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The Multi-Product Multi-Constraint Newsboy Problem with Incremental Discount and Batch Order

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Abstract: This study points out the real-world prevalence of the multiple-product multiple-constraint Newsboy problem, i.e., the Newsstand problem, in which not only there are incremental discounts on the purchasing prices, but also the orders are placed in batch forms. The constraints are the service levels and warehouse capacity. Moreover, the quantities of the orders are integer multiples of packets, each containing more than one product. The objective of this problem is to find the order quantities such that the expected sum of the shortage, holding and purchasing costs is minimized. We assume that the holding and shortage costs occur at the end of the period; they are modeled by a quadratic function and that the decision variables are integer. We present a formulation to the problem and show that it is a integer nonlinear programming model. Finally, we provide an efficient algorithm to solve the new problem and illustrate the results with a numerical example.

Key words: Newsboy problem, inventory control, mixed integer nonlinear programming, genetic algorithm

INTRODUCTION

The Newsboy problem is a classical inventory problem that is very significant in terms of both theoretical and practical considerations. This problem is a tool to decide the stock quantity of an item when there is a single purchasing opportunity before the start of the selling period and the demand for the item is random. In one hand, the ordered items that remain unsold or unused at the end of the period either become obsolete or are sold at a single discounted price. On the other hand, if the buyer initially decides to buy smaller amounts of these commodities, shortages may occur, causing loss of revenue. In this problem, the commodity has the most important characteristic of a single-period product and the question becomes how to determine the quantity to be ordered to minimize (maximize) the costs (profit) incurred. Answering this is the main objective of the classical Newsboy model.

In real world situations, many products have a limited selling period; so the model of Newsboy problem is often used to aid decision-making in fashion, sporting, service industries, etc. to manage capacity or evaluate advanced booking of ordering. The classical Newsboy problem and its various extensions are widely studied by Khouja (1999) and Silver and Pyke (1998).

The five out of ten types of extensions to the Newsboy problem introduced by Khouja (1999) are:

- Different objectives and utility functions may be considered
- Different discount policies may be employed
- Multi-product constraint models may be used
- Multi-period models may be considered
- Other extensions

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For the first type of extension, Silver and Pyke (1998), Lau (1980), Lau and Lau (1988) and Anvari (1987) proposed models to the Newsboy problem in which the goal was to maximize the probability of achieving a target profit. Anvari (1987), Anvari and Kusy (1990), Chung (1990) and Atkinson (1979) used different effectiveness and risk tolerance criteria.

While in the classical Newsboy problem, we assume that the purchase cost per unit is fixed, sometimes the vendor gives more price discount to stimulate the buyer to purchase earlier to decrease the inventory level. As a result, in the second type of extension, Anvari (1987), Anvari and Kusy (1990), Chung (1990) and Pfeifer (1989) formulated different sales discount at different sales quantities.

For the third type of extension, Lau and Lau (1995, 1996), Abdel-Malek and Areeratchakul (2007), Khouja (1999) and Vairaktarakis (2000) considered a multi-product problem with budget constraint. Abdel-Malek *et al*. (2004) considered the multi-product Newsboy problem with budget constraint and presented exact solution formulae when the demand probability density function was uniform. They proposed a generic iterative method to find the optimum or near optimum solution for general continuous density functions of the demand. As a sequel of this research, Abdel-Malek and Montanari (2005a) examined the solution space of the problem in order to provide the necessary insight into this phenomenon. Furthermore, Abdel-Malek and Montanari (2005b) developed a methodology to examine the dual of the solution space of the multi-product Newsboy problem with two constraints.

In the fourth type of extension, Matsuyama (2006) analyzed the Newsboy model in which a fraction of the shortage was back-ordered. Moreover, Alfares and Elmorra (2005) analyzed the Newsboy problem in both single-periodic and multi-periodic frames in which random yield and fixed order cost were considered.

