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Indirect Boundary Element Method for Calculation of Inviscid Compressible Flow Past a Joukowski Aerofoil with Linear Element Approach Using Doublet Distribution Alone

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Abstract: In this paper, an indirect boundary element method is used to give solution for surface as well as ground water bodies which are major nutrition fluids. The inviscid compressible flow (i.e., velocity distribution) over the surface of the joukowski aerofoil has been calculated with linear element approach using doublet distribution alone whereas in our previous research papers, we applied constant boundary element approach for this purpose. To check the accuracy of the method, the computed flow velocity is compared with the exact velocity. The comparison of these results has been given in the tables and graphs. It is found that the computed results are in good agreement with the analytical results.

Key words: Indirect boundary element method, Inviscid compressible flow, Joukowski aerofoil, linear element

INTRODUCTION

Boundary element methods are normally used to find the velocity distribution over the surface of water body which is main nutrition source. In addition to problem related to water resources, this technique can be used to calculate inviscid compressible flow (i.e., velocity distribution) over the surface of a Joukowski aerofoil.

It has always been a struggle in fluid flow modeling to find more efficient numerical methods that can be used to solve a complicated system of Partial Differential Equations (PDE) of fluid flows. The calculation of practical flows was made possible over time by the development of many numerical techniques, such as the finite difference method, the finite element method, the finite volume method and the boundary element method. These methods which have evolved with the discovery of new algorithms and the availability of faster computers, are CPU time and storage hungry. One of the advantages of the boundary element method is that the entire surface of the body has to be discretized, whereas with domain methods it is essential to discretize the entire region of the flow field. The most important characteristics of the boundary element method are the much smaller system of equations and the considerable reduction in data, with the latter being a prerequisite to run a computer program efficiently. These methods have been successfully applied in a number of fields, including elasticity, potential theory, elastostatics and elastodynamics (Brebbia, 1978; Brebbia and Walker, 1980). Furthermore, this method is well suited to problems having an infinite domain. Thus, it is concluded that the boundary element method is a time-saving, accurate and efficient numerical technique compared with other numerical techniques.

The boundary element method can be classified into two categories i.e., direct and indirect. The indirect method utilizes a distribution of singularities over the boundary of the body and computes this distribution as the solution of integral equation. The equation of indirect method can be derived from that of direct method. (Lamb, 1932; Milne-Thomson, 1968; Ramsey, 1942; Kellogg, 1929; Brebbia and Walker, 1980). The indirect method has been used in the past for flow field calculations around arbitrary bodies (Hess and Smith, 1967; Muhammad, 2008; Luminita, 2008; Mushtaq *et al*., 2008, 2009; Mushtaq and Shah, 2010a,b; Mushtaq, 2011; Mushtaq and Shah, 2012). Most of the work on fluid flow calculations using boundary element methods has been done in the field of incompressible flow. Very few attempts have been made on flow field calculations using boundary element methods in the field of compressible flow. In this paper, the indirect boundary element method has been used for the solution of inviscid compressible flows around a Joukowski aerofoil with linear element approach using doublet distribution alone.

MATERIALS AND METHODS

Mathematical formulation: We know that equation of motion for two-dimensional, steady, irrotational and isentropic flow (Mushtaq and Shah, 2010a,b, Mushtaq, 2011; Mushtaq and Shah, 2012) is:

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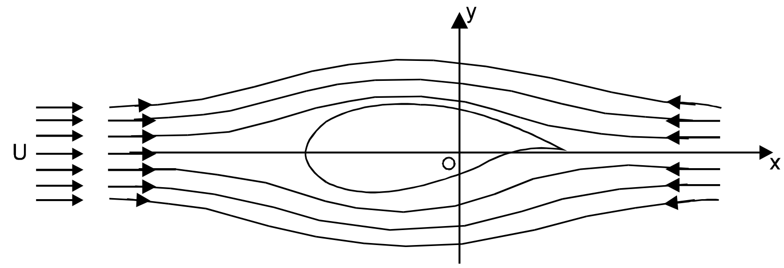

Fig. 1: Flow past a Joukowski aerofoil

$$\left(1-Ma^2\right)\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2} = 0 \quad (1)$$

where, Ma is the Mach number and Φ is the total velocity potential of the flow.
Using the dimension less variables, x = X, y = β Y, where $\beta = \sqrt{1-Ma^2}$ Eq. (1) becomes:

$$\nabla^2\Phi = 0 \quad (2)$$

which is Laplace's equation.

