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Context-Dependent DEA and Weighted-Russell Measures to Evaluate Progress and Regress

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Abstract: This study develops the context-dependent DEA by incorporating value judgment, inspired by Russell measure of technical efficiency to measurement technical efficiency. Next, a way of identifying progress or regress from a current period to next one is extended. This extension of DEA models, is illustrated by an empirical application to bank branches.

Key words: DEA, Russell measure, context-dependent

INTRODUCTION

Data Envelopment Analysis (DEA), developed by Charnes *et al*. (1978), provides a non-parametric methodology for evaluating the efficiency of each of a set of comparable Decision Making Units (DMUs), relative to one another. In the original model of Charnes *et al*. (1978) efficiency is represented by the ratio of weighted sum of outputs to the weighted sum of inputs in a specific time period. Many additional theoretical papers in the field have adapted models and applications (Tone, 2001; Sengopta, 2005; Sueyoshi and Sekitani, 2005).

In previous research efforts, Färe and Lovell (1978) approached the measurement of technical efficiency by suggesting some desirable properties that an ideal technical efficiency measure should satisfy and then, proposed a measure which satisfied them. This measure was called Russell measure of technical efficiency. Russell measure was extended to the multiple output case by Färe *et al*. (1983). Unfortunately, this approach has a difficulty in the efficiency measurement, because, the objective function is formulated as a non-linear programming problem. Sueyoshi and Sekitani (2007) proposed a re-formulation of the Russell measure by a second-order cone programming model and applied the primal-dual interior point algorithm to solve the Russell measure.

In the applications of DEA presented in the literature, the models presented are designed to obtain a measure of efficiency in a single period. In many instances, however, the DMUs involved may examine in several periods. In such situations, we are often interested to know if there is a progress or regress from period t to t+1. Tulkens and Eeckaut (1995) have presented a way to measure non-parametrically efficiency, progress and regress from panel data.

In this study, we consider the measurement of efficiency and progress or regress of DMUs from DEA perspective. We assume that the productive activities of DMUs are observed in T periods. Over the time periods, it is important to know that whether a specific DMU_o has progress or regress from period t to t+1. We extend the context-dependent DEA by incorporating value judgment to determine progress and regress measures. The objective here is two-fold: first, this study proposes a re-formulation of Russell measure by incorporating value judgment into the inputs and outputs. Next, we extend the context-dependent DEA by incorporating value judgment into the inputs and outputs to determine progress and regress measures of DMUs in two successive periods.

A RE-FORMULATION OF RUSSELL MEASURE

Assume we have n decision making units, each consumes m inputs to produce s outputs. We denote by y_{rj} the level of the r-th output, r = 1,..., s and by x_{ij} the level of the i-th input, i = 1,..., m to the j-th unit. The Russell graph measure of technical efficiency was defined as a combination of the input and output Russell measures of technical efficiency (Färe *et al*., 1983). For a given DMU_o: (x_o, y_o), the value of this measure can be obtained from the following nonlinear programming problem:

$$
\begin{aligned}
h_o^* &= \text{Min} \quad \frac{1}{m+s}\Big[\sum_{i=1}^{m}\theta_i + \sum_{r=1}^{s}\frac{1}{\varphi_r}\Big] \\
\text{s.t.} & \\
& \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{io}, \quad i = 1,\cdots,m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \ge \varphi_r y_{ro}, \quad r = 1,\cdots,s, \\
& \theta_i \le 1,\ \varphi_r \ge 1, \quad \text{for all } i,r, \\
& \lambda_j \ge 0, \quad j = 1,\cdots,n
\end{aligned}
\tag{1}
$$

In this formulation, the constraints $\theta_i \le 1$ and $\varphi_r \ge 1$ are the requirements for dominance. As we can see, the dominance factors θ_i and φ_r appear in the objective function in an additive way. Instead of combining the input and output Russell measures in an additive way as in (1), Pastor *et al*. (1999) defined a measure as the ratio between them. They proposed the following model:

$$
\begin{aligned}
& \text{Min} \quad \frac{\frac{1}{m}\sum_{i=1}^{m}\theta_i}{\frac{1}{s}\sum_{r=1}^{s}\varphi_r} \\
\text{s.t.} & \\
& \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{io},\ i = 1,\cdots,m, \\
& \sum_{j=1}^{n}\lambda_j y_{rj} \le \varphi_r y_{ro},\ r = 1,\cdots,s, \\
& \theta_i \le 1,\ \varphi_r \ge 1,\ \text{for all } i,r, \\
& \lambda_j \ge 0,\ j = 1,\cdots,n
\end{aligned}
\tag{2}
$$

These formulations are nonlinear and do not require a priori information on the importance of the attributes (inputs and outputs). However, different attributes play different roles in the evaluation of a DMU's performance. In order to incorporate such a priori information, let ρ_{io} and γ_{ro} are weights related to the inputs and outputs of DMU_o, respectively, such that

$$\sum_{i=1}^{m}\rho_{io} = 1 \quad \text{and} \quad \sum_{r=1}^{s}\gamma_{ro} = 1$$