Finally, in the fifth type of extension, Reyes (2005) used the Newsboy model in a supply chain in which both sides had incomplete information on the demand. Mostard and Teunter (2006) considered a single-period model in which a percentage of the sold products were returned, assuming that these products could be returned in a specific range of time and were able to be sold again if they were not damaged. In their model, the shortages were considered as lost sales. Kogan and Lou (2003) formulated the Newsboy problem in a dynamic model with constant time, believing that their proposed model was applicable in production environments. Keren and Pliskin (2006) presented the Newsboy as a risk-averse model and calculated the optimal order quantity by using the utility theory. Chen and Chuang (2006) analyzed the Newsboy problem along with the shortage level constraint. Abdel-Malek and Montanari (2005a) presented the Newsboy problem with budget constraint and proposed different formulae to obtain the order quantity for three ranges of the budget quantity. Furthermore, Abdel-Malek and Areeratchakul (2007) used the quadratic programming approach in a multi-product Newsboy problem with budget, capacity and order constraints.

Using quadratic concave holding and shortage cost functions, Lin and Tsai (2006) proposed two models to assign more increase to the costs of excess inventories and shortage at the end of a period. They derived precise expressions for the expected total profit of these two Newsboy problems. Casimir (2002) used the multi-item Newsboy model to determine the value of two types of incomplete information, global information and product-mix information. He showed that this value depended on the number of products, the existence of a budget restriction and the degree of substitutability. Sen and Zhang (1999) considered a single item Newsboy problem where the item could be sold to different demand classes at different prices. The demands were assumed sequentially over time. Dekker *et al.* (2000) analyzed the effect of a cutoff transaction-size on the average inventory cost in a simple Newsboy setting. In this research, it was assumed that customers with an order larger than a pre-specified cutoff transaction size were satisfied in an alternative way, against additional cost. Weng (2004) developed a generalized newsvendor model to analyze the coordinated quantity decision between the manufacturer and the buyer. The manufacturer and the buyer operated to meet random demand of one product with a short lifecycle.

In this study, we consider a multi-product Newsboy problem in which the demand follows a Poisson distribution. In this problem, service levels and warehouse capacity are considered constraints and the incremental discount policy is used to purchase the items. Furthermore, the quantity of the orders must be integer multiples of packets, each containing more than one product.

PROBLEM DEFINITION

Consider a company that orders products to a supplier with the following rules:

- The opportunity to order is only once and only at the start of a period
- The demand of each product (j) by the customers follows a Poisson distribution with parameter λ_j
- The order quantity of each product should only be integer multiples of packets each with n_j items
- There is no enforced constraint by the supplier to supply an order
- The entire capacity of the warehouse is limited for the products
- The company intends to satisfy customers with minimum service levels
- Shortage is licensable and takes the lost sale condition
- The shortage and holding costs are known and are deployed at the end of the period and increase in quadratic fashion
- Discount for purchasing items is allowed and follows the incremental discount rule
- Lead-time is considered zero

We are to determine the order quantity of each product such that the constraints are satisfied and the expected total costs of the inventory model is minimized.

PROBLEM MODELING

Since the orders are placed only once and at the beginning of each period, we may take advantage of the classical Newsboy problem and develop a mathematical model for the problem at hand. To do this, we first define the variables and the parameters of the model. Then, we determine the costs and the constraints. Finally, the model is presented.

The Parameters and Variables

For i = 1, 2,..., n and j = 1, 2,..., T the parameters and the variables of the model are:

T	:	Is the number of products
X_j	:	Is the stochastic demand of the jth product
λ_j	:	Is the expected demand of the jth product
$f_{X_j}(x_j)$	:	Is the probability mass function of the jth product demand
n_j	:	Is the number of items in the packets of the jth product
$H_j(x)$	:	Is the holding cost function of the jth product at the end of a period
h_{1j}	:	Is the linear coefficient of the quadratic holding cost function of the jth product
h_{2j}	:	Is the quadratic coefficient of the quadratic holding cost function of the jth product
$\pi_j(x)$	:	Is the shortage cost function of the jth product at the end of a period
π_{1j}	:	Is the linear coefficient of the quadratic shortage cost function of the jth product
π_{2j}	:	Is the quadratic coefficient of the quadratic shortage cost function of the jth product
Q_j	:	Is a decision variable representing the order quantity of the jth product
M_j	:	Is a decision variable representing the number of packets that have been ordered for the jth product
α_j	:	Is the minimum service level of the jth product

f_j : Is the space required for each packet of the jth product
q_{ij} : Is the ith discount break point of the jth product
C_{ij} : Is the purchase cost of the jth product in the ith break point
F : Is the total available warehouse space
C_{H_j} : Is the expected holding cost of the jth product at the end of a period
C_{B_j} : Is the expected shortage cost of the jth product at the end of a period
C_{P_j} : Is the expected purchasing cost of the jth product
Z : Is the expected total cost

Here, we first model a single-product problem and then extend it to the multi-product case.

MODELING THE COSTS

The costs associated with a single-product problem are the holding, shortage and purchase costs, determined as follows:

Holding Cost

For a specific product j in a period, since we assumed that the holding cost occurs at the end of the period, at the starting point of the period we need to determine the end-point expected inventory of the product. If the total demand quantity is more than the order quantity of the period, i.e., $X_j \geq Q_j$, then the inventory quantity at the end of the period is zero. However, if the total demand quantity is less than the order quantity, then the inventory quantity at the end of the period is $Q_j - X_j$. In other words

$$\text{Inventory level at the end of a period} = \begin{cases} 0 & \text{if} \quad X_j \geq Q_j \\ (Q_j - X_j) & \text{if} \quad X_j < Q_j \end{cases} \tag{1}$$

Since the probability mass function of the demand for product j is $f_{X_j}(x_j)$, the expected inventory at the end of the period is determined as:

$$\text{Expected inventory at the end of the period} = \sum_{X_j=0}^{Q_j-1} (Q_j - X_j) f_{X_j}(x_j) \tag{2}$$

Finally, considering the quadratic increase of the holding cost (Lin and Tsai, 2006), the expected holding cost at the end of the period which is calculated at the start of the period is

$$C_{H_j} = \sum_{X_j=0}^{Q_j-1} \left(h_{1_j}(Q_j - X_j) + h_{2_j}(Q_j - X_j)^2 \right) f_{X_j}(x_j) \tag{3}$$

Shortage Cost

If shortage occurs during a period, it will take the lost sale condition. Since the holding cost is calculated at the end of the period, the expected shortage is calculated at the same time. In this case, if the total demand quantity is more than the ordered quantity, i.e., $X_j > Q_j$, then at the end of the period the shortage quantity will be $X_j - Q_j$. However, if the total demand quantity at the end of the period is less than the order quantity, the shortage quantity at the end of the period will be zero. In other words,

$$\text{Shortage quantity at the end of the period} = \begin{cases} (X_j - Q_j) & \text{if} \quad X_j > Q_j \\ 0 & \text{if} \quad X_j \le Q_j \end{cases} \tag{4}$$

Accordingly, the expected shortage at the end of the period is:

$$\text{Expected shortage at the end of the period} = \sum_{X_j = Q_j + 1}^{+\infty} (X_j - Q_j) f_{X_j}(x_j) \tag{5}$$

Taking into account the quadratic shortage cost function of Lin and Tsai (2006), we have

$$C_{B_j} = \sum_{X_j = Q_j + 1}^{+\infty} \left(\pi_{1_j}(X_j - Q_j) + \pi_{2_j}(X_j - Q_j)^2 \right) f_{X_j}(x_j) \tag{6}$$