Flow past a joukowski aerofoil: Consider the flow past a Joukowski aerofoil and let the onset flow be the uniform stream with velocity U in the positive direction of the x-axis as shown in Fig. (1).

Exact velocity: The magnitude of the exact velocity distribution over the boundary of a Joukowski aerofoil is given by Chow (1979) and Mushtaq (2011, 2012) is:

$$V = U\left|\frac{1-\dfrac{r^2}{(z-z_1)^2}+\dfrac{2ic}{(z-z_1)}}{1-\left(\dfrac{a}{z}\right)^2}\right|$$

Where:
r = Radius of the cylinder,
a = Joukowski transformation constant
z = x+iy, z_1 = b+I c,

$$b = a-\sqrt{r^2-c^2}$$

In Cartesian coordinates the exact velocity becomes:

$$V = U\frac{\dfrac{[[\{(x-b)^2+(y-c)^2\}^2-r^2\{(x-b)^2-(y-c)^2\}+2c(y-c)\{(x-b)^2+(y-c)^2\}]^2+[2c(x-b)\{(x-b)^2+(y-c)^2\}+2r^2(x-b)(y-c)]^2]^{1/2}}{\left[(x-b)^2+(y-c)^2\right]^2}}{\dfrac{\sqrt{\left[\left(x^2+y^2\right)^2-a^2\left(x^2-y^2\right)\right]^2+4a^4x^2y^2}}{\left(x^2-y^2-a^2\right)^2+4x^2y^2}} \quad (3)$$

Boundary conditions: Now the condition to be satisfied on the boundary of a Joukowski aerofoil is:

$$\overset{\otimes}{V}.\hat{n} = 0 \quad (4)$$

where n̂ is the unit normal vector to the boundary of the aerofoil. Since the motion is irrotational:

$$\overset{\otimes}{V} = -\nabla\Phi$$

where, Φ is the total velocity potential. Thus Eq. (4) becomes:

$$(-\nabla\Phi).\hat{n} = 0$$

or:

$$\frac{\partial\Phi}{\partial n} = 0 \quad (5)$$

Now the total velocity potential Φ is the sum of the perturbation velocity potential $\phi_{j.a}$ where the subscript j.a stands for Joukowski aerofoil and the velocity potential of the uniform stream $\phi_{u.s}$.
i.e.:

$$\Phi = \phi_{u.s}+\phi_{j.a} \quad (6)$$

or:

$$\frac{\partial\Phi}{\partial n} = \frac{\partial\phi_{u.s}}{\partial n}+\frac{\partial\phi_{j.a}}{\partial n} \quad (7)$$

From Eq. (5) and (7), we get:

$$\frac{\partial\phi_{j.a}}{\partial n}+\frac{\partial\phi_{u.s}}{\partial n} = 0$$

or:

$$\frac{\partial\phi_{j.a}}{\partial n} = -\frac{\partial\phi_{u.s}}{\partial n} \quad (8)$$

But the velocity potential of the uniform stream, given in Milne-Thomson (1968), Shah (2008), is:

$$\phi_{u.s} = -U\,x \quad (9)$$

then:

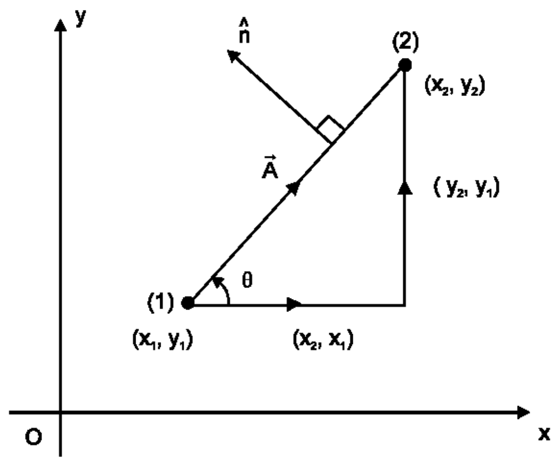

Fig. 2: Unit normal to any vector in the plane

$$\frac{\partial \phi_{u.s}}{\partial n} = -U\frac{\partial x}{\partial n} = -U(\hat{n}.\hat{i}) \tag{10}$$

Thus from Eq. (8) and (10), we get:

$$\frac{\partial \phi_{j.a}}{\partial n} = U(\hat{n}.\hat{i}) \tag{11}$$

Now from the Fig. (2):

$$\overset{\otimes}{A} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

Therefore the unit vector in the direction of the vector $\overset{\otimes}{A}$ is given by:

$$\overset{\otimes}{A} = \frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

The outward unit normal vector $\hat{n}$ to the vector $\overset{\otimes}{A}$ is given by:

$$\hat{n} = \frac{-(y_2 - y_1)\hat{n} + (x_2 - x_1)\hat{j}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Thus:

$$\hat{n}.\hat{i} = \frac{(y_2 - y_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \tag{12}$$

From Eq. (11) and (12), we get:

$$\frac{\partial \phi_{j.a}}{\partial n} = U\frac{(y_1 - y_2)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \tag{13}$$

Equation (13) is the boundary condition which must be satisfied over the boundary of a Joukowski aerofoil.

