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# Non-Linear Regression Models to Predict the Lamb and Sheep Weight Growth

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**Abstract:** The study of living organism growth is the hot issue now a days in biological sciences. In this research, we explore the descriptive analysis of Lamb and Sheep weight growth in Pakistan. Also the non-linear regression models are identified on the basis of model selection criteria's for lamb and sheep weight growth prediction. From the analysis, we have found that for commercial sheep, spring born lambs and twin born lambs, the best model is the Von Bertalanffy model. While for predicting the weight of male commercial sheep, autumn born and single born lambs, the best model is the Brody model.

**Key words:** Growth curve, sheep weight, lamb weight, non-linear regression modeling

## INTRODUCTION

Growth of living cells, tissues, organs and organisms is a biological phenomenon and can be explained in terms of mathematical terms. Increase in number of cells and increase in size of cells results in overall increase in mass of living tissue. This increase is not always steady and linear but has some non-linear fashion. Therefore, it may be divided into phases to explain it.

Many researcher defined growth in their own way. Growth, one of the most essential traits for animals, is defined as an increase in tissues and organs of the animals per unit time and effected by genetic and environmental factors (Tariq *et al*., 2011). The growth that has sigmoid form is explained reliably by non-linear growth models such as Gompertz, logistic, Richards, Weibull, Monomolecular, Brody and von Bertalanffy. Information about parameters of these non linear models enables researchers to obtain beneficial clues for selection studies.

Growth is a trait of interest in the domestic complex subject which has been studied through many different approaches. A widely used approach is to fit growth data with mathematical equations or growth curve equations. Those functions are based on deterministic differential equations that seek a biological interpretation. Even though growth is a variable among individuals, it follows a well defined course in populations of animals with age. Generally; growth follows a sigmoid or s-shaped curve through the growth rate which varies with age. The rate slowly declines to zero reaching a plateau when the animal achieves mature weight (Arango and van Vleck, 2002).

Different models have been developed by researchers for studying growth and other such attributes. For instance Benjamin Gompertz (1825) developed Gompertz model to calculate mortality rates. Today this model is most frequently applied to study biological growth. A gompertz curve or gompertz function is a sigmoid function. It is a type of mathematical model for a time series, where growth is slowest at the start and of a time period.

Other important function is the Logistic curve. It was developed by Verhulst (1838) as a model for population growth. In this function inflection point is independent on measurement. It often applied for sigmoid growth where the inflection is located at approximately half of the ultimate value and is closely related to the Hubert Curve. In statistics, logistic regression (sometimes called the logistic model or logit model) is used for prediction of the probability of occurrence of an event by fitting data to a logistic function. It is a generalized linear model used for binomial regression.

Bertalanffy model was developed by Bertalanffy (1957) as a model for body weight growth. The point of inflection is fixed at 8/27 or 29.63% of the maximum value. It is suitable for sigmoid growth with inflection points around 30% of the ultimate.

In Brody curve the inflection points occurs between the two curves. It was derived by S. Brody as models for Piecewise growth process of exponential type. The increasing function is only valid in temporarily limited intervals and can be extrapolated. The decreasing function can be extrapolated and is suitable for monotonous decrease.

Wildeus (1997) conducted the study to explore the growth performance of different breeds of sheep in US. Lancelot *et al*. (2000) explored the different factors which are affecting the growth sheep. They use multi-level modeling and have found that Age, litter-size, age xlitter-size, litter-sizextreatment and age xlitter-sizex treatment are the significant factors for growth of sheep. Lamb *et al*. (2008) fitted logistic, Gompertz, Richards and exponential models and linear regression models to describe the growth of two breeds of lambs from birth to slaughter. On the basis of model selection criteria's $R^2$

