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## Research Article

# Using an Algorithm to Fit a GAMLSS Model on Dry Matter Data from *Brachiaria brizantha*

<sup>1</sup>Danny Villegas Rivas, <sup>2</sup>Nora Valbuena Torres, <sup>3</sup>Manuel Milla Pino, <sup>2</sup>Martín Grados Vásquez, <sup>5</sup>Ydalia Velásquez Casana, <sup>3</sup>Erick Delgado Bazan, <sup>4</sup>César Osorio Carrera, <sup>6</sup>Ricardo Shimabuku Ysa, <sup>7</sup>River Chávez Santos, <sup>4</sup>Luis Ramírez Calderón and <sup>8</sup>Clelia Jima Chamiquit

<sup>1</sup>Faculty of Forestry and Environmental Engineering, Universidad Nacional de Jaén, Cajamarca, Peru

<sup>2</sup>Agricultural and Marine Sciences Program, Universidad Nacional Experimental de los Llanos Occidentales Ezequiel Zamora, Guanare, Venezuela

<sup>3</sup>Faculty of Civil Engineering, Universidad Nacional de Jaén, Cajamarca, Peru

<sup>4</sup>Graduate School, Universidad César Vallejo, Peru

<sup>5</sup>Graduate School, Universidad Nacional de Trujillo, Peru

<sup>6</sup>Faculty of Mechanical and Electrical Engineering, Universidad Nacional de Jaén, Cajamarca, Peru

<sup>7</sup>Faculty of Economic and Administrative Sciences, Universidad Nacional Toribio Rodríguez de Mendoza de Amazonas, Peru

<sup>8</sup>Faculty of Health Sciences, Universidad Nacional Toribio Rodríguez de Mendoza de Amazonas, Peru

## Abstract

**Background and Objective:** Forage production in the tropics is generally asymmetrically distributed. Hence the need to use more complex models, especially when multiple comparisons are made and there are very large deviations from normality. The objective of this research is to fit a Generalized Additive Model for Location, Scale and Shape (GAMLSS) model on accumulated dry matter data from *Brachiaria brizantha* using a model selection algorithm. **Materials and Methods:** A Box-Cox Power Exponential (BCPE) distribution was adjusted on the dry matter from *Brachiaria brizantha* data implementing GAMLSS in R (programming language). The accumulated dry matter data for *B. brizantha* were obtained from a study carried out on a farm in the state of Portuguesa, Venezuela. The explanatory covariate  $x$  was the interval between cuts (21, 28, 35 and 42 days). **Results:** The dependent variable (dry matter) exhibited both skewness and kurtosis. GAMLSS allowed flexible modeling of both the distribution of the dry matter yield from *B. brizantha* and the dependence of all the parameters of the distribution on intervals between cuttings. For the dry matter yield from *B. brizantha*, which exhibited skewness and leptokurtosis, the BCPE distribution, provided the best fit. **Conclusion:** The interval between cuttings showed an effect that is reflected in the average yield of dry matter from *B. brizantha*. The interval between cuts affected the skewness and the kurtosis of the distribution.

**Key words:** Pastures, yield, Box-Cox, power, exponential, skewness, kurtosis

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**Corresponding Author:** Danny Villegas Rivas, Faculty of Forestry and Environmental Engineering, National University of Jaen, Cajamarca, Peru

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**Competing Interest:** The authors have declared that no competing interest exists.

**Data Availability:** All relevant data are within the paper and its supporting information files.

## INTRODUCTION

In tropical livestock systems, poor grassland productivity is one of the most important limitations due to the adaptability and persistence in these environments. However, new cultivars of the genus *Brachiaria* have been released to the market as options to overcome the problems observed in traditional forages, thus providing better fodder options<sup>1</sup>. *Brachiaria brizantha* is one of the superior introducing grasses that has been adapted and known by farmers. These grasses are compatible with the tropical climate and are tolerant of various types of soil, including acidic soils<sup>2</sup>. Knowledge of the distribution of production and quality of forage during the year is a tool to plan utilization<sup>3</sup>. Forage production in the tropics is related to the distribution of rainfall, which is generally asymmetrically distributed. This fact raises the need to use more complex models, especially when multiple comparisons are made and there are very large deviations from normality, the risk of error increases<sup>4,5</sup>.