Equation of indirect boundary element method: The equation of indirect boundary element method for two-dimensional flow in the case of doublet alone (Muhammad, 2008 and Mushtaq, 2008, 2009, 2010, 2011 and 2012) is:

$$-c_i\Phi_i + \frac{1}{2\pi}\int_{\Gamma-i}\Phi\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\Gamma + \phi_\infty = -(\phi_{u.s})_i \tag{14}$$

Where:

c_i = 0 when I is exterior to Γ
= 1 when I is interior to Γ
= When I lies on Γ and Γ is smooth

Matrix formulation for linear element approach: Let the boundary of the region be discretized into m elements, then Eq. (14) can be written as:

$$-c_i\Phi_i + \sum_{j=1}^{m}\left[\frac{1}{2\pi}\int_{\Gamma_j-i}\Phi\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\Gamma\right] + \Phi_\infty = -(\phi_{u.s})_i \tag{15}$$

where Γ_j- I is the length of the element 'j' excluding the point 'I'.

For the linear boundary element approach, the number of nodes will be more than the number of elements. Suppose that m is the number of nodes in this case. Since Φ varies linearly over the element, its value at any point can be defined in terms of the nodal values and the two shape functions N_1, N_2, that is:

$$\Phi = N_1\Phi_1 + N_2\Phi_2 = [N_1\ N_2]\begin{Bmatrix}\Phi_1\\ \Phi_2\end{Bmatrix} \tag{16}$$

Where:

$$N_1 = \frac{1}{2}(1-\delta)$$

and:

$$N_2 = \frac{1}{2}(1+\delta),\ -1 \le \delta \le 1$$

The integrals along the element 'j' i.e.:

$$\frac{1}{2}\int_{\Gamma_j-i}\Phi\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\Gamma$$

can be written as:

$$\frac{1}{2}\int_{\Gamma_j-i}\Phi\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\Gamma = \frac{1}{2\pi}\int_{\Gamma_j-i}[N_1\ N_2]\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\Gamma\begin{Bmatrix}\Phi_1\\ \Phi_2\end{Bmatrix} = \left[h_{ij}^1\ h_{ij}^2\right]\begin{Bmatrix}\Phi_1\\ \Phi_2\end{Bmatrix} \tag{17}$$

Where:

$$h_{ij}^1 = \frac{1}{2}\int_{\Gamma_j-i}\Phi_1\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\Gamma,\ h_{ij}^2 = \frac{1}{2\pi}\int_{\Gamma_j-i}\Phi_2\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\Gamma$$

or:

$$h_{ij}^{k} = \frac{1}{2}\int_{\Gamma_{j-i}} k\frac{\partial}{\partial n}\left(\log\frac{1}{r}\right)d\Gamma \qquad k = 1, 2 \qquad (18)$$

The h^{k}_{ij} are influence coefficients during the interaction between the point 'I' under consideration and a particular node k on an element 'j'.

To write the Eq. (15) corresponding to the node 'I' the contributions from all elements associated with the node 'I' are to be added into one term, defining the nodal coefficients. This will give the following Eq.:

$$-c_i\Phi_i + \left[\hat{H}_{i1}\ \hat{H}_{i2}..........\hat{H}_{im}\right]\begin{Bmatrix}\Phi_1\\ \Phi_2\\ .\\ .\\ .\\ \Phi_m\end{Bmatrix} + \phi_\infty = -(\phi_{u.s})_i \qquad (19)$$

where, $\hat{H}_{ij}$ term is the sum of the contributions from all the adjoining elements of the node 'I'. Hence Eq. (19) represents the assembled equation for node 'I' and can be written as:

$$-c_i\Phi_i + \sum_{j=1}^{m}\hat{H}_{ij}\Phi_j + \phi_\infty = -(\phi_{u.s})_i \qquad (20)$$

or:

$$\sum_{j=1}^{m} H_{ij} + \phi_\infty = -(\phi_{u.s})_i \qquad (21)$$

Where:

$$H_{ij} = \begin{cases}\hat{H}_{ij} & \text{when } i \neq j\\ \hat{H}_{ij} - c_i & \text{when } i = j\end{cases}$$

