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Calculation of Compressible Flow Past a Joukowski Aerofoil Using Direct Boundary Element Method with Linear Element Approach

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Abstract: In this study, the Direct Boundary Element Method (DBEM) is being applied to present solution for surface and ground water bodies which are important nutrition fluids. To calculate a steady, irrotational, inviscid compressible flow past a Joukowski aerofoil using Direct Boundary Element Method with linear element approach. The results obtained using this method has been compared with the analytical solutions for the body under consideration.

Key words: Direct boundary element method, compressible flow, joukowski aerofoil

INTRODUCTION

The DBEM is necessarily applied to find the velocity distribution over the surface of water body which is main nutrition source. Therefore the DBEM with linear approach can be used to solve the water flow problems usually encountered in food preparation factories and municipal water supplies. Along with problems related to water resources, this technique can be applied to calculate velocity distribution over the boundary of a Joukowski aerofoil.

Now a days, the boundary element method is successfully applied by numerical community. The technique of the boundary element method consists of sub-dividing the boundary of the body into a series of discrete elements, over which the function can vary. The technique offers important advantages over domain type methods such as finite elements and finite differences. One of the advantages is that with boundary elements one only has to define the surface of the body, whereas with field methods it is necessary to mesh the entire flow field. The amount of input data for a boundary element method is therefore significantly less than for a field method which is a very important advantage in practice, as many hours can be spent in preparing and checking the data for finite element or finite difference programs. Furthermore, the method is well-suited to problems with an infinite domain. Boundary element methods can be formulated using two different approaches called the direct and the indirect methods. The direct method takes the form of a statement which provides the values of the unknown variables at any field point in terms of the complete set of all the boundary data. On the other hand, the indirect method utilizes a distribution of singularities over the boundary of the body and computes this distribution as the solution of integral equation (Brebbia, 1978 and 1980). The direct and indirect methods have been used in the past for flow field calculations around bodies (Morino *et al*., 1975; Hess and Smith, 1967; Kohr, 2000; Luminita *et al*., 2008a; Luminita, 2008b; Muhammad *et al*., 2008; Mushtaq *et al*., 2008; 2009; Mushtaq and Shah, 2010a,b; Mushtaq, 2011 and Mushtaq *et al*., 2012a,b,c,d). Most of the work on fluid flow calculations using boundary element methods has been done in the field of incompressible flow. Very few attempts have been made on flow field calculations using boundary element methods in the field of compressible flow. In this paper, the DBEM has been used for the solution of inviscid compressible flows around a Joukowski aerofoil.

MATERIALS AND METHODS

Mathematical formulation: We know that equation of motion for two-dimensional, steady, irrotational and isentropic flow (Mushtaq and Shah, 2010a; Mushtaq 2011 and Mushtaq *et al*., 2012c,d; Shah, 2011) is:

$$\left(1-Ma^2\right)\frac{\partial^2\Phi}{\partial X2}+\frac{\partial^2\Phi}{\partial Y2} = 0 \quad (1)$$

where, Ma is the Mach number and Φ is the total velocity potential of the flow. Here X and Y are the space coordinates.

Using the dimensionless variables, x = X, y = β Y,

where $\beta = \sqrt{1-Ma^2}$ Eq. (1) becomes:

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2} = 0$$

or:

$$\nabla^2\Phi = 0 \quad (2)$$

which is Laplace's equation.