Generalized Additive Models for Location, Scale and Shape (GAMLSS) are semi-parametric regression type models. They are parametric, in that they require a parametric distribution assumption for the response variable and "semi" in the sense that the modeling of the parameters of the distribution, as functions of explanatory variables, may involve using non-parametric smoothing functions. GAMLSS were introduced by researchers<sup>6</sup> as a way of overcoming some of the limitations associated with the popular generalized linear models, GLM and generalized additive models, GAM<sup>7,8</sup>. In GAMLSS the exponential family distribution assumption for the response variable ( $y$ ) is relaxed and replaced by a general distribution family, including highly skew and/or kurtotic continuous and discrete distributions<sup>9</sup>. The systematic part of the model is expanded to allow modeling not only of the mean (or location) but other parameters of the distribution of  $y$  as, linear and/or non-linear, parametric and/or additive non-parametric functions of explanatory variables and/or random effects<sup>6</sup>. Hence GAMLSS is especially suited to modeling a response variable that does not follow an exponential family distribution, (e.g., leptokurtic or platykurtic and/or positive or negative skew response data, or over-dispersed counts) or which exhibit heterogeneity, (e.g., where the scale or shape of the distribution of the response variable changes with explanatory variables(s))<sup>9</sup>.

In this paper, a GALMSS model for skewed data, where the exponential family assumption is relaxed and replaced by a very general distribution family is considered. Within this new framework, the systematic part of the model is expanded to allow not only the mean (or location) but all the parameters

of the conditional distribution of  $y$  to be modeled as parametric and/or additive nonparametric (smooth) functions of explanatory variables and/or random-effects terms. The objective of this research is to fit a GAMLSS model on accumulated dry matter data from *Brachiaria brizantha* using a model selection algorithm.

## MATERIALS AND METHODS

**Study area:** The study was carried out at a farm in Portuguesa state, Venezuela (Fig. 1) from November, 2016-October, 2017). The farm is located between 1008020-1004802 West Longitude and 425400-427420 North Latitude. The response variable 'y' is the accumulated dry matter (kg ha<sup>-1</sup>). The explanatory variable 'x' is the interval between cutting (21, 28, 35 and 42 days).

**Box-Cox power exponential distribution:** The Box-Cox power exponential distribution (BCPE) is defined by (for details see<sup>11</sup>).

Let  $Y$  be a positive random variable having a Box-Cox power exponential distribution, denoted by, defined through the transformed random variable  $Z$  given by:

$$Z = \begin{cases} \frac{1}{\sigma v} \left[ \left( \frac{Y}{\mu} \right)^v - 1 \right] & \text{if } v \neq 0 \\ \frac{1}{\sigma} \log \left( \frac{Y}{\mu} \right) & \text{if } v = 0 \end{cases} \quad (1)$$

for  $0 < Y < \infty$  where  $y$  and where the random variable  $Z$  is assumed to follow a standard power exponential distribution with power parameter,  $\tau > 0$ , treated as a continuous parameter (Parameterization (1) assumes a standard normal distribution for  $Z$ )<sup>11</sup>.

The probability density function of  $Z$ , a standard power exponential variable, is given by:

$$f_z(z) = \frac{\tau}{c^2 \binom{1+\tau}{1} \Gamma(1/\tau)} \exp\{-0,5 |z/c|^\tau\} \quad (2)$$

for  $-\infty < Z < \infty$   $y$   $\tau > 0$ , where  $c^2 = 2^{-2/\tau} \Gamma(1/\tau) [\Gamma(3/\tau)]^{-1}$ . This parameterization ensures that  $Z$  has mean 0 and standard deviation 1 for all  $\tau > 0$ . Note that  $\tau = 1$  and 2 correspond to the Laplace (i.e., two-sided exponential) and normal distributions respectively, while the uniform distribution is the limiting distribution as  $\tau \rightarrow \infty$ <sup>12</sup>.