When all nodes are taken into consideration, Eq. (21) is M x (M+1) system of equations . Which can put in the matrix form in case of linear element as:

[H] {U} = {R} (22)

where as usual [H] is a matrix of influence coefficients, {U} is a vector of unknown total potentials Φ_i and {R} on the R.H.S. is a known vector whose elements are the negative of the values of the velocity potential of the uniform stream at the nodes on the region of the body. Note that {U} in Eq. (22) has (M+1) unknowns Φ_1, Φ_2, …., Φ_m, ϕ_∞. To solve precisely this system of equations, the value of Φ at some position must be specified. For convenience ϕ_∞ is chosen as zero. Thus M x (M+1) system reduces to an M x M system of equations which can be solved as before but now the diagonal coefficients of [H] will be found by:

$$H_{ij} = -\sum_{\substack{j=1\\ j\neq 1}} H_{ij} - 1 \qquad (23)$$

Process of discretization: Now for the discretization of the boundary of the Joukowski aerofoil, the coordinates of the extreme points of the boundary elements can be generated within computer programme using Fortran language as follows:

Divide the boundary of the circular cylinder into m elements in the clockwise direction by using the formula:

$$\theta_k = \left[(m+2) - 2k\right]\frac{\pi}{m}, \quad k = 1, 2, ..., m \qquad (24)$$

Then the extreme points of these m elements of circular cylinder are found by:

$$\xi\ k = -b + r\cos\theta_k$$

$$\eta\ k = c + r\sin\theta_k$$

Now by using Joukowski transformation in Eq. (3), the extreme points of the Joukowski aerofoil are:

$$x_k = \xi_k\left(1 + \frac{a^2}{\xi_k^2 + \eta_k^2}\right)$$

$$y_k = \eta_k\left(1 - \frac{a^2}{\xi_k^2 + \eta_k^2}\right)$$

where, k = 1, 2, ……., m.

The coordinates of the middle node of each boundary element are given by:

$$\left.\begin{aligned} x_m &= \frac{x_k + x_k + 1}{2}\\ y_m &= \frac{y_k + y_k + 1}{2}\end{aligned}\right\} \quad k, m = 1, 2,....., n \qquad (25)$$

and therefore the boundary condition (13) in this case takes the form:

$$\frac{\partial\phi_{j.a}}{\partial n} = U\frac{(y_1)_m - (y_2)_m}{\sqrt{\left[(x_2)_m - (x_1)_m\right]^2 + \left[(y_2)_m - (y_1)_m\right]^2}} \qquad (26)$$

RESULTS AND DISCUSSION

The FORTRAN language has been used to compute data which are given in Tables 1-4. This data has been applied for various conditions such as R = 7.5, a = 0.2, Ma = 0.7 and c = 0.15

The boundary has been divided in to 8 elements given in Table 1. On the basis of these elements, two relations have been established (Fig. 3, 4) which shows the comparison of computed and analytical results over the boundary of a Joukowski Aerofoil.

With the increase of elements up to 16 given in Table 2, the relation generated for computed and analytical velocity has been developed given in Fig. 5-6.

The total increase was taken up to 32 and 64 elements and the results composed in Fig. 7-10 are indicating the

Table 1: Computed and analytical velocity distribution comparison over the surface of a Joukowski aerofoil for 8 linear boundary elements

Element	X	Y	$R = \sqrt{X_2 + Y_2}$	Computed velocity	Exact velocity
1	-13.70	2.80	13.99	0.71995E+00	0.88787E+00
2	-9.95	6.55	11.91	0.17376E+01	0.20507E+01
3	-4.65	6.55	8.03	0.17355E+01	0.20519E+01
4	-0.83	2.75	2.87	0.71497E+00	0.85796E+00
5	-0.83	-2.55	2.68	0.73896E+00	0.79150E+00
6	-4.65	-6.25	7.79	0.17342E+01	0.19655E+01
7	-9.95	-6.25	11.75	0.17372E+01	0.19642E+01
8	-13.70	-2.50	13.93	0.71962E+00	0.80282E+00

Table 2: Computed and analytical velocity distribution comparison over the surface of a Joukowski aerofoil for 16 linear boundary elements