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and AIC they have found that the best fitted models for both types of breeds were logistic, Gompertz and Richards models. Gbangboche *et al*. (2008) conducted a study on application of non-linear models to growth. The objective of this study were to compare the goodness of fit of four non-linear growth models, i.e., Brody, Gompertz, Logistic and von Bertalanffy, in west African dwarf (WAD) sheep. On the basis of model selection criteria's they have found that good fitted model for WAD sheep is the brody model. Kucuk and Eyduran (2010) aimed to determine the most suitable growth model for Akkaraman and German Blackheaded Mutton x Akkaraman B1 cross breed lambs using various growth models i.e., Monomolecular, Logistic and Gompertz growth models. They have found that the most suitable model based on model selection criteria's is the monomolecular model. Ozdemir and Dellal (2009) conducted a study to analyze the growth curves of young Angora goats by using live weight data during period between birth and 12th months. For this purpose, they use Logistic and Gompertz models which are non-linear growth models. Behzadi and Aslaminejad (2010) had applied Artificial Neural Network (ANN) for predicting a growth of baluchi sheep. They compare ANN with other regression models and found that ANN is best as compared to other model for predicting baluchi sheep growth. Ulutas *et al*. (2010) reported that type of birth affect the growth of sheep. They compare the growth characteristics of single and twin birth lambs in Karayaka sheep which is an indigenous breed of the northern part of Turkey. They have found that Single birth lamb of both sexes showed lower asymptotic weight than the twin birth ones. There were a noticeable difference in the absolute growth rate between birth types before inflection point but decline after the inflection point was slower for twins than that for singles. Similarly, the decrease in relative growth rate was higher for singles than that for twins. Gurcan *et al*. (2012) conducted the study to determine the relationship between growth and live body weight in Turkey. For this purpose, they use negative exponential, Brody, Gompertz, Logistic and Von Bertalanffy models to determine the growth characteristics of quail. On the basis of model selection criteria's $R^2$, Mean Squared Error (MSE) etc., they have found that the best model for quail is logistic model. In Pakistan little work is done on growth modeling in animals and/or livestock species especially the commercial animals. Therefore, the present study was planned to achieve the following objectives: In general application of mathematical models to study growth pattern in living organisms.

**Objectives of present study**
**In particular, the following points were focused:**

1. To estimate growth curve parameters of different non-linear regression models in commercial sheep
2. To compare non-linear regression models using suitable criteria
3. To find out the best fit model for studying growth
4. To compute non-linear prediction equation for commercial sheep

## MATERIALS AND METHODS

**Experimental material:** The study was conducted on commercial sheep found in commercial markets. Growth performance data were collected on monthly weights from birth to one year of age for fitting growth curve. The live weights of in some lambs were recorded from birth till two years of age.

There were three years and two seasons for birth of lambs i.e., spring and autumn and two types of births were found in lambs: single and twin. Male and female were the sexes of lambs. These data structure is presented in Table 1.

Table 1: Data structure of performance records of commercial sheep

| Performance record | No. of observations |
|---|---|
| Over all | 5265 |
| Male | 2600 |
| Female | 2665 |
| Spring season | 780 |
| Autumn | 4485 |
| Single | 4550 |
| Twin | 715 |

**Statistical tools for Analysis:** For the analysis purpose to explore the data of lamb's weight, we use descriptive statistics i.e., mean, Standard Deviation (SD) and CV for overall and for each type and sex and for each year. We also use some growth models to predict the lamb's weights. These models are given in the Table 2.

We use SPSS20 statistical software for the analysis purposes.

Table 2: Growth models

| Model | Equation |
|---|---|
| Gompertz | Wt = A*exp (-b * exp (-ct) |
| Logistic | Wt = a/[1+b*exp (-ct)] |
| Von bertalanffy | Wt= a* [1-b*exp (-ct)]$^3$ |
| Brody | Wt = a* (1-b*exp (-ct) |

Where a, b, c are parameters of growth curve. "a" represents asymptotic weight, "b" point of inflection/rate of body weight gained after birth to mature body weight and "c" rate of growth/maturity rate. Model selection criteria's $R^2$, MSE, MAPE and MAD are used for selecting suitable model

## RESULTS AND DISCUSSION

Data on growth performance were analyzed and results about descriptive statistics, parameters estimates and prediction equations are presented here in numeric and figure forms. Over all means for growth performance traits presented in Table 3 show that average weight of lambs ranged from 3.5-33.6 kilograms with a CV of 8.9-