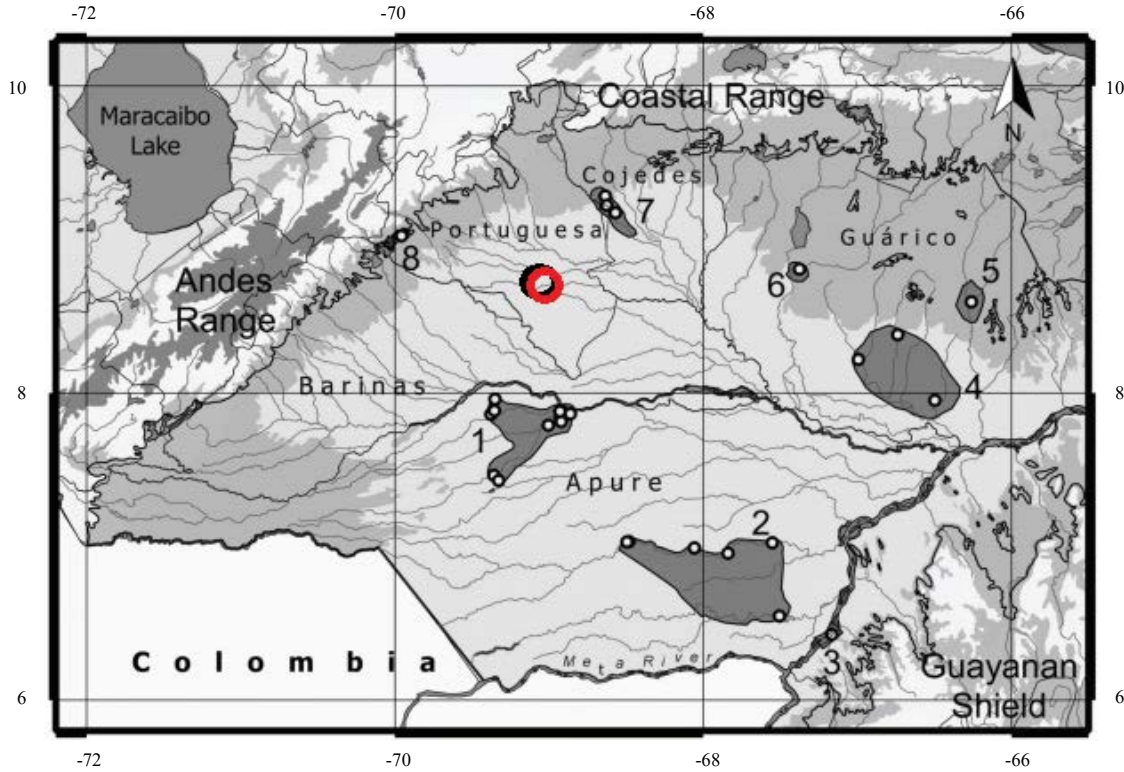


Fig. 1: Relative location of the Portuguese state, Venezuela

Source: Seijas<sup>10</sup>

From 2, the probability density function of  $Y$ , a BCPE ( $\mu, \sigma, \nu, \tau$ ), random variable, is given by:

$$f_Y(y) = f_Z(z) \left| \frac{dz}{dy} \right| = \frac{y^{\nu-1}}{\mu^\nu \sigma} f_Z(z) \quad (3)$$

**Modeled data using GAMLSS:** Let  $y^T = (y_1, y_2, \dots, y_n)$  be the vector of the response variable observations. Also, for  $k = 1, 2, \dots, p$ , let  $g_k(\cdot)$  be a known monotonic link function relating  $\theta_k$  to explanatory variables and random effects through an additive model given by:

$$g_k(\theta_k) = \eta_k = X_k \beta_k + \sum_{(j=1)}^{(j_k)} Z_{jk} \gamma_{jk} \quad (4)$$

where,  $\theta_k$  and  $\eta_k$  are vectors of length  $n$ , e.g.,  $\theta_k^T = (\theta_{1k}, \theta_{2k}, \dots, \theta_{nk})$ ,  $\beta_k^T = (\beta_{1k}, \beta_{2k}, \dots, \beta_{j_k k})$  is a parameter vector of length  $J'_k$ ,  $X_k$  is a known design matrix of order  $n \times J'_k$ ,  $Z_{jk}$  is a fixed known  $n \times q_{jk}$  design matrix and  $\gamma_{jk}$  is a  $q_{jk}$ -dimensional random variable. Model (4) is called the GAMLSS.

The vectors  $\gamma_{jk}$  for  $j = 1, 2, \dots, j_k$  could be combined into a single vector with a single design matrix  $Z_k$ .

If, for  $k = 1, 2, \dots, p$ ,  $J_k = 0$  then model (4) reduces to a fully parametric model given by:

$$g_k(\theta_k) = \eta_k = X_k \beta_k \quad (5)$$

If  $Z_{jk} = I_n$ , where  $I_n$  is an  $n \times n$  identity matrix and  $\gamma_{jk} = h_{jk} = h_{jk}(x_{jk})$  for all combinations of  $j$  and  $k$  in a model (4), this gives:

$$g_k(\theta_k) = \eta_k = X_k \beta_k + \sum_{j=1}^{j_k} h_{jk}(x_{jk}) \quad (6)$$

where,  $x_{jk}$  for  $j = 1, 2, \dots, j_k$  and  $k = 1, 2, \dots, p$  are vectors of length  $n$ . The function  $h_{jk}$  is an unknown function of the explanatory variable  $x_{jk}$  and  $h_{jk} = h_{jk}(x_{jk})$  is the vector that evaluates the function  $h_{jk}$  at  $x_{jk}$ . The explanatory vectors are assumed to be known. We call the model in equation (5) the semiparametric GAMLSS. Model (6) is an important special case of model (4). If  $Z_{jk} = I_n$  and  $\gamma_{jk} = h_{jk} = h_{jk}(x_{jk})$  for specific combinations of  $j$  and  $k$  in a model (4), then the resulting model contains parametric, nonparametric and random-effects terms.

The first two population parameters  $\theta_1$  and  $\theta_2$  in a model (4) is usually characterized as location and scale parameters, denoted here by  $\mu$  and  $\sigma$ , whereas the remaining parameter(s), if any, are characterized as shape parameters, although the model may be applied more generally to the parameters of any population distribution.