Element	X	Y	$R = \sqrt{X_2 + Y_2}$	Computed velocity	Exact velocity
1	-14.52	1.58	14.60	0.38469E+00	0.44004E+00
2	-13.42	4.24	14.07	0.10954E+01	0.11738E+01
3	-11.39	6.26	13.00	0.16394E+01	0.17365E+01
4	-8.74	7.36	11.42	0.19336E+01	0.20416E+01
5	-5.87	7.36	9.41	0.19333E+01	0.20422E+01
6	-3.21	6.26	7.04	0.16381E+01	0.17372E+01
7	-1.18	4.23	4.39	0.10908E+01	0.11719E+01
8	-0.02	1.53	1.53	0.38073E+00	0.41176E+00
9	-0.02	-1.33	1.33	0.40688E+00	0.35349E+00
10	-1.18	-3.93	4.10	0.10897E+01	0.10902E+01
11	-3.21	-5.96	6.77	0.16379E+01	0.16557E+01
12	-5.87	-7.06	9.18	0.19332E+01	0.19606E+01
13	-8.74	-7.06	11.23	0.19336E+01	0.19600E+01
14	-11.39	-5.96	12.86	0.16394E+01	0.16549E+01
15	-13.42	-3.94	13.98	0.10954E+01	0.10922E+01
16	-14.52	-1.28	14.57	0.38466E+00	0.35887E+00

Table 3: Computed and analytical velocity distribution comparison over the surface of a Joukowski aerofoil for 32 linear boundary elements

Element	X	Y	$R = \sqrt{X_2 + Y_2}$	Computed velocity	Exact velocity
1	-14.73	0.88	14.76	0.19540E+00	0.23719E+00
2	-14.44	2.32	14.63	0.57869E+00	0.62324E+00
3	-13.88	3.67	14.36	0.93973E+00	0.98704E+00
4	-13.07	4.88	13.95	0.12646E+01	0.13146E+01
5	-12.04	5.92	13.41	0.15410E+01	0.15933E+01
6	-10.82	6.73	12.74	0.17580E+01	0.18124E+01
7	-9.47	7.29	11.95	0.19075E+01	0.19636E+01
8	-8.03	7.58	11.04	0.19836E+01	0.20408E+01
9	-6.57	7.57	10.03	0.19834E+01	0.20411E+01
10	-5.13	7.29	8.92	0.19070E+01	0.19643E+01
11	-3.78	6.73	7.72	0.17572E+01	0.18132E+01
12	-2.57	5.91	6.45	0.15396E+01	0.15937E+01
13	-1.53	4.88	5.11	0.12627E+01	0.13138E+01
14	-0.72	3.66	3.73	0.93663E+00	0.98382E+00
15	-0.16	2.30	2.30	0.57168E+00	0.61492E+00
16	0.19	0.82	0.84	0.19080E+00	0.20753E+00
17	0.19	-0.61	0.64	0.21771E+00	0.15032E+00
18	-0.16	-1.99	2.00	0.56986E+00	0.53302E+00
19	-0.72	-3.36	3.43	0.93621E+00	0.90311E+00
20	-1.53	-4.58	4.83	0.12626E+01	0.12333E+01
21	-2.57	-5.61	6.17	0.15396E+01	0.15133E+01
22	-3.78	-6.43	7.46	0.17572E+01	0.17329E+01
23	-5.13	-6.99	8.67	0.19070E+01	0.18839E+01
24	-6.57	-7.27	9.80	0.19835E+01	0.19607E+01
25	-8.03	-7.28	10.84	0.19836E+01	0.19604E+01
26	-9.47	-6.99	11.77	0.19075E+01	0.18832E+01
27	-10.82	-6.43	12.59	0.17580E+01	0.17321E+01
28	-12.04	-5.62	13.28	0.15410E+01	0.15129E+01
29	-13.07	-4.58	13.85	0.12647E+01	0.12342E+01
30	-13.88	-3.37	14.29	0.93974E+00	0.90667E+00
31	-14.44	-2.02	14.58	0.57869E+00	0.54288E+00
32	-14.73	-0.58	14.74	0.19541E+00	0.15692E+00

Table 4: Computed and analytical velocity distribution comparison over the surface of a Joukowski aerofoil for 64 linear boundary elements