Purchasing Cost

The purchasing cost of the company for the jth product at the beginning of a period is calculated using the incremental discount policy. Let the incremental discount policy be

$$C_{P_j} = \begin{cases} C_{1_j} Q_j & ; \quad 0 < Q_j \le q_{1_j} \\ C_{1_j} q_{1_j} + C_{2_j}(Q_j - q_{2_j}) & ; \quad q_{1_j} < Q_j \le q_{2_j} \\ \vdots & \\ C_{1_j} q_{1_j} + C_{2_j}(q_{2_j} - q_{1_j}) + \ldots + C_{n_j}(Q_j - q_{n-1j}) & ; \quad Q_j \ge q_{n_j} \end{cases} \tag{7}$$

where, q_{l_j} and C_{l_j}; $l = 1, 2, \ldots, n$ are the discount points and the purchasing costs for each unit of the jth product that corresponds to its lth discount break point, respectively. Figure 1 shows this policy.

In order to include the discount policy in the inventory model, we use Eq. 8 to model the incremental discount policy.

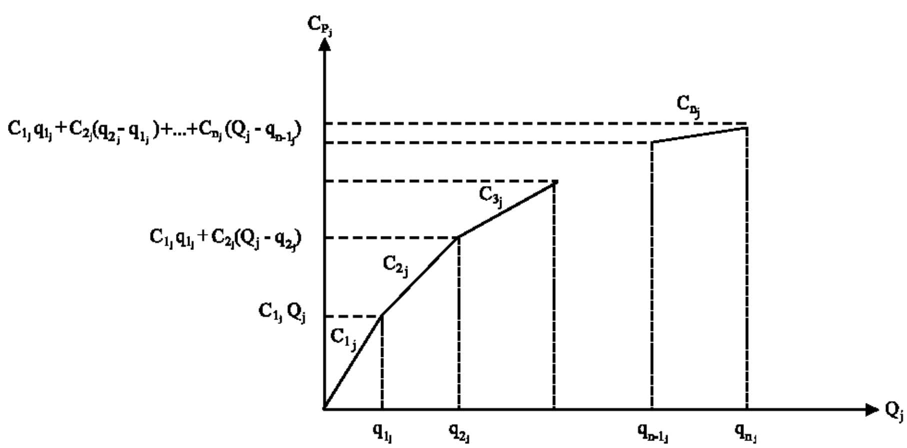

Fig. 1: Incremental discount policy to purchase the jth product

$$
\begin{aligned}
&C_{P_j} = C_{1_j}W_1 + C_{2_j}W_2 + \ldots + C_{n_j}W_n \\
&Q_j = W_{1_j} + W_{2_j} + \ldots + W_{n_j} \\
&q_{1_j}Y_{2_j} \le W_{1_j} \le q_{1_j}Y_{1_j} \\
&\left(q_{2_j} - q_{1_j}\right)Y_{3_j} \le W_{2_j} \le \left(q_{2_j} - q_{1_j}\right)Y_{2_j} \\
&\quad . \\
&\quad . \\
&\quad . \\
&0 \le W_{n_j} \le MY_{n_j} \\
&Y_{i_j} = 0,1 \qquad \forall i, \qquad i = 1,2,\cdots,n
\end{aligned}
\tag{8}
$$

And M is a very big number.

CONSTRAINTS

Three constraints of warehouse space, batch order and service level are modeled in the following subsections.

Warehouse Space Constraint

Since the space required for each packet of the jth product is f_j square meters, the number of packets that have been ordered for the jth product is M_j and that the total available warehouse space is F square meters, the warehouse space constraint becomes

$$f_j M_j \le F \tag{9}$$

Service Level Constraint

Suppose that a maximum $1 - \alpha_j$ percent of the customers' average shortage to the costumers' average demand for the jth product has to occur in a period. In other words, since we determine the shortage at the end of a period, using Eq. 5 we have

$$\frac{\sum_{X_j=Q_j+1}^{+\infty} (X_j - Q_j) f_{X_j}(x_j)}{\lambda_j} \le 1 - \alpha_j \tag{10}$$

Or

$$\frac{\sum_{X_j=0}^{Q_j} (Q_j - X_j) f_{X_j}(x_j)}{\lambda_j} \ge \alpha_j \tag{11}$$