For many families of population distributions a maximum of two shape parameters  $\nu$  ( $=$ ) and  $\tau$  ( $=\theta_3$ ) suffice, giving the model:

$$\begin{aligned}
 g_1(\mu) &= \eta_1 = X_1\beta_1 + \sum_{j=1}^{J_1} Z_{j1}\gamma_{j1} \\
 g_2(\sigma) &= \eta_2 = X_2\beta_2 + \sum_{j=1}^{J_2} Z_{j2}\gamma_{j2} \\
 g_3(\nu) &= \eta_3 = X_3\beta_3 + \sum_{j=1}^{J_3} Z_{j3}\gamma_{j3} \\
 g_4(\tau) &= \eta_4 = X_4\beta_4 + \sum_{j=1}^{J_4} Z_{j4}\gamma_{j4}
 \end{aligned} \tag{7}$$

The GAMLSS model (4) is more general in that the distribution of the dependent variable is not limited to the exponential family and all parameters (not just the mean) are modeled in terms of both fixed and random effects, for details see Rigby and Stasinopoulos<sup>11</sup>.

**Model for the four parameters of the Box-Cox power exponential distribution:** The parameters  $(\mu, \sigma, \nu, \tau)$  of the Box-Cox power exponential distribution may be modeled as functions of many explanatory variables using the generalized additive model for location, scale and shape (denoted by GAMLSS) of Rigby and Stasinopoulos<sup>11</sup>. Here we consider a special case of the GAMLSS model where there is a single explanatory variate  $X$ . Given  $X = x$ ;  $Y$  is modeled by a Box-Cox power exponential variable, BCPE  $(\mu, \sigma, \nu, \tau)$ , with probability density function  $f_Y(y)$ , defined by (4) where the parameters  $\mu, \sigma, \nu$  and  $\tau$  are modeled as smooth non-parametric functions of  $x$ , i.e.  $Y \sim \text{BCPE}(\mu, \sigma, \nu, \tau)$  where:

$$\begin{aligned}
 g_1(\mu) &= h_1(x) \\
 g_2(\sigma) &= h_2(x) \\
 g_3(\nu) &= h_3(x) \\
 g_4(\tau) &= h_4(x)
 \end{aligned} \tag{8}$$

and for  $k = 1, 2, 3, 4$ ,  $g_k(\cdot)$  are known monotonic link functions, usually the identity for  $\mu$  and  $\nu$  and the log for  $\sigma$  and  $\tau$  and  $h_k(x)$  are smooth non-parametric functions of  $x$ .

For  $i = 1, 2, \dots, n$ , given  $X = x_i$ , observations  $Y_i$  are assumed to be independent BCPE  $(\mu_i, \sigma_i, \nu_i, \tau_i)$  variables with probability density functions  $f_{Y_i}(y_i)$  obtained from (3) and parameters

obtained from (8). This model is appropriate for independent observations of  $Y$  (e.g. cross-sectional data) rather than correlated observations (e.g. longitudinal data), for details see Rigby and Stasinopoulos<sup>11</sup>.

**Model estimation and selection:** The non-parametric functions  $h_k$  for  $k = 1, 2, 3, 4$  are estimated by maximizing the penalized log-likelihood function  $l_p$  defined by:

$$l_p = l_d - \frac{1}{2} \sum_{k=1}^4 \lambda_k \int_{-\infty}^{\infty} \{h''_k(u)\}^2 du \tag{9}$$

where,  $h''_k(u)$  is the second derivative of  $h_k(u)$  to  $u$  and  $\sum_{i=1}^n l_i$  is the log-likelihood function of the data and  $l_i$  is the log-likelihood function of observation  $y_i$  from a Box-Cox power exponential distribution, BCPE  $(\mu_i, \sigma_i, \nu_i, \tau_i)$ , obtained from (4).

The penalized log-likelihood function (9) is maximized iteratively using either the RS or CG algorithm of Rigby and Stasinopoulos<sup>11</sup>, which in turn uses a back-fitting algorithm to perform each step of the Fisher scoring procedure, requiring the log-likelihood of the data,  $l_d$  and first and expected second derivatives to  $\mu, \sigma, \nu$  and  $\tau$ .

A general criterion for model selection is the generalized Akaike Information Criterion (GAIC), obtained by adding to the fitted deviance a fixed penalty  $\#$  for each effective degree of freedom used in a model, i.e.  $\text{GAIC}(\#) = \hat{D} + \# \cdot \text{df}$ , where  $\text{df}$  denotes the total effective degrees of freedom used in the model,  $\hat{D} = -2\hat{l}$  denotes the fitted deviance and denotes the fitted log-likelihood.