Table 3: Weight growth of commercial sheep

| | Commercial sheep | | | Male sheep | | | Female sheep | | |
|---|---|---|---|---|---|---|---|---|---|
| Weight | Mean | SD | CV | Mean | SD | CV | Mean | SD | CV |
| Birth weight | 3.53 | 0.31 | 8.94 | 3.61 | 0.33 | 9.10 | 3.47 | 0.29 | 8.32 |
| 30 days | 7.94 | 0.78 | 9.84 | 8.19 | 0.81 | 9.95 | 7.7 | 0.67 | 8.67 |
| 60 days | 11.89 | 1.19 | 10.03 | 12.36 | 1.23 | 9.96 | 11.43 | 0.96 | 8.37 |
| 90 days | 15.86 | 1.64 | 10.39 | 16.49 | 1.72 | 10.40 | 15.24 | 1.32 | 8.64 |
| 120 days | 19.1 | 2.12 | 11.11 | 20.1 | 2.16 | 10.77 | 18.12 | 1.55 | 8.56 |
| 150 days | 21.00 | 2.54 | 12.10 | 22.21 | 2.67 | 12.03 | 19.83 | 1.73 | 8.74 |
| 180 days | 23.68 | 3.54 | 14.95 | 25.36 | 3.67 | 14.49 | 22.05 | 2.5 | 11.34 |
| 210 days | 26.24 | 3.38 | 12.90 | 28.02 | 3.32 | 11.83 | 24.51 | 2.42 | 9.86 |
| 240 days | 27.35 | 3.52 | 12.89 | 29.21 | 3.36 | 11.52 | 25.54 | 2.63 | 10.31 |
| 270 days | 28.18 | 3.47 | 13.30 | 30.3 | 3.6 | 11.89 | 26.11 | 2.54 | 9.72 |
| 300 days | 30.78 | 3.73 | 12.13 | 33.01 | 3.44 | 10.44 | 28.61 | 2.54 | 8.88 |
| 330 days | 32.53 | 3.79 | 11.65 | 34.81 | 3.45 | 9.91 | 30.31 | 2.61 | 8.62 |
| Wt 1 year | 33.66 | 3.77 | 11.21 | 35.98 | 3.46 | 9.62 | 31.4 | 2.47 | 7.86 |
| 1 ½ Year | | | | 42.25 | 4.23 | 10.02 | 35.12 | 2.78 | 7.92 |
| 2 Year | | | | 45.65 | 8.65 | 18.95 | 37.88 | 4.85 | 12.80 |

Table 4: Weight growth of born sheep with season and types of birth

| | Spring born sheep | | | Autumn born sheep | | | Single born sheep | | | Twins born sheep | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Weight | Mean | SD | CV | Mean | SD | CV | Mean | SD | CV | Mean | SD | CV |
| Birth weight | 3.51 | 0.27 | 7.65 | 3.54 | 0.32 | 9.15 | 3.62 | 0.24 | 6.49 | 2.96 | 0.11 | 3.83 |
| 30 days | 7.77 | 0.7 | 8.97 | 7.97 | 0.79 | 9.94 | 8.13 | 0.65 | 8.02 | 6.75 | 0.43 | 6.31 |
| 60 days | 11.58 | 1.14 | 9.87 | 11.95 | 1.2 | 10.01 | 12.13 | 1.07 | 8.83 | 10.36 | 0.73 | 7.01 |
| 90 days | 15.36 | 1.53 | 9.93 | 15.95 | 1.66 | 10.38 | 16.17 | 1.51 | 9.35 | 13.87 | 0.96 | 6.94 |
| 120 days | 18.48 | 1.73 | 9.38 | 19.21 | 2.17 | 11.30 | 19.46 | 1.98 | 10.16 | 16.79 | 1.5 | 8.96 |
| 150 days | 20.24 | 1.99 | 9.81 | 21.14 | 2.61 | 12.33 | 21.38 | 2.42 | 11.32 | 18.57 | 1.87 | 10.10 |
| 180 days | 22.78 | 2.94 | 12.89 | 23.85 | 3.62 | 15.18 | 24.08 | 3.52 | 14.63 | 21.16 | 2.49 | 11.76 |
| 210 days | 25.29 | 3 | 11.85 | 26.42 | 3.43 | 12.97 | 26.64 | 3.3 | 12.40 | 23.72 | 2.79 | 11.78 |
| 240 days | 26.12 | 2.99 | 11.46 | 27.58 | 3.57 | 12.96 | 27.76 | 3.46 | 12.48 | 24.78 | 2.81 | 11.34 |
| 270 days | 26.75 | 3.14 | 11.74 | 28.44 | 3.8 | 13.35 | 28.56 | 3.73 | 13.05 | 25.74 | 2.89 | 11.24 |
| 300 days | 29.47 | 3.07 | 10.42 | 31.02 | 3.8 | 12.25 | 31.13 | 3.71 | 11.93 | 28.54 | 3.06 | 10.73 |
| 330 days | 31.31 | 3.15 | 10.05 | 32.76 | 3.86 | 11.79 | 32.86 | 3.8 | 11.56 | 30.43 | 3.02 | 9.91 |
| Wt 1 year | 32.87 | 3.56 | 10.83 | 33.81 | 3.8 | 11.23 | 33.94 | 3.83 | 11.29 | 31.93 | 2.85 | 8.94 |
| 1 ½ Year | 37.78 | 4.75 | 12.57 | 38.81 | 5.09 | 13.11 | 38.88 | 5.16 | 13.28 | 37.21 | 3.96 | 10.64 |
| 2 Year | 41.2 | 5.19 | 12.60 | 41.7 | 8.35 | 20.02 | 41.85 | 7.95 | 18.99 | 40.15 | 7.91 | 19.70 |