However, we need the orders to be placed in packets of size n_j. In this case, we have

$$Q_j = n_j M_j \tag{12}$$

In short, since the probability mass function of X_j is $f_{X_j}(x_j) = \frac{\lambda^{x_j} e^{-\lambda_j}}{x_j!}$, the single-product model becomes

$$\text{Min } Z = \sum_{X_j=0}^{Q_j-1}\left(h_{1_j}(Q_j - X_j) + h_{2_j}(Q_j - X_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!} +$$

$$\sum_{X_j=Q_j+1}^{+\infty}\left(\pi_{1_j}(X_j - Q_j) + \pi_{2_j}(X_j - Q_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!} + \sum_{i=1}^{n} C_{i_j} W_{i_j}$$

s.t.:

$$Q_j = W_{1_j} + W_{2_j} + \ldots + W_{n_j}$$

$$Q_j = n_j M_j$$

$$f_j M_j \le F$$

$$\frac{\sum_{X_j=0}^{Q_j}(Q_j - X_j) f_{X_j}(x_j)}{\lambda_j} \ge \alpha_j \tag{13}$$

$$q_{1_j} Y_{2_j} \le W_{1_j} \le q_{1_j} Y_{1_j}$$

$$\left(q_{2_j} - q_{1_j}\right) Y_{3_j} \le W_{2_j} \le \left(q_{2_j} - q_{1_j}\right) Y_{2_j}$$

.

.

.

$$0 \le W_{n_j} \le M Y_{n_j}$$

$$Y_{i_j} = 0,1 \qquad \forall i, \qquad i = 1, 2, \cdots, n$$

$$Y_{1_j} \ge Y_{2_j} \ge \cdots \ge Y_{n_j}$$

M is a very large number

$M_j \ge 0$ and Integer

Finally, the single product model in (13) can be easily extended to a multi-product model in (14).

$$\text{Min } Z = \sum_{j=1}^{T}\sum_{X_j=0}^{Q_{j}-1}\left(h_{1j}(Q_j - X_j) + h_{2j}(Q_j - X_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!}$$

$$+\sum_{j=1}^{T}\sum_{X_j=Q_j+1}^{\infty}\left(\pi_{1j}(X_j - Q_j) + \pi_{2j}(X_j - Q_j)^2\right)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!} + \sum_{j=1}^{T}\sum_{i=1}^{n} C_{i_j} W_{i_j}$$

s.t.:

$$\sum_{j=1}^{T} f_j M_j \le F$$

$$\frac{\sum_{X_j=0}^{Q_j}(Q_j - X_j)\frac{e^{-\lambda_j}\lambda_j^{X_j}}{X_j!}}{\lambda_j} \ge \alpha_j \qquad \forall j, \qquad j = 1, 2, \ldots, T$$

$$Q_j = \sum_{i=1}^{n} W_{i_j} \qquad \forall j, \qquad j = 1, 2, ..., T$$

$$Q_j = n_j M_j \qquad \forall j, \quad j = 1, 2, ..., T$$

$$q_{1_j} Y_{2_j} \le W_{1_j} \le q_{1_j} Y_{1_j} \qquad \forall j, \qquad j = 1, 2, ..., T$$

$$\vdots \tag{14}$$

$$\left(q_{i_j} - q_{i-1_j}\right) Y_{i_j} \le W_{i_j} \le \left(q_{i_j} - q_{i-1_j}\right) Y_{i-1_j} \qquad \forall i, \quad i = 2, ..., n_j - 1 \quad \text{and} \quad \forall j, \quad j = 1, 2, ..., T$$

$$0 \le W_{n_{j_j}} \le MY_{n_{j_j}} \qquad \forall j, \qquad j = 1, 2, ..., T \qquad \text{M is a big number}$$

$$Y_{1_j} \ge Y_{2_j} \ge \cdots \ge Y_{n_{j_j}} \qquad \forall j, \qquad j = 1, 2, \cdots, T$$