The total effective degrees of freedom  $\text{df}$  combines the effective degrees of freedom used in the smooth functions  $h_1(x)$  to  $h_4(x)$  in (8), for modeling  $\mu, \sigma, \nu$  and  $\tau$ , denoted by  $\text{df}_\mu, \text{df}_\sigma, \text{df}_\nu$  and  $\text{df}_\tau$ , respectively. Each effective degrees of freedom (e.g.  $\text{df}_\mu$ ) is defined by the trace of the corresponding smoothing matrix in the fitting algorithm, which is in turn directly related to the corresponding smoothing parameter (e.g.  $\lambda_1$ ), see Rigby and Stasinopoulos<sup>11</sup> for more details. The model with the smallest value of the criterion  $\text{GAIC}(\#)$  is then selected. The Akaike information criterion (AIC) and the Schwartz Bayesian criterion (SBC) are special cases of the  $\text{GAIC}(\#)$  criterion corresponding to  $\# = 2$  and  $\# = \log(n)$ , respectively. Let BCPE  $(\text{df}_\mu, \text{df}_\sigma, \text{df}_\nu, \text{df}_\tau, \lambda)$  represent the BCPE model (8), where the first four values inside the brackets denote the total effective degrees of freedom used in the smooth non-parametric functions  $h_1(x)$  to  $h_4(x)$  for modeling  $\mu, \sigma, \nu$  and  $\tau$ , respectively and the fifth value  $\lambda$  denotes the power transform parameter in the transform  $x = x \text{var}^\lambda$ , where  $x$  is the explanatory variate recorded in the data set.

For cumulated dry matter data set the three-step procedure for selecting a model of form (8) proposed by Rigby and Stasinopoulos<sup>11</sup> is used:

### Initial choices:

- Choose link functions  $g_k(\cdot)$  For  $k = 1, 2, 3, 4$  in (4)
- Choose an initial parameter value  $\lambda_0$  for  $\lambda$  in the transformation  $x = x \text{ var}^\lambda$ , assuming  $x \text{ var} > 0$  for all observations. The initial value  $\lambda_0$  is chosen to provide an approximate constant absolute gradient between  $y$  and  $x$ . (A shifted (or offset) power transformation of  $x \text{ var}$ , i.e.,  $x = (x \text{ var} + \delta)^\lambda$  could be considered if  $x \text{ var}$  takes negative values)
- Choose a single penalty # for each effective degree of freedom used in the models for  $\mu, \sigma, \nu$  and  $\tau$

### Model selection:

- From model BCPE  $(1, 1, 1, 1; \lambda_0)$  (where the first four values  $df_\mu = df_\sigma = df_\nu = df_\tau = 1$  indicate fitting a constant for parameters), apply forward selection of to give the chosen value which minimizes criterion GAIC (#)
- In model BCPE  $(df_{\mu_1}, 1, 1, 1, \lambda)$ , estimate  $\lambda$ , (e.g. using a grid search over values of  $\lambda$ ) to give the value  $\lambda_1$  which minimizes GAIC (#)

- From model BCPE  $(1, 1, 1, 1; \lambda_1)$ , apply forward selection of  $df_\sigma$  to give chosen value  $df_{\sigma_1}$  which minimizes GAIC (#)
- From model BCPE  $(df_{\mu_1}, df_{\sigma_1}, 1, 1, \lambda_1)$  create a two-way table of values of GAIC (#) for combinations  $(df_\nu, df_\tau)$  and search for the combination  $(df_{\nu_1}, df_{\tau_1})$  which minimizes GAIC (#)

### Fine-tuning of model BCPE $(df_{\mu_1}, df_{\sigma_1}, df_{\nu_1}, df_{\tau_1}, \lambda_1)$ :

- Fine-tuning of  $df_{\sigma_1}$  by changing  $df_{\sigma_1}$  in steps of 1, if the value of GAIC (#) decreases
- Similar fine tuning of  $df_{\mu_1}$
- Fine-tuning of  $\lambda$  by re-estimating  $\lambda$  as in step 2 (b)

The R code to use the algorithm described above is shown in the Appendix.

## RESULTS AND DISCUSSION

**Dry matter of *Brachiaria brizantha*:** In Fig. 2a, the distribution of the dependent variable (accumulated dry matter) of *B. brizantha* showed asymmetry and kurtosis. Figure 2b the box plots show an asymmetric distribution of the accumulated dry matter data for intervals between cuts of 35 and 42 days. Figure 2c the densities associated with the accumulated dry matter data of *B. brizantha* show leptokurtic