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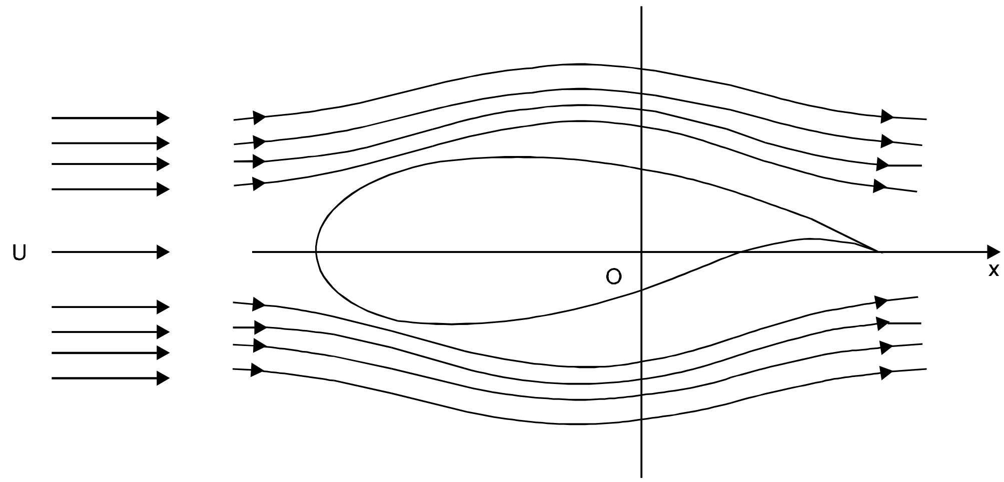

Fig. 1: Flow past a Joukowski aerofoil

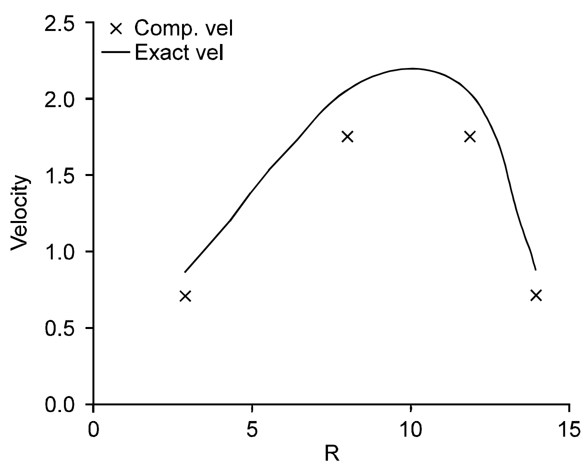

Fig. 2: Comparison of computed and analytical velocity distributions over the boundary of a Joukowski aerofoil using upper 4 values of 8 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

Inviscid compressible flow past a joukowski aerofoil: Consider the flow past a Joukowski aerofoil and let the onset flow be the uniform stream with velocity U in the positive direction of the x-axis as shown in Fig. 1.

Exact velocity: The magnitude of the exact velocity distribution over the surface of a Joukowski aerofoil is given by Chow (1979); Mushtaq (2011) and Mushtaq *et al.*, 2012c) is:

$$V = U\left|\frac{1-\dfrac{r^2}{(z-z_1)2}+\dfrac{2ic}{(z-z_1)}}{1-\left(\dfrac{a}{z}\right)^2}\right|$$

Where:

r = Radius of the cylinder,
a = Joukowski transformation constant
z = x+I y, z_1 = b+I c
b = a- $\sqrt{r^2-c^2}$

In Cartesian coordinates the exact velocity becomes:

$$V = U\frac{\Big[\big[\{(x-b)^2+(y-c)^2\}^2-r^2\{(x-b)^2-(y-c)^2\}+2c(y-c)\{(x-b)^2+(y-c)^2\}\big]^2+\big[2c(x-b)\{(x-b)^2+(y-c)^2\}+2r^2(x-b)(y-c)\big]^2\Big]^{1/2}}{\left[(x-b)^2+(y-c)^2\right]^2}\times\frac{\sqrt{\left[(x^2+y^2)^2-a^2(x^2-y^2)\right]^2+4a^4x^2y^2}}{(x^2-y^2-a^2)^2+4x^2y^2}$$

Boundary conditions: Now the condition to be satisfied on the boundary of a Joukowski aerofoil is (Mushtaq, 2011):

$$\frac{\partial\phi_{j.a}}{\partial n} = U\frac{(x+b)}{\sqrt{(x+b)^2+(y-c)^2}} \quad (3)$$

Where, the subscript j.a stands for Joukowski aerofoil. Eq. (3) is the boundary condition which must be satisfied over the boundary of a Joukowski aerofoil.