$$Y_{i_j} = 0, 1 \qquad \forall j, \quad j = 1, 2, \cdots T \quad \text{and} \quad \forall i, \quad i = 1, 2, \cdots, n$$

$$M_j \ge 0 \text{ and integer} \qquad \forall j, \qquad j = 1, 2, \cdots, T$$

A SOLUTION ALGORITHM

Since the model in Eq. 14 is integer-nonlinear in nature, reaching an analytical solution (if any) to the problem is difficult (Gen and Cheng, 1997). As a result, in this section we will try to solve the model by a stochastic search algorithm. Melanie (1996) indicates that the genetic algorithm has been a powerful search technique that can effectively solve similar models. As a result, in this section we apply this algorithm.

Genetic Algorithm

The main information unit of any living organism is the gene, which is a part of a chromosome that determines specific characteristics such as eye-color, complexion, hair-color, etc.

The fundamental principal of Genetic Algorithms (GA), which was inspired by the concept of survival of fittest, first was introduced by Holland (1975). Since then many researchers have applied and expanded this concept in different fields of study. In genetic algorithms, the optimal solution is the winner of the genetic game and any potential solution is assumed to be a creature that is determined by different parameters. These parameters are considered as genes of chromosomes that could be assumed to be binary strings. In this algorithm, the better chromosome is one that is nearer to the optimal solution. In applied applications of genetic algorithms, populations of chromosomes are created randomly. The number of these populations is different in each problem. Some hints about choosing the proper number of population exist in different reports (Man *et al*., 1997).

Genetic algorithms imitate the evolutionary process of species that reproduce. They therefore do not operate on a single current solution, but on a set of current solutions called population. New candidates for the solution are generated with a mechanism called crossover that combines part of the genetic patrimony of each parent and then applies a random mutation. If the new individual, called child or offspring, inherits good characteristics from his parents the probability of its survival increases. This process will continue until a stopping criterion is satisfied. Then, the best offspring is chosen as a near optimum solution.

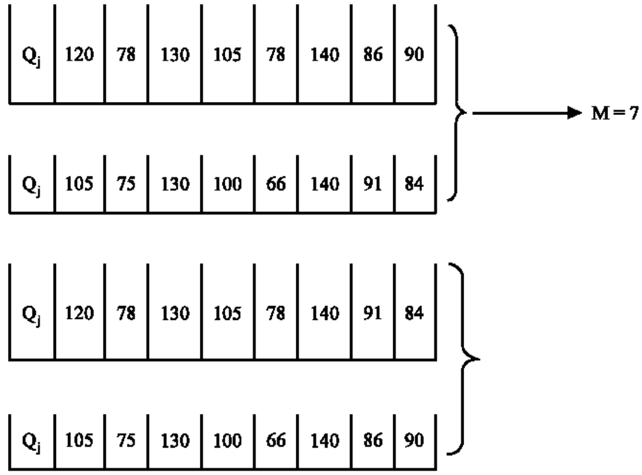

Fig. 2: The single-point crossover operation

Chromosomes

In a GA a chromosome is a string or trail of genes, which is considered as the coded figure of a possible solution (appropriate or none-appropriate). In this study, the chromosomes are strings of the order quantities of the products (Q_j ' s).

Population

A group of chromosomes is called population. One of the characteristics of a GA is that instead of focusing on a single point of the search space (or one chromosome) it works on a population of chromosomes. Therefore, in each step, the algorithm has a population of chromosomes that holds specific qualities more than the previous population. Each population or generation of chromosomes has the same size, which is well-known as the population size and is denoted by N. If N is small, then a small search space will be investigated and the GA algorithm will be very slow. In this research, 10, 100 and 1000 are chosen as different population sizes.