Element	X	Y	$R = \sqrt{X_2 + Y_2}$	Computed velocity	Exact velocity
1	-14.78	0.52	14.79	0.98076E-01	0.13823E+00
2	-14.71	1.25	14.76	0.29328E+00	0.33365E+00
3	-14.57	1.97	14.70	0.48563E+00	0.52625E+00
4	-14.35	2.67	14.60	0.67333E+00	0.71420E+00
5	-14.07	3.35	14.47	0.85454E+00	0.89567E+00
6	-13.73	4.00	14.30	0.10275E+01	0.10689E+01
7	-13.32	4.61	14.09	0.11906E+01	0.12323E+01
8	-12.85	5.18	13.86	0.13422E+01	0.13842E+01
9	-12.33	5.70	13.59	0.14809E+01	0.15233E+01
10	-11.76	6.17	13.28	0.16053E+01	0.16480E+01
11	-11.15	6.57	12.95	0.17142E+01	0.17573E+01
12	-10.50	6.92	12.58	0.18066E+01	0.18501E+01
13	-9.82	7.20	12.18	0.18817E+01	0.19255E+01
14	-9.12	7.41	11.75	0.19386E+01	0.19828E+01
15	-8.40	7.56	11.30	0.19768E+01	0.20213E+01
16	-7.67	7.63	10.82	0.19959E+01	0.20407E+01
17	-6.93	7.63	10.31	0.19959E+01	0.20408E+01
18	-6.20	7.56	9.78	0.19765E+01	0.20217E+01
19	-5.48	7.41	9.22	0.19382E+01	0.19833E+01
20	-4.78	7.20	8.64	0.18811E+01	0.19263E+01
21	-4.10	6.92	8.04	0.18059E+01	0.18509E+01
22	-3.45	6.57	7.42	0.17133E+01	0.17581E+01
23	-2.84	6.16	6.78	0.16041E+01	0.16485E+01
24	-2.27	5.69	6.13	0.14795E+01	0.15234E+01
25	-1.75	5.17	5.46	0.13405E+01	0.13838E+01
26	-1.28	4.60	4.78	0.11886E+01	0.12310E+01
27	-0.88	3.99	4.09	0.10250E+01	0.10665E+01
28	-0.53	3.34	3.38	0.85143E+00	0.89163E+00
29	-0.25	2.66	2.67	0.66929E+00	0.70787E+00
30	-0.03	1.95	1.95	0.47998E+00	0.51626E+00
31	0.12	1.21	1.22	0.28385E+00	0.31642E+00
32	0.25	0.45	0.51	0.91189E-01	0.10948E+00
33	0.26	-0.23	0.35	0.11812E+00	0.61689E-01
34	0.12	-0.90	0.91	0.28089E+00	0.23031E+00
35	-0.03	-1.65	1.65	0.47814E+00	0.43415E+00
36	-0.25	-2.36	2.37	0.66849E+00	0.62688E+00
37	-0.53	-3.04	3.09	0.85101E+00	0.81110E+00
38	-0.88	-3.69	3.79	0.10248E+01	0.98617E+00
39	-1.28	-4.30	4.49	0.11884E+01	0.11508E+01
40	-1.75	-4.87	5.18	0.13405E+01	0.13037E+01
41	-2.27	-5.39	5.85	0.14795E+01	0.14433E+01
42	-2.84	-5.86	6.51	0.16041E+01	0.15685E+01
43	-3.45	-6.27	7.16	0.17133E+01	0.16780E+01
44	-4.10	-6.62	7.78	0.18059E+01	0.17709E+01
45	-4.78	-6.90	8.39	0.18812E+01	0.18462E+01
46	-5.48	-7.11	8.98	0.19382E+01	0.19033E+01
47	-6.20	-7.26	9.55	0.19766E+01	0.19416E+01
48	-6.93	-7.33	10.09	0.19959E+01	0.19607E+01
49	-7.67	-7.33	10.61	0.19960E+01	0.19606E+01
50	-8.40	-7.26	11.10	0.19768E+01	0.19412E+01
51	-9.12	-7.11	11.57	0.19386E+01	0.19026E+01
52	-9.82	-6.90	12.01	0.18817E+01	0.18454E+01
53	-10.50	-6.62	12.42	0.18067E+01	0.17700E+01
54	-11.15	-6.27	12.80	0.17143E+01	0.16772E+01
55	-11.76	-5.87	13.14	0.16053E+01	0.15679E+01
56	-12.33	-5.40	13.46	0.14809E+01	0.14432E+01
57	-12.85	-4.88	13.75	0.13422E+01	0.13042E+01
58	-13.32	-4.31	14.00	0.11906E+01	0.11522E+01
59	-13.73	-3.70	14.22	0.10275E+01	0.98885E+00
60	-14.07	-3.05	14.40	0.85453E+00	0.81559E+00
61	-14.35	-2.37	14.55	0.67337E+00	0.63412E+00
62	-14.57	-1.67	14.66	0.48564E+00	0.44618E+00
63	-14.71	-0.95	14.74	0.29328E+00	0.25358E+00
64	-14.78	-0.22	14.78	0.98072E-01	0.58174E-01