Appendix: R code to fit a GAMLSS model on dry matter data in *Brachiaria brizantha*

```

dataa
min(KGMS)
max(KGMS)
library(gamlss.dist)
library(gamlss.data)
library(tidyverse)
library(ggpubr)
library(ggforce)
p3 <- ggplot(data = datos, aes(x = edad)) + geom_density(alpha = 0.7, fill = "gray20") + labs(title = "Distribución KGMS") + theme_bw()
##
library(gamlss)
distribuciones <- fitDist(y = edad, k = log(length(edad)), type = "realplus", trace = FALSE, try.gamlss = TRUE, parallel = "multicore", ncpus = 3L)
##
distribuciones$fits %>% enframe(name = "distribucion", value = "GAIC") %>% arrange(GAIC)
summary(distribuciones)
modelo <- gamlss(formula = KGMS ~ pb(edad), sigma.formula = ~ pb(edad), nu.formula = ~ pb(edad), family = BCPE, data = datos, control = gamlss.control(trace = FALSE)
)
##
summary(modelo)
##
grid_predictor.1 <- edad
grid_predictor.1
predicciones <- predictAll(modelo, newdata = data.frame(edad = grid_predictor.1), type = "response")
##
predicciones <- as.data.frame(predicciones)
predicciones <- bind_cols(data.frame(edad = grid_predictor.1), predicciones)
predicciones %>% head()
predicciones

```

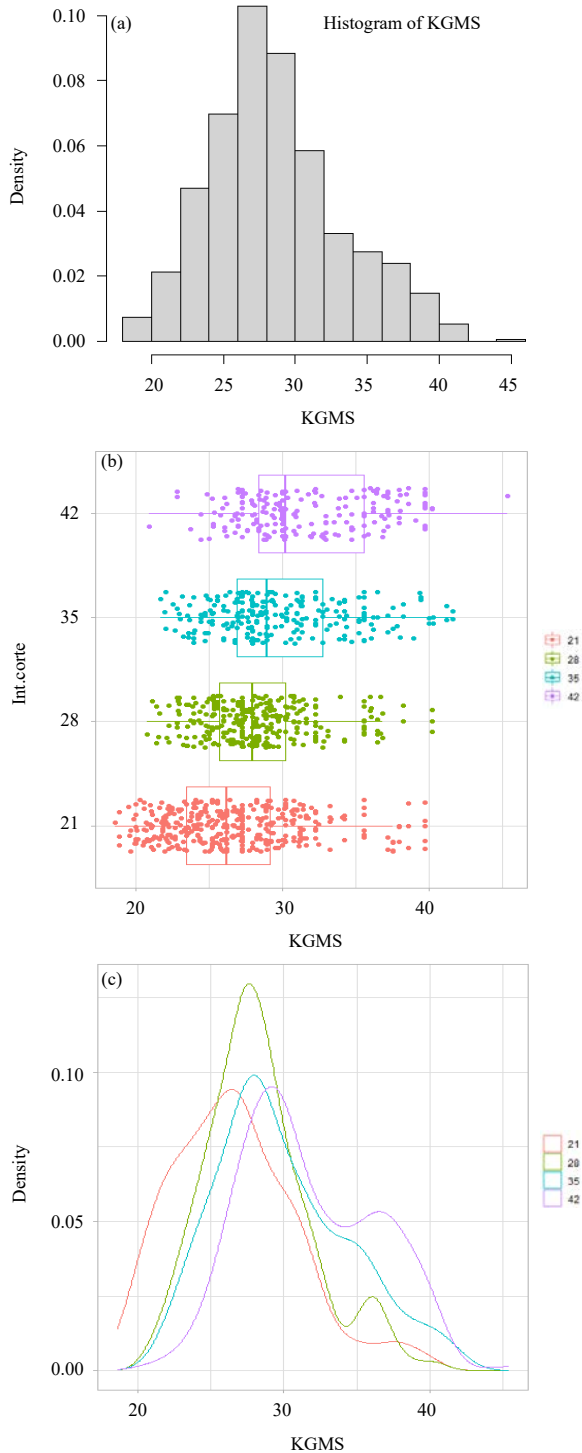


Fig. 2(a-c): Dry matter distribution of *Brachiaria brizantha*  
 (a) Histogram, (b) Box-plot and (c) Density at four intervals between cutting

distributions for the 4 intervals between cuts. The data of Table 1 shows how the accumulated dry matter of *B. brizantha* increases as the interval between cuts increases. The data of

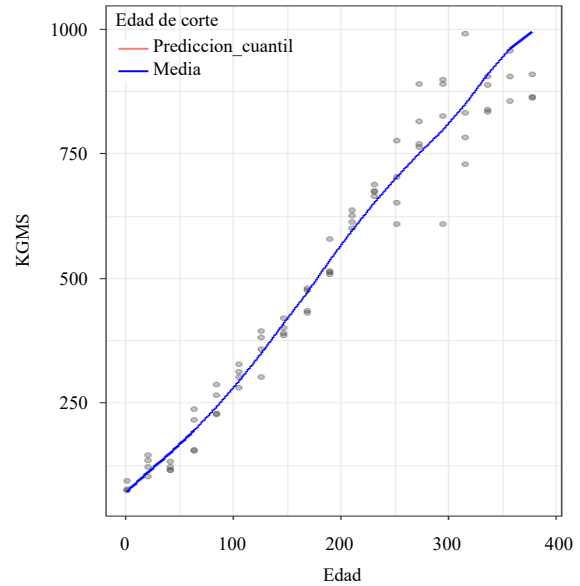


Fig. 3: Box-Cox Power Exponential Distribution (BCPE) fitted on *Brachiaria brizantha* dry matter data at four intervals between cuttings