Crossovers

In a crossover operation, it is necessary to mate pairs of chromosomes to create offspring. There are three types of crossover operations: single-point, multi-point and uniform. In a single-point crossover, we break two chromosomes from one point (M) randomly and exchange their broken parts, resulting in two chromosomes. The initial chromosomes are called parents and the chromosomes resulted from the exchange are called offspring. Crossover operates on the parents chromosomes with the possibility of P_c, meaning that with the possibility of P_c the crossover action will occur. If no crossover occurs, the offspring's chromosomes will be the same as their parents. Figure 2 shows a single-point crossover operation in which Q_j shows the chromosome containing the order quantities of the products and the break point is chosen as M = 7.

In this research, we use single point crossover with 0.80, 0.85 and 0.90 as different values of the P_c parameter.

Mutation

Mutation is the second operation in a GA method for exploring new solutions and it operates on each of the chromosomes resulted from the crossover operation. In mutation, we replace a gene with a randomly selected number within the boundaries of the parameter (Gen and Cheng, 1997). We create

Q_j	120	78	130	105	78	140	86	90
RN	0.573	0.131	0.485	0.684	0.743	0.972	0.371	0.824

↓

Q	120	75	140	105	78	140	80	90

Fig. 3: A sample of mutation operation

a random number RN between (0,1) for each gene. If RN is less than a predetermined mutation probability P_m, then the mutation occurs in the gene; otherwise, the mutation operation is not performed in that gene.

More precisely, assume that for a specific gene such as a_j in a chromosome Q_j the generated random number is less than P_m and hence the gene is selected for mutation. Then, we change the value of a_j to the new value a_j^* according to Eq. 15 and 16, randomly and with the same probability:

$$a_j^* = a_j + (u_j - a_j) \times r \times \left(1 - \frac{i}{\max gen}\right) \quad (15)$$

$$a_j^* = a_j - (a_j - l_j) \times r \times \left(1 - \frac{i}{\max gen}\right) \quad (16)$$

where, l_j and u_j are the lower and upper limits of the specified gene, r is a uniform random variable between 0 and 1, i is the number of current generation and maxgen is the maximum number of generations. Note that the value of a_j is transferred to its right or left randomly by Eq. 15 and 16, respectively and r is this percentage. Furthermore,

$$1 - \frac{i}{\max gen}$$

is an index with a value close to one in the first generation and close to zero in the last generation that makes large mutations in the early generations and almost no mutation in the last generations. Figure 3 depicts a mutation operation in which P_m is chosen 0.5.

In this study, 0.01, 0.05 and 0.1 are employed as different values of the P_m parameter.

Objective Function Evaluation

After producing the new chromosomes by crossover and mutation operations, we need to evaluate them. Whether a solution (represented by a chromosome) is appropriate or not depends on the objective function evaluation. In a minimization problem, the more appropriate the solution is the smaller the amount of the objective function (fitness value) will be. The chromosomes that are the fittest will take part in offspring generation with more probability.

Stopping Criterion

The last step in a GA method is to check if the algorithm has found a solution that is good enough to meet the user's expectations. Stopping criteria is a set of conditions such that when satisfied a good solution is obtained. Different criteria used in literature are as follows: (1) Stopping the algorithm after a specific number of generations, (2) no improvement in the objective function and (3) Reaching a specific value of the objective function. In this research, we stop when a predetermined number of consecutive generations is reached. The number of sequential generations depends on the specified problem and the expectations of the user.

In short, the steps involved in the G.A algorithm used in this research are:

- Setting the parameters P_c, P_m and N
- Initializing the population randomly
- Evaluating the objective function
- Selecting individual for mating pool
- Applying the crossover operation for each pair of chromosomes with probability P_c
- Applying mutation operation for each chromosome with probability P_m
- Replacing the current population by the resulting mating pool
- Evaluating the objective function
- If stopping criteria is met, then stop. Otherwise, go to step 5

In order to demonstrate the application of the proposed GA algorithm and evaluate its performances, in the next section we bring a numerical example.