14.9%. Ranges for all performance traits were having two clear-cut bounds (lower and upper) show wider gaps and lot of variation. The overall growth performance of commercial sheep is presented in Table 4.1. It shows that a total growth of 30.13 kg has taken place. C.V of growth performance ranged from 8.9-14.9%.

The growth performance of commercial male sheep is presented also in Table 3. It shows that a total growth of 42.04 kg has taken place. C.V of growth performance ranged from 9.10-18.95%. The growth rate appeared to be 0.115 kg. The growth performance of commercial female sheep is presented in Table 3. It shows that a total growth of 34.41 kg has taken place. C.V of growth performance ranged from 8.32-12.80%. The growth performance in commercial spring born sheep is presented in Table 4. It shows that a total growth of 24.07 kg has taken place. C.V of growth performance ranged from 7.65-12.89%. The growth performance in commercial autumn born sheep is presented in Table 4. It shows that a total growth of 38.16 kg has taken place. C.V of growth performance ranged from 9.15-20.02%. The growth performance in single born sheep is presented in Table 4. It shows that a total growth of 38.23 kg has taken place. C.V of growth performance ranged from 6.49-18.99%. The growth performance in commercial twin born sheep is presented in Table 4. It shows that a total growth of 37.19 kg has taken place. C.V of growth performance ranged from 3.83-19.70%.

**Growth modeling for sheep and lambs:** From the Table 5, we have found that Von Bertalanffy model is the most suitable model to predict commercial sheep weight because it has a greatest $R^2$ and lower MSE and MAD as compared to others growth models.

Table 6 indicated that The Brody model was found to be most suitable model for predicting the weight of commercial male sheep because it has larger $R^2$ and lower MAD as compared to other growths models.

Table 7 shows that based on $R^2$, the best fitted model is the Brody model for predicting commercial female sheep weight and also lower MAPE as compared to other non-linear regression models. While on the basis of MSE the best fitted model is VonBertalanffy because it has lower MSE and MAD as compared to other non-linear growth models.

Table 5: Parameters for growth curve in commercial sheep using non-linear regression models

| Models | A | b | C | $R^2$ | BIAS | MAPE | MSE | MAD |
|---|---|---|---|---|---|---|---|---|
| Gompertz | 37.71 | 1.86 | 0.24 | 0.996 | 0.04 | 0.02 | 0.32 | 0.49 |
| Logistic | 35.65 | 4.00 | 0.36 | 0.991 | 0.18 | 0.03 | 0.64 | 0.65 |
| Von bertalanffy | 38.95 | 0.48 | 0.20 | 0.997 | 0.24 | 0.02 | 0.29 | 0.46 |
| Brody | 43.64 | 0.92 | 0.12 | 0.99 | 0.47 | 0.01 | 0.34 | 0.49 |

