interior and boundary of the region or the interior, boundary and external of the region. But sometimes, the boundaries of regions do not have to take into account. In this situation, the E4-IM of this study is very applicable.

Similarly, from the perspective of RCC model, RCC model is a qualitative description model and it is lack of formal description and hard to calculate. If there are some topological relations modeled by RCC-5 that need formal description and calculation, it can be replaced by E4-IM.

So, the E4-IM can be used to model spatial relations under the circumstance that when boundaries of regions are not within the scope of consideration or some spatial relations modeling by RCC-5 need to formally describe or calculate.

CONCLUSION

In this study, based on the RCC theory, an extend 4-intersection model of RCC-5 for modeling spatial relationships of two simple regions is presented and it is proved to be exclusive and complete. Then as an application, an extended 6-intersection model based on the E4-IM is further explored to represent the topological relations of a simple region and a region with a hole, 17 kinds of topological relations of a simple region and a region with a hole in practice are calculated out by program under the condition of three constraints. The result shows that the E4-IM is easy to be extended further and can be used to represent topological relations of complex regions.

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