Equation of direct boundary element method: The equation of DBEM for two-dimensional flow (Mushtaq, 2008-2012) is:

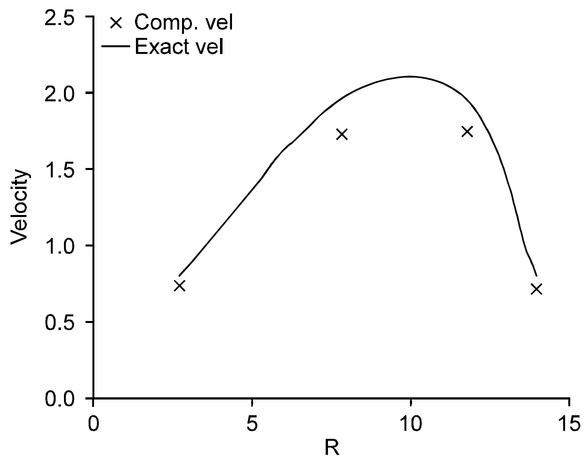

Fig. 3: Comparison of computed and analytical velocity distributions over the boundary of a Joukowski aerofoil using lower 4 values of 8 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

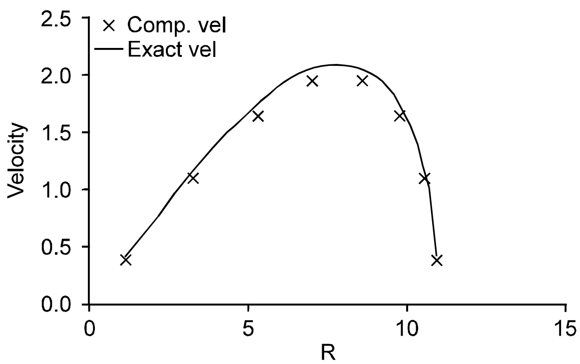

Fig. 4: Comparison of computed and analytical velocity distributions over the boundary of a Joukowski aerofoil using upper 8 values of 16 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

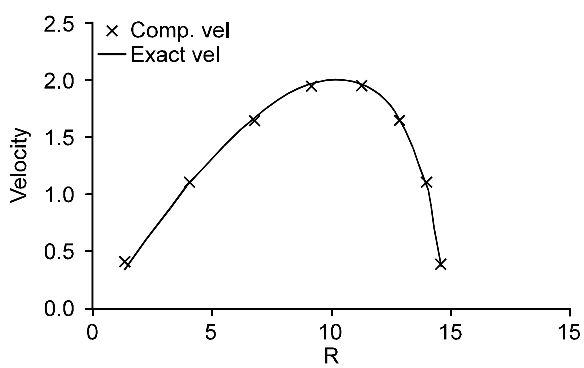

Fig. 5: Comparison of computed and analytical velocity distributions over the boundary of a Joukowski aerofoil using lower 8 values of 16 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

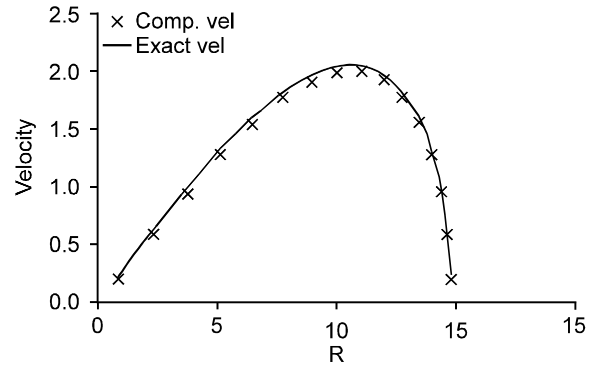

Fig. 6: Comparison of computed and analytical velocity distributions over the boundary of a Joukowski aerofoil using upper 16 values of 32 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7.