Table 6: Parameters for growth curve of commercial male sheep using non-linear regression models

| Models | a | b | C | $R^2$ | BIAS | MAPE | MSE | MAD |
|---|---|---|---|---|---|---|---|---|
| Gompertz | 38.46 | 1.78 | 0.62 | 0.992 | -0.37 | 0.03 | 0.74 | 0.73 |
| Logistic | 36.78 | 3.67 | 0.37 | 0.987 | -0.01 | 0.04 | 0.95 | 0.83 |
| Von bertalanffy | 39.43 | 0.47 | 0.22 | 0.994 | 0.09 | 0.02 | 0.46 | 0.56 |
| Brody | 42.91 | 0.91 | 0.14 | 0.996 | 0.29 | 0.01 | 0.43 | 0.43 |

Table 7: Parameters for growth curve of commercial female sheep using non-linear regression models

| Models | a | b | C | $R^2$ | BIAS | MAPE | MSE | MAD |
|---|---|---|---|---|---|---|---|---|
| Gompertz | 34.81 | 1.76 | 0.253 | 0.993 | -0.18 | 0.03 | 0.43 | 0.56 |
| Logistic | 33.18 | 3.62 | 0.36 | 0.989 | 0.09 | 0.03 | 0.645 | 0.63 |
| Von bertalanffy | 35.76 | 0.46 | 0.21 | 0.995 | 0.07 | 0.02 | 0.33 | 0.46 |
| Brody | 39.15 | 0.90 | 0.13 | 0.996 | 0.47 | 0.02 | 0.47 | 0.52 |

Table 8: Parameters for growth curve of spring born lambs

| Models | a | b | C | $R^2$ | BIAS | MAPE | MSE | MAD |
|---|---|---|---|---|---|---|---|---|
| Gompertz | 37.98 | 1.88 | 0.262 | 0.996 | -0.34 | 0.03 | 0.42 | 0.52 |
| Logistic | 36.07 | 4.07 | 0.38 | 0.992 | 0.11 | 0.03 | 0.62 | 0.60 |
| Von bertalanffy | 39.11 | 0.49 | 0.22 | 0.997 | -0.04 | 0.016 | 0.19 | 0.32 |
| Brody | 43.32 | 0.93 | 0.13 | 0.998 | 0.58 | 0.02 | 0.47 | 0.62 |

Table 9: Parameters for growth curve of autumn born lambs

| Models | a | b | C | $R^2$ | BIAS | MAPE | MSE | MAD |
|---|---|---|---|---|---|---|---|---|
| Gompertz | 36.54 | 1.77 | 0.257 | 0.993 | -0.04 | 0.03 | 0.50 | 0.58 |
| Logistic | 34.89 | 3.63 | 0.36 | 0.988 | 0.20 | 0.04 | 0.84 | 0.73 |
| Von bertalanffy | 37.49 | 0.47 | 0.21 | 0.994 | 0.47 | 0.03 | 0.63 | 0.62 |
| Brody | 40.88 | 0.91 | 0.14 | 0.996 | 0.11 | 0.01 | 0.26 | 0.40 |

Table 10: Parameters for growth curve of single born lambs

| Models | a | b | C | $R^2$ | BIAS | MAPE | MSE | MAD |
|---|---|---|---|---|---|---|---|---|
| Gompertz | 36.88 | 1.76 | 0.25 | 0.993 | 0.06 | 0.03 | 0.52 | 0.59 |
| Logistic | 35.26 | 3.59 | 0.37 | 0.988 | -0.02 | 0.04 | 0.80 | 0.75 |
| Von bertalanffy | 37.82 | 0.46 | 0.22 | 0.994 | -0.13 | 0.02 | 0.41 | 0.57 |
| Brody | 41.13 | 0.90 | 0.14 | 0.996 | 0.11 | 0.02 | 0.28 | 0.43 |

Table 11: Parameters for growth curve twin born lambs

| Models | a | b | C | $R^2$ | BIAS | MAPE | MSE | MAD |
|---|---|---|---|---|---|---|---|---|
| Gompertz | 34.99 | 1.88 | 0.25 | 0.995 | -0.32 | 0.03 | 0.43 | 0.53 |
| Logistic | 33.14 | 4.05 | 0.37 | 0.990 | -0.01 | 0.03 | 0.57 | 0.63 |
| Von bertalanffy | 36.10 | 0.49 | 0.21 | 0.996 | 0.003 | 0.02 | 0.23 | 0.38 |
| Brody | 40.31 | 0.92 | 0.12 | 0.997 | 0.60 | 0.02 | 0.58 | 0.61 |

From the Table 8, we have found that the VonBertalanffy model is the most suitable model foe predicting the weight of spring born lambs because it has a greatest $R^2$, lower MAPE, MAD, MSE and bias as compared to other non-linear growth models.