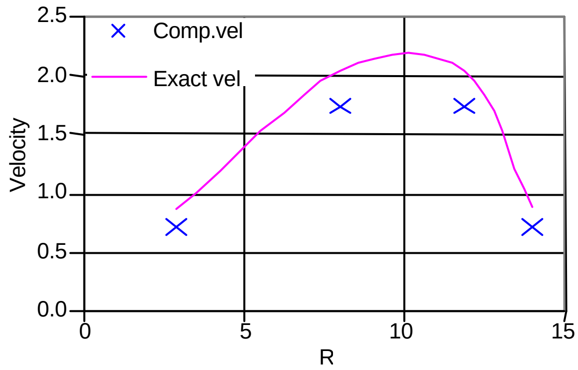

Fig. 3: Computed and analytical velocity distributions comparison over the boundary of a Joukowski aerofoil using upper 4 values of 8 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

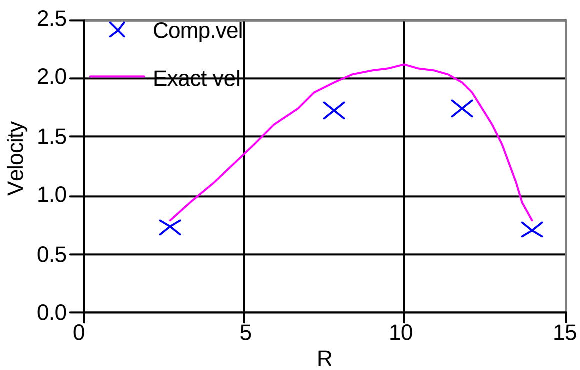

Fig. 4: Computed and analytical velocity distributions comparison over the boundary of a Joukowski aerofoil using lower 4 values of 8 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

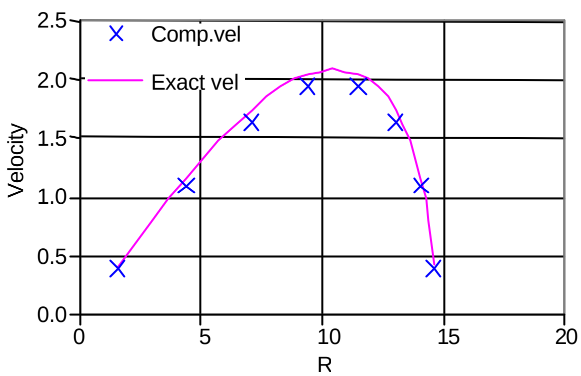

Fig. 5: Computed and analytical velocity distributions comparison over the boundary of a Joukowski aerofoil using upper 8 values of 16 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

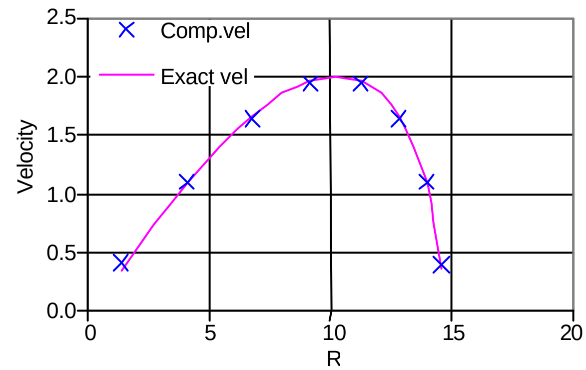

Fig. 6: Computed and analytical velocity distributions comparison over the boundary of a Joukowski aerofoil using lower 8 values of 16 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

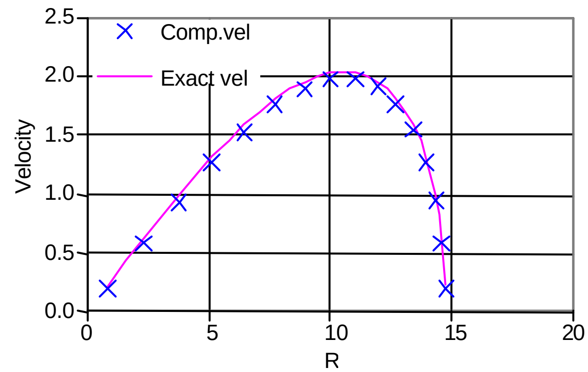

Fig. 7: Computed and analytical velocity distributions comparison over the boundary of a Joukowski aerofoil using upper 16 values of 32 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