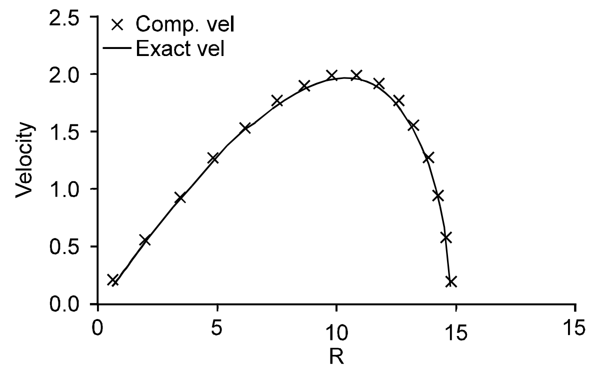

Fig. 7: Comparison of computed and analytical velocity distributions over the boundary of a Joukowski aerofoil using lower 16 values of 32 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

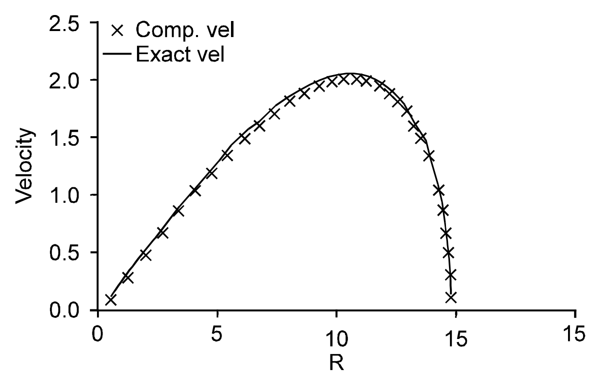

Fig. 8: Comparison of computed and analytical velocity distributions over the boundary of a Joukowski aerofoil using upper 32 values of 64 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

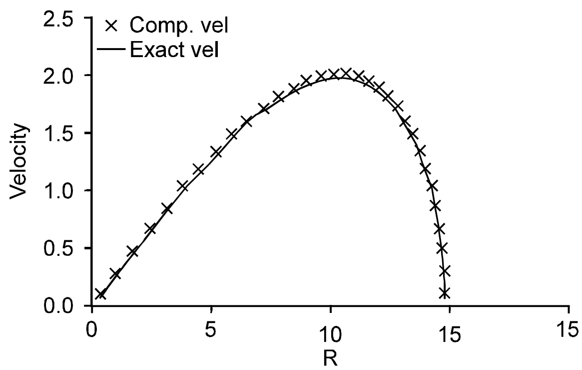

Fig. 9: Comparison of computed and analytical velocity distributions over the boundary of a Joukowski aerofoil using lower 32 values of 64 boundary elements with indirect linear element approach for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7

$$-c_i\phi_i + \frac{1}{2\pi}\int_{\Gamma-i} \phi\frac{\partial}{\partial n}\left[\log\left(\frac{1}{r}\right)\right]d\Gamma + \phi\infty = \frac{1}{2\pi}\int_{\Gamma} \log\left(\frac{1}{r}\right)\frac{\partial\phi}{\partial n}d\Gamma \quad (4)$$

Where:

c_i = 0 when I is exterior to Γ
= 1 when I is interior to Γ
= ½ when i lies on Γ and Γ is smooth

Matrix formulation with linear element approach: The Eq. DBEM (4) can be written for this case as:

$$-c_i\phi_i + \sum_{j=1}^{m}\int_{\Gamma_j-i} \phi\frac{\partial}{\partial n}\left[\log\left(\frac{1}{r}\right)\right]d\Gamma + \phi\infty = \sum_{j=1}^{m}\int_{\Gamma_j} \frac{\partial\phi}{\partial n}\left(\frac{1}{2\pi}\right)d\Gamma \quad (5)$$

Since ϕ and vary linearly over the element, their values at any point on the element can be defined in terms of their nodal values and the shape functions N_1 and N_2 as:

$$\phi = [N_1 \;\; N_2]\begin{Bmatrix}\phi_1\\ \phi_2\end{Bmatrix}$$
$$\frac{\partial\phi}{\partial n} = [N_1 \;\; N_2]\begin{Bmatrix}\frac{\partial\phi_1}{\partial n}\\ \frac{\partial\phi_2}{\partial n}\end{Bmatrix} \quad (6)$$