Table 1: Yield (kg) of dry matter of *Brachiaria brizantha* at four intervals between cuttings

Statistics	Intervals between cutting (days)			
	21	28	35	42
Mean	467.17	546.3935	569.453	623.185
Median	456.01	514.18	611.25	674.32
Standard deviation	294.4591	320.8447	314.3087	308.0972

Table 2: Assumptions on dry matter data from *Brachiaria brizantha* at four intervals between cutting

Assumption	Test	p-value
Normality	Wilk-Shapiro	0.002337
Homogeneity of variances	Levene	0.9773
Autocorrelation	Durbin-Watson	<0.0000

Table 3: GAMLSS model adjusted on dry matter data of *Brachiaria brizantha* at four intervals between cutting

Fit distribution	GAIC
BCPE	865
BCPEo	962
GG	968
GB2	987
BCCG	993
BCCGo	993
BCT	997
BCTo	997
exGAUS	1006
WEI2	1006

Table 2 shows the non-normality or asymmetry ( $p < 0.05$ ) of the accumulated dry matter distribution and serial autocorrelation of errors ( $p < 0.05$ ). From Fig. 3 and Table 3, the chosen BCPE

Table 4: Box-Cox power exponential distribution (BCPE) fitted on *Brachiaria brizantha* dry matter data at four intervals between cutting

$\mu$ model	Estimation	Standard error	p-value
Intercept	430.39159	0.35389	<0.0000
Intervals between cuttings	5.02916	0.01091	<0.0000
Log( $\sigma$ ) model	Estimation	Standard error	p-value
Intercept	-0,3453	0,0008818	<0.0000
Intervals between cuttings	-0,01066	0,00002717	<0.0000
Log( $\nu$ ) model	Estimation	Standard error	p-value
Intercept	0,7739	0,001938	<0.0000
Intervals between cuttings	0,003976	0,00006215	<0.0000
$\tau$ model	Estimation	Standard error	p-value
Intercept	12,60	12,79	0.328

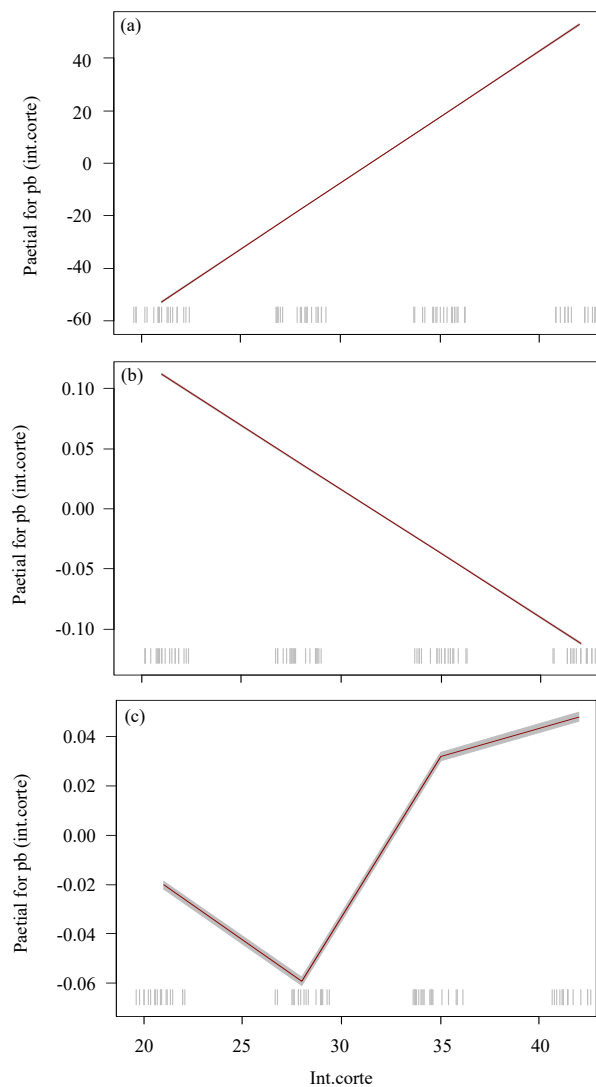


Fig.4(a-c): Fitted values for the parameters (a)  $\mu$ ,(b)  $\sigma$  and (c)  $\nu$  in the final chosen BCPE model on dry matter data in *Brachiaria brizantha*

model provides a substantially better fit than the previous models both in terms of the GAIC (865). The fitted models for the  $\mu$ ,  $\sigma$  and  $\nu$  in the chosen BCPE model have plotted in