From the Table 9, it is observed that the Brody model is the most suitable model for predicting autumn lamb's weight because it has a greatest $R^2$ and lower MAPE, MSE and MAD as compared to others non-linear regression models.

Table 10 indicated that the Brody model is the most suitable model for predicting the single born lamb's weight because it has a greatest $R^2$ and lower MSE, MAPE and MAD as compared to others no-linear models.

From the Table 11, we have found that the VonBertalanffy model is the most suitable model for predicting the twin lamb's weight because it has lower MAPE, MSE and MAD as compared to others non-linear regression models.

**Conclusion:** Growth is natural phenomenon and possesses underlying mathematical basis. In living things the growth happens in sigmoid fashion. Non-linear regression models were appropriate for explaining this phenomenon. According the model selection criteria's, we have found among growth models the most suitable model for predicting the weight of commercial sheep, spring born lambs and twin born lambs, is the Von Bertalanffy model. While for predicting the weight of male commercial sheep, autumn born and single born lambs, the best model is the Brody model. Hence we can say that in Pakistan for predicting sheep and lamb weights the most suitable non-linear models are Von Bertalanffy and the Brody model.

## REFERENCES


Arango, J.A. and L.D. Van Vleck, 2002. Size of beef cows; Early ideas new developments. Genetic and Molecular Res., 1: 51-63.

Behzadi, M.R.B. and A.A. Aslaminejad, 2010. A comparison of neural network and nonlinear regression predictions of sheep growth. J. Anim. Vet. Adv., 9: 2128-2131.

Bertalanffy, L.V., 1957. Quantitative laws in metabolism and growth. Q. Rev. Biol., 32: 217-230.

Gbangboche, A.B., R. Glele-Kakai, S. Salifou, L.G. Albuquerque and P.L. Leroy, 2008. Comparison of Non-linear growth models to describe the growth curve in west African dwarf Sheep. Animal, 2: 1003-1012.

Gompertz, B., 1825. On the nature of thefucntion of the law of human mortality and a new mode of determining the value of live contingencies. Phil. Trans. Roy. Soc., 182: 513-585.

Gurcan, E.K., O. Cobanuglu and S. Genc, 2012. Determination of Body Weight-Age Relationship by Non-Linear Models in Japanese Quail. J. Anim. Vet. Adv., 11: 314-317.

Kucuk, M. and E. Eyduran, 2010. The determination of the best growth model for Akkaraman and German Blackheaded Mutton x Akkaraman B1 cross breed lambs. Bulgarian J. Agric. Sci., 15: 90-92.

Lamb, N.R., E.A. Navajas, G. Simm and L. Bunger, 2008. A genetic investigation of various growth models to describe growth of lambs of two contrasting breeds. J. Anim. Sci., 86: 2642-54.

Lancelot, R., M. Lensnoff, E. Tillard and J.J. McDermott, 2000. Graphical approaches to support the analysis of linear-multilevel models of latib pre-weaning growth in Kolda (Senegal). Preventing Vet. Med., 46: 225-247.

Ozdemir, H. and G. Dellal, 2009. Determination of growth curves in young Angora goats. Tarim Bilimleri Gergisi, 15: 358-362.

Tariq, M.M., M.A. Bajwa, A. Waheed, E. Eyduran, F. Abbas, F.A. Bokhari and A. Akbar, 2011. Growth curve in mengali sheep breed of Balochistan. J. Anim. Plant Sci., 21: 5-7.

Ulutas, Z., 2010. The effect of birth types on growth curve parameters of Karayaka Lambs. J. Anim. Vet. Adv., 9: 1384-1388.

Wildeus, S., 1997. Hair sheep genetic resources and their contribution to diversified small ruminant production in the United States. J. Anim. Sci., 75: 630-640.

Verhulst, P.F., 1838. Notice sur la loi que la population suit dans son accroissement. Corr. Math. Phy., 10: 113-121.