The integrals along an element 'j' on the L.H.S. of Eq. (5) can now be written as:

$$\int_{\Gamma_j-i} \phi\frac{\partial}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right)d\Gamma$$
$$= \int_{\Gamma_j-i} [N_1 \;\; N_2]\frac{\partial}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right)d\Gamma\begin{Bmatrix}\phi_1\\ \phi_2\end{Bmatrix}$$
$$= \left[h_{ij}^1 \;\; h_{ij}^2\right]\begin{Bmatrix}\phi_1\\ \phi_2\end{Bmatrix}$$

Where:

$$h_{ij}^k = \int_{\Gamma_j-i} N_k\frac{\partial}{\partial n}\left(\frac{1}{2\pi}\log\right)d\Gamma,\; k = 1, 2 \quad (7)$$

The integrals on the R.H.S. of Eq. (5) can be written as:

$$\int_{\Gamma_j} \frac{\partial\phi}{\partial n}\left(\frac{1}{2\pi}\log\frac{1}{r}\right)d\Gamma$$
$$= \int_{\Gamma_j} [N_1 \;\; N_2]\left(\frac{1}{2\pi}\log\frac{1}{r}\right)d\Gamma\begin{Bmatrix}\frac{\partial\phi_1}{\partial n}\\ \frac{\partial\phi_2}{\partial n}\end{Bmatrix}$$
$$= \left[g_{ij}^1 \;\; g_{ij}^2\right]\begin{Bmatrix}\frac{\partial\phi_1}{\partial n}\\ \frac{\partial\phi_2}{\partial n}\end{Bmatrix}$$

Where:

$$g_{ij}^k \int_{\Gamma_j} N_k\left(\frac{1}{2\pi}\log\frac{1}{r}\right)d\Gamma,\quad k = 1, 2 \quad (8)$$

Again the integrals in Eq. (7) and (8) are calculated numerically as before except for the element on which the fixed point 'I' is lying. For this element the integrals are calculated analytically. The integrals h_{ii}^1 and h_{ii}^2 are zero because r and n̂ are orthogonal to each other over the element. The value of the integrals g_{ii}^1 and g_{ii}^2 are given by:

$$g_{ii}^{(1)} = \frac{\ell}{8\pi}\left[3-2\log\ell\right]$$

and:

$$g_{ii}^{(2)} = \frac{\ell}{8\pi}\left[1-2\log\ell\right]$$

Also the velocity midway between two nodes on the boundary can then be approximated by using the formula:

$$\text{Velocity } \overset{\otimes}{V} = \frac{\Phi_{k+1}-\Phi_k}{\text{Length from node k to k+1}} \quad (9)$$

where, the total velocity potential Φ is the sum of the perturbation velocity potential $\phi_{j.a}$ and the velocity potential of the uniform stream $\phi_{u.s.}$

Process of discretization: Now for the discretization of the boundary of the Joukowski aerofoil, the coordinates of the extreme points of the boundary elements can be generated within computer program using Fortran language as follows.

Divide the boundary of the circular cylinder into m elements in the clockwise direction by using the formula (Mushtaq *et al*., 2009; Mushtaq and Shah, 2010a,b; Mushtaq, 2011; Mushtaq and Shah 2012a; Mushtaq *et al*., 2012c, d):

$$\theta_k = \left[(m+2)-2k\right]\frac{\pi}{m}, \quad k = 1, 2, \ldots, m \qquad (10)$$

Then the extreme points of these m elements of circular cylinder are found by:

$$\xi_k = -b + r \ \mathrm{cose}\ \theta_k$$
$$\eta_k = c + r \sin\theta_k$$

Now by using Joukowski transformation:

$$z = \zeta + \frac{a^2}{\zeta}$$

the extreme points of the Joukowski aerofoil are:

$$x_k = \xi_k\left(1+\frac{a^2}{\xi_k^2+\eta_k^2}\right)$$
$$y_k = \eta_k\left(1-\frac{a^2}{\xi_k^2+\eta_k^2}\right)$$

where, k = 1, 2, ……., m .