Fig. 4a-c, respectively. The estimated parameters for Box-Cox power exponential distribution (BCPE) fitted on *Brachiaria brizantha* dry matter data at four intervals between cuttings are shown in Table 4. The parameters  $\mu$ ,  $\sigma$ ,  $\nu$  and  $\tau$  of the BCPE distribution may be interpreted as relating to the location (median), scale (coefficient of variation), skewness (power transformation to the symmetry) and kurtosis, respectively<sup>9</sup>. The fitted value for  $\tau$  in Equation (13) indicates leptokurtosis as  $\tau > 3$ . From Table 5 for the different intervals between cutting, the distribution of dry matter from *B. brizantha* is asymmetric as  $\nu_i > 0$ . From Eq. 10 and Fig. 4a, it is observed that the longer the interval between cuts, the average yield of dry matter increases. Similar results are reported by Hare *et al.*<sup>13</sup> in a study on the effect of cutting interval on yield and quality of three brachiaria hybrids in Thailand. From Fig. 4b it is observed that the longer the interval between cuts, the variability in the dry matter yield decreases and from Fig. 4c it is observed that for intervals between the cutting of 21-28 days the asymmetry of the distribution of dry matter yield decreases, while for intervals between the cutting of 35-42 days it increases.

This indicates that cutting stimulates regrowth of the plant and consequently its dry matter production since a significant increase in dry matter is observed when going from the first to the third cut. There is enough information in the literature to explain the effect of the cutting stimulus<sup>13</sup> or grazing<sup>1,14</sup> in the dry matter production of pastures. These variations in the average dry matter production between cuts of the genotypes as a whole were the product of the environmental effect (heterogeneous distribution of rainfall within the same dry season), rather than by effects of the genotype itself ( $p > 0.05$ ). In this regard, some studies show that environmental effects are often more important than the effects of the genotype on the productive traits of forage plants<sup>15,16</sup>. In this way, the modeling of the accumulated dry matter using the BCPE distribution allows to show the effect of the interval between cut on the accumulated dry matter and to make decisions in the management of this species that allow better yields.



Table 5: Estimated parameters from the GAMLSS model on the accumulated dry matter from *B. brizantha* at four intervals between cuttings

Interval between cutting (days)	Mean $\hat{\mu}_i$	Standard deviation $\hat{\sigma}_i = e^{\text{Log}(\hat{\sigma})}$	Skewness $\hat{\phi}_i = e^{\text{Log}(\hat{\phi})}$	Kurtosis $\hat{\tau}_i$
21	536.0040	0.5660	2.3570	12.6
28	571.2081	0.5253	2.4235	12.6
35	606.4122	0.4875	2.4919	12.6
42	641.6163	0.4524	2.5622	12.6

In the parameterization of the Box-Cox Power Exponential Distribution (BCPE) fitted on *Brachiaria brizantha* dry matter data at four intervals between cutting with parameters used in the GAMLSS methodology, its expectation is given by:  $E[Y] = \mu$  and its variance by  $\text{Var}[Y] = \sigma^2\mu^2$ :

$$\hat{\mu} = 430,39159+5,02916 \text{ (interval between cutting)} \quad (10)$$

$$\text{Log}(\hat{\sigma}) = -0,3453-0,01066 \text{ (interval between cutting)} \quad (11)$$

$$\text{Log}(\hat{\phi}) = 0,7739+0,003976 \text{ (interval between cutting)} \quad (12)$$

$$\hat{\tau} = 12,6 \quad (13)$$

### CONCLUSION

GAMLSS allowed flexible modeling of both the distribution of the dry matter yield of *B. brizantha* and the dependence of all the parameters of the distribution on intervals between cutting. The BCPE distribution provides a flexible model for the dry matter yield of *B. brizantha*, exhibiting both skewness and leptokurtosis. For the dry matter yield from *Brachiaria brizantha*, exhibiting skewness and platykurtosis, the BCPE distribution, provided the best fit. Finally, the results of this research suggested that the interval between cutting has an effect that is reflected in the average yield of dry matter of *B. brizantha*, that is, the yield of dry matter increases as the interval between cutting increases. . Similarly, the interval between cuts affects the skewness and the kurtosis of the distribution.

### SIGNIFICANCE STATEMENT

This study uncovers the potential uses of GAMLSS models in the dynamics of dry matter production that may be beneficial to pasture and forage researchers. This study uncovers the potential uses of GAMLSS models in the dynamics of dry matter production that may be beneficial to pasture and forage researchers. This study will help the pasture and forage researcher to uncover critical areas of accumulated dry matter production of

*Brachiaria brizantha* that many researchers were unable to explore. Therefore, a new theory can be reached on modeling the dynamics of accumulated dry matter production of *B. brizantha*.

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