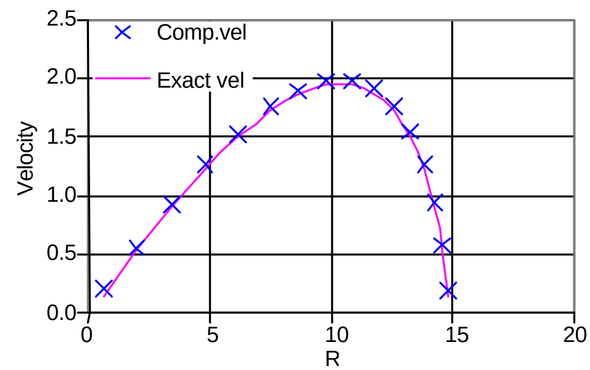

Fig. 8: Computed and analytical velocity distributions comparison over the boundary of a Joukowski aerofoil using lower 16 values of 32 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

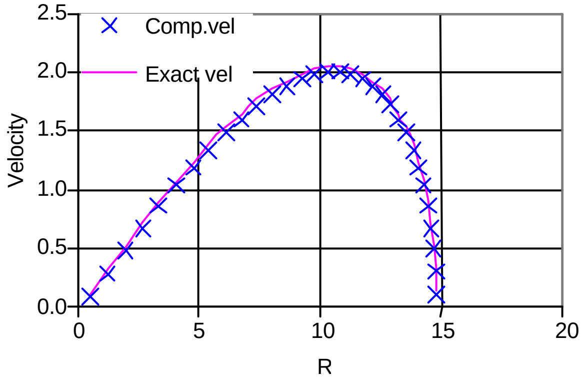

Fig. 9: Computed and analytical velocity distributions comparison over the boundary of a Joukowski aerofoil using upper 32 values of 64 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

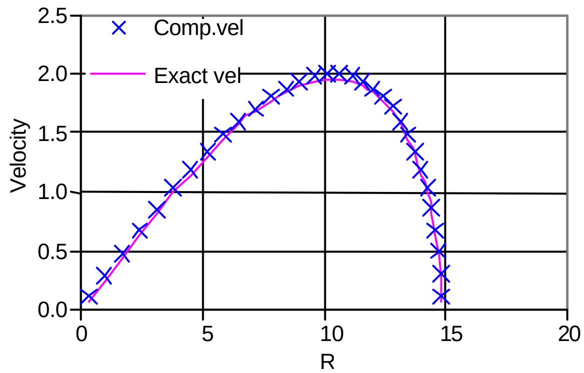

Fig. 10: Computed and analytical velocity distributions comparison over the boundary of a Joukowski aerofoil using lower 32 values of 64 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

efficiency of boundary element method in computing the velocity which is reflected by smoothness of the curve.

Boundary element methods are very frequently used for inviscid fluids to estimate various aspects using linear approach. These types of solutions are very efficient and can be directly applied to study surface as well as ground water bodies which are mostly under threat of contamination. Hence to solve the contamination problem, its movements in the major nutrition body are being studied to control water pollution to save life.

Chow (1979) has proposed analytical methods to solve the problems related to fluid flow such as flow towards wells and migration of contaminants in groundwater movements during pumping out from deep horizons. These methods are very lengthy and involve complicated calculations. In this research paper, we are proposing a new numerical approach to solve the compressible fluid flow problems. The comparison made into Chow (1979) results is very similar and given in Table 1-4 and graphs 3-10 for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7. Our method is time saving and the degree of accuracy is very good.

Hence the tables and figures generated for different conditions show that the results are very well comparable with the analytically results. Therefore the indirect boundary element method can be applied to surface and underground water bodies which are main nutrition source to solve contamination problems.

Conclusion and recommendations: An indirect boundary element method has been applied for the calculation of inviscid compressible flow past a Joukowski aerofoil with linear element approach using doublet distribution alone. The calculated flow velocities obtained using this method is compared with the analytical solutions for flow over the boundary of a Joukowski aerofoil. It is found that from the tables and graphs, the computed results obtained by this method are good in agreement with the analytical ones for the body under consideration and the accuracy of the result increases due to increase of number of boundary elements.

Thus the indirect boundary element method is being suggested to deal the fluid flow problems such as tides, turbulent or laminar thrust related to the submarines and other shipping vehicles.

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