Therefore the boundary condition (3) in this case takes the form:

$$\frac{\partial\phi_{j.a}}{\partial n} = U\frac{(x_k+b)}{\sqrt{(x_k+b)^2+(y_k-c)^2}} = \frac{(x_k+b)}{\sqrt{(x_k+b)^2+(y_k-c)^2}}$$

taking:

$$U = 1 \qquad (11)$$

RESULTS AND DISCUSSION

The following Tables and Figures show the comparison of computed and analytical velocity distribution over the boundary of a Joukowski aerofoil for 8, 16, 32 and 64 direct linear boundary elements.

In this research study, we are introducing a direct boundary element method with linear element approach to solve fluid flow problems. In the past analytical techniques have been developed by Chow (1979) to solve fluid flow problems which are lengthy and time consuming. Compared to these existing methods, we have calculated velocity distribution over the boundary of a Joukowski Aerofoil for r = 7.5, a = 0.2, c = 0.15 and Ma = 0.7 and made comparison with Chow (1979) in Tables 1-4 and Graphs 3-10. Our results are time saving and accurate in solving problems related to fluid movements. On the basis of results calculated through computer using FORTRAN language shows that there is high

Table 1: Comparison of computed and analytical velocity distribution over the boundary of a Joukowski Aerofoil for 8 direct linear boundary elements

Element	X	Y	R =	Velocity	Exact velocity
1	-13.70	2.80	13.99	0.71995E+00	0.88787E+00
2	-9.95	6.55	11.91	0.17376E+01	0.20507E+01
3	-4.65	6.55	8.03	0.17355E+01	0.20519E+01
4	-.83	2.75	2.87	0.71497E+00	0.85796E+00
5	-.83	-2.55	2.68	0.73896E+00	0.79150E+00
6	-4.65	-6.25	7.79	0.17342E+01	0.19655E+01
7	-9.95	-6.25	11.75	0.17372E+01	0.19642E+01
8	-13.70	-2.50	13.93	0.71962E+00	0.80282E+00

Table 2: Comparison of computed and analytical velocity distribution over the boundary of a Joukowski Aerofoil for 16 direct linear boundary elements

Element	X	Y	R =	Velocity	Exact velocity
1	-14.52	1.58	14.60	0.38469E+00	0.44004E+00
2	-13.42	4.24	14.07	0.10954E+01	0.11738E+01
3	-11.39	6.26	13.00	0.16394E+01	0.17365E+01
4	-8.74	7.36	11.42	0.19336E+01	0.20416E+01
5	-5.87	7.36	9.41	0.19333E+01	0.20422E+01
6	-3.21	6.26	7.04	0.16381E+01	0.17372E+01
7	-1.18	4.23	4.39	0.10908E+01	0.11719E+01
8	-0.02	1.53	1.53	0.38073E+00	0.41176E+00
9	-0.02	-1.33	1.33	0.40688E+00	0.35349E+00
10	-1.18	-3.93	4.10	0.10897E+01	0.10902E+01
11	-3.21	-5.96	6.77	0.16379E+01	0.16557E+01
12	-5.87	-7.06	9.18	0.19332E+01	0.19606E+01
13	-8.74	-7.06	11.23	0.19336E+01	0.19600E+01
14	-11.39	-5.96	12.86	0.16394E+01	0.16549E+01
15	-13.42	-3.94	13.98	0.10954E+01	0.10922E+01
16	-14.52	-1.28	14.57	0.38466E+00	0.35887E+00

Table 3: Comparison of computed and analytical velocity distribution over the boundary of a Joukowski Aerofoil for 32 direct linear boundary elements