A Numerical Example

Consider a multi-product Newsboy problem with fifteen products and general data given in Table 1. The total available warehouse space is 1750 square meters. Table 2 shows different values of the GA parameters used to obtain the solution. In this research all the possible combinations of the GA parameters (P_c, P_m and N) are employed and using the max(max) criterion the best combination of the parameters has been selected. Table 3 shows the best solution of the algorithm. The best combination of the GA algorithm is shown in Table 4. Furthermore, the convergence path of the best result of the objective function values in different generations is shown in Fig. 4.

Table 1: General data of the numerical example

Product (j)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n_j	5	3	10	5	3	10	1	6	10	6	5	3	1	6	1
α_j	0.8	0.8	0.9	0.75	0.7	0.7	0.8	0.85	0.85	0.7	0.8	0.7	0.7	0.75	0.6
f_j	3	5	2	3	4	5	3	4	2	5	3	6	4	3	1
h_{1j}	1	2	4	4	2	5	1	2	3	5	3	5	3	3	2
h_{2j}	2	3	5	5	3	6	2	3	4	6	4	6	4	4	3
π_{1j}	7	12	30	30	40	45	16	21	42	34	20	15	10	20	47
π_{2j}	12	17	35	35	45	50	21	26	47	39	25	20	15	25	52
C_{1j}	18	20	30	20	10	35	15	25	33	30	12	28	100	90	100
C_{2j}	15	17	25	18	8	30	12	21	30	27	10	25	90	80	90
C_{3j}	12	15	23	15	7	28	10	18	27	24	8	22	80	70	80
C_{4j}	10	8	18	13	5	20	8	16	21	21	7	20	60	60	75
q_{1j}	30	20	40	30	15	40	15	15	50	40	50	40	20	25	30
q_{2j}	90	35	70	90	30	70	35	35	90	70	70	70	40	45	45
q_{3j}	100	90	110	100	50	140	55	70	130	100	120	100	60	70	70
λ_j	102	73	123	95	62	19	69	83	120	89	115	91	52	76	66

Table 2: The parameters of the GA method

P_c	P_m	N
0.80	0.01	10
0.85	0.05	100
0.90	0.10	1000

Table 3: The best order quantities (Q_j 's)

Product	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q_j	110	78	130	100	69	140	77	90	130	96	125	96	51	78	72
Z	45197.7														

Table 4: The best combination of the GA parameters

P_c	P_m	N
0.90	0.05	1000

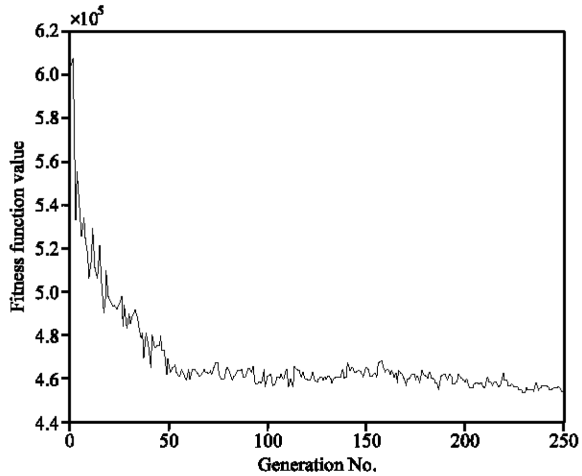

Fig. 4: The convergence path of the best result

CONCLUSION AND RECOMMENDATIONS FOR FUTURE RESEARCH

In this study, a multi-product Newsboy model with incremental discounts and batch orders in which there are minimum service level and warehouse space are constraints, was developed. Then, the meta-heuristic solution algorithm of GA has been proposed to solve the obtained non-linear integer model. At the end, a numerical example was given to demonstrate the application of the proposed method. Some of the future works of this research are:

- Some parameters of the model can be considered fuzzy. In this case, a fuzzy system may be employed to analyze the problem
- In addition to GA algorithm, some other meta-heuristic algorithms like Simulated Annealing, Tabu-Search and Ant-Colony optimization may be employed to solve the integer non-linear model
- The lead-time may be considered nonzero or a random variable
- The holding and shortage costs may be considered to occur during a period

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