Element	X	Y	R =	Velocity	Exact velocity
1	-14.73	0.88	14.76	0.19540E+00	0.23719E+00
2	-14.44	2.32	14.63	0.57869E+00	0.62324E+00
3	-13.88	3.67	14.36	0.93973E+00	0.98704E+00
4	-13.07	4.88	13.95	0.12646E+01	0.13146E+01
5	-12.04	5.92	13.41	0.15410E+01	0.15933E+01
6	-10.82	6.73	12.74	0.17580E+01	0.18124E+01
7	-9.47	7.29	11.95	0.19075E+01	0.19636E+01
8	-8.03	7.58	11.04	0.19836E+01	0.20408E+01
9	-6.57	7.57	10.03	0.19834E+01	0.20411E+01
10	-5.13	7.29	8.92	0.19070E+01	0.19643E+01
11	-3.78	6.73	7.72	0.17572E+01	0.18132E+01
12	-2.57	5.91	6.45	0.15396E+01	0.15937E+01
13	-1.53	4.88	5.11	0.12627E+01	0.13138E+01
14	-0.72	3.66	3.73	0.93663E+00	0.98382E+00
15	-0.16	2.30	2.30	0.57168E+00	0.61492E+00
16	0.19	0.82	0.84	0.19080E+00	0.20753E+00
17	0.19	-0.61	0.64	0.21771E+00	0.15032E+00
18	-0.16	-1.99	2.00	0.56986E+00	0.53302E+00
19	-0.72	-3.36	3.43	0.93621E+00	0.90311E+00
20	-1.53	-4.58	4.83	0.12626E+01	0.12333E+01
21	-2.57	-5.61	6.17	0.15396E+01	0.15133E+01
22	-3.78	-6.43	7.46	0.17572E+01	0.17329E+01
23	-5.13	-6.99	8.67	0.19070E+01	0.18839E+01
24	-6.57	-7.27	9.80	0.19835E+01	0.19607E+01
25	-8.03	-7.28	10.84	0.19836E+01	0.19604E+01
26	-9.47	-6.99	11.77	0.19075E+01	0.18832E+01
27	-10.82	-6.43	12.59	0.17580E+01	0.17321E+01
28	-12.04	-5.62	13.28	0.15410E+01	0.15129E+01
29	-13.07	-4.58	13.85	0.12647E+01	0.12342E+01
30	-13.88	-3.37	14.29	0.93974E+00	0.90667E+00
31	-14.44	-2.02	14.58	0.57869E+00	0.54288E+00
32	-14.73	-0.58	14.74	0.19541E+00	0.15692E+00

Table 4: Comparison of computed and analytical velocity distribution over the boundary of a Joukowski Aerofoil for 64 direct linear boundary elements

Element	X	Y	R =	Velocity	Exact velocity
1	-14.78	0.52	14.79	0.98076E-01	0.13823E+00
2	-14.71	1.25	14.76	0.29328E+00	0.33365E+00
3	-14.57	1.97	14.70	0.48563E+00	0.52625E+00
4	-14.35	2.67	14.60	0.67333E+00	0.71420E+00
5	-14.07	3.35	14.47	0.85454E+00	0.89567E+00
6	-13.73	4.00	14.30	0.10275E+01	0.10689E+01
7	-13.32	4.61	14.09	0.11906E+01	0.12323E+01
8	-12.85	5.18	13.86	0.13422E+01	0.13842E+01
9	-12.33	5.70	13.59	0.14809E+01	0.15233E+01
10	-11.76	6.17	13.28	0.16053E+01	0.16480E+01
11	-11.15	6.57	12.95	0.17142E+01	0.17573E+01
12	-10.50	6.92	12.58	0.18066E+01	0.18501E+01
13	-9.82	7.20	12.18	0.18817E+01	0.19255E+01
14	-9.12	7.41	11.75	0.19386E+01	0.19828E+01
15	-8.40	7.56	11.30	0.19768E+01	0.20213E+01
16	-7.67	7.63	10.82	0.19959E+01	0.20407E+01
17	-6.93	7.63	10.31	0.19959E+01	0.20408E+01
18	-6.20	7.56	9.78	0.19765E+01	0.20217E+01
19	-5.48	7.41	9.22	0.19382E+01	0.19833E+01
20	-4.78	7.20	8.64	0.18811E+01	0.19263E+01
21	-4.10	6.92	8.04	0.18059E+01	0.18509E+01
22	-3.45	6.57	7.42	0.17133E+01	0.17581E+01
23	-2.84	6.16	6.78	0.16041E+01	0.16485E+01
24	-2.27	5.69	6.13	0.14795E+01	0.15234E+01

Table 4: Continue

25	-1.75	5.17	5.46	0.13405E+01	0.13838E+01
26	-1.28	4.60	4.78	0.11886E+01	0.12310E+01
27	-0.88	3.99	4.09	0.10250E+01	0.10665E+01
28	-0.53	3.34	3.38	0.85143E+00	0.89163E+00
29	-0.25	2.66	2.67	0.66929E+00	0.70787E+00
30	-0.03	1.95	1.95	0.47998E+00	0.51626E+00
31	0.12	1.21	1.22	0.28385E+00	0.31642E+00
32	0.25	0.45	0.51	0.91189E-01	0.10948E+00
33	0.26	-0.23	0.35	0.11812E+00	0.61689E-01
34	0.12	-0.90	0.91	0.28089E+00	0.23031E+00
35	-0.03	-1.65	1.65	0.47814E+00	0.43415E+00
36	-0.25	-2.36	2.37	0.66849E+00	0.62688E+00
37	-0.53	-3.04	3.09	0.85101E+00	0.81110E+00
38	-0.88	-3.69	3.79	0.10248E+01	0.98617E+00
39	-1.28	-4.30	4.49	0.11884E+01	0.11508E+01
40	-1.75	-4.87	5.18	0.13405E+01	0.13037E+01
41	-2.27	-5.39	5.85	0.14795E+01	0.14433E+01
42	-2.84	-5.86	6.51	0.16041E+01	0.15685E+01
43	-3.45	-6.27	7.16	0.17133E+01	0.16780E+01
44	-4.10	-6.62	7.78	0.18059E+01	0.17709E+01
45	-4.78	-6.90	8.39	0.18812E+01	0.18462E+01
46	-5.48	-7.11	8.98	0.19382E+01	0.19033E+01
47	-6.20	-7.26	9.55	0.19766E+01	0.19416E+01
48	-6.93	-7.33	10.09	0.19959E+01	0.19607E+01
49	-7.67	-7.33	10.61	0.19960E+01	0.19606E+01
50	-8.40	-7.26	11.10	0.19768E+01	0.19412E+01
51	-9.12	-7.11	11.57	0.19386E+01	0.19026E+01
52	-9.82	-6.90	12.01	0.18817E+01	0.18454E+01
53	-10.50	-6.62	12.42	0.18067E+01	0.17700E+01
54	-11.15	-6.27	12.80	0.17143E+01	0.16772E+01
55	-11.76	-5.87	13.14	0.16053E+01	0.15679E+01
56	-12.33	-5.40	13.46	0.14809E+01	0.14432E+01
57	-12.85	-4.88	13.75	0.13422E+01	0.13042E+01
58	-13.32	-4.31	14.00	0.11906E+01	0.11522E+01
59	-13.73	-3.70	14.22	0.10275E+01	0.98885E+00
60	-14.07	-3.05	14.40	0.85453E+00	0.81559E+00
61	-14.35	-2.37	14.55	0.67337E+00	0.63412E+00
62	-14.57	-1.67	14.66	0.48564E+00	0.44618E+00
63	-14.71	-0.95	14.74	0.29328E+00	0.25358E+00
64	-14.78	-0.22	14.78	0.98072E-01	0.58174E-01

degree of similarity between computed and analytical results, with the increase of numbers of boundary elements the degree of accuracy is increased and become more reliable, efficient and economical compared to other domain methods.

Conclusions: A direct boundary element method has been applied for the calculation of inviscid compressible flow past a Joukowski Aerofoil with linear element approach. The calculated flow velocities obtained using this method is compared with the analytical solutions for flow over the boundary of a Joukowski Aerofoil. The tables and figures, indicate that the computed results obtained by this method are good in agreement with the analytical ones for the body under consideration and an increase in the number of elements leads to an increase in accuracy.

Recommendations: The direct boundary element method is being suggested to deal the problems such as tides, turbulent or laminar thrust related to the submarines and other shipping vehicles. Keeping in view of the sever electricity crises and introductive new architecture with water cooling system is being initiative in many countries, the boundary element method with linear approach can provide good solution for predicted complications.

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