We define an efficiency index μ_o as follows:

$$\mu_o = \frac{\sum_{i=1}^{m}\rho_{io}\theta_i}{\sum_{r=1}^{s}\gamma_{ro}\varphi_r} \tag{3}$$

The numerator in (3) is a weighted sum of the dominance factors θ_i and the denominator is a weighted sum of the dominance factors φ_r. The larger the γ_{ro} is the more importance the y_{ro} and the smaller the ρ_{io} is the more importance the x_{io}. In an effort to estimate the efficiency of DMU $_o$, we formulate the following fractional program:

$$\mu^*_o = \text{Min} \quad \frac{\sum_{i=1}^{m} \rho_{io}\theta_i}{\sum_{r=1}^{s} \gamma_{ro}\varphi_r}$$

$$\begin{aligned} \text{s.t.} & \\ & \sum_{j=1}^{n} \lambda_j x_{ij} = \theta_i x_{io},\ i = 1,\cdots,m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} = \varphi_r y_{ro},\ r = 1,\cdots,s, \\ & \theta_i \le 1,\ \varphi_r \ge 1, \text{ for all } i, r, \\ & \lambda_j \ge 0,\ j = 1,\cdots,n, \end{aligned} \tag{4}$$

Minimizing μ_o in (4) means that the numerator is minimized and simultaneously, the denominator is maximized and hence (4) measures how far DMU_o is from the frontier. Hence, we are looking for a point on the frontier so that the weighted distance from x_o to frontier is minimized and simultaneously, the weighted distance from y_o to frontier is maximized. Let μ_o* be the optimal objective value to (4). Based on this value, we define an efficient DMU_o as follows:

Definition 1

A DMU_o: (x_o, y_o) is efficient if and only if $\mu^*_o = 1$.

The linear fractional model (4) can be converted to a non-fractional form in the usual manner of Charnes and Cooper (1962). Specifically, make the transformation

$$\beta = [\sum_{r=1}^{s} \gamma_{ro}\varphi_r]^{-1}$$

and let $\beta\lambda_j = \bar{\lambda}_j, \beta\theta_i = \bar{\theta}_i, \beta\varphi_r = \bar{\varphi}_r$, we have

$$\mu^*_o = \text{Min} \quad \sum_{i=1}^{m} \rho_{io}\bar{\theta}_i$$

$$\begin{aligned} \text{s.t.} & \\ & \sum_{r=1}^{s} \gamma_{ro}\bar{\varphi}_r = 1, \\ & \sum_{j=1}^{n} \bar{\lambda}_j x_{ij} = \bar{\theta}_i x_{io},\ i = 1,\cdots,m, \\ & \sum_{j=1}^{n} \bar{\lambda}_j y_{rj} = \bar{\varphi}_r y_{ro},\ r = 1,\cdots,s, \\ & \bar{\theta}_i \le \beta,\ \bar{\varphi}_r \ge \beta, \text{ for all } i, r, \\ & \bar{\lambda}_j \ge 0,\ j = 1,\cdots,n, \end{aligned} \tag{5}$$

MEASURING PROGRESS AND REGRESS

Context-Dependent DEA

It is assumed that there are n decision making units (DMU_j: j = 1,...,n) and their productive activities are examined in T periods. In the t-th period, each DMU_o uses $x_o^{(t)}$

(an m-dimensional vector of inputs) in order to produce $y_o^{(t)}$ (an s-dimensional vector of outputs). Based upon (4), we propose a stratification procedure for period t in the same manner to the original context-dependent DEA (Seiford and Zhu, 2003; Morita *et al*., 2005) as $J_{l+1}^{(t)} = J_l^t - E_l^t$, where $E_l^{(t)} = \{DMU_o \in J_l^{(t)}: \pi_{ol}^{(t)} = 1\}$ and $\pi_{ol}^{(t)}$ is the optimal value to the following linear programming model for each l:

$$\begin{aligned}
&\pi_{ol}^{(t)} = \text{Min} \quad \sum_{i=1}^{m} \rho_{io}\bar{\theta}_i \\
&\text{s.t.} \\
&\qquad \sum_{r=1}^{s} \gamma_{ro}\bar{\varphi}_r = 1, \\
&\qquad \sum_{j\in J_l^{(t)}} \bar{\lambda}_j x_{ij}^{(t)} = \bar{\theta}_i x_{io}^{(t)},\ i = 1,\cdots,m, \\
&\qquad \sum_{j\in J_l^{(t)}} \bar{\lambda}_j y_{rj}^{(t)} = \bar{\varphi}_r y_{ro}^{(t)},\ r = 1,\cdots,s, \\
&\qquad \bar{\theta}_i \le \beta,\ \bar{\varphi}_r \ge \beta,\ \text{for all } i, r, \\
&\qquad \bar{\lambda}_j \ge 0,\ j \in J_l^{(t)},
\end{aligned} \tag{6}$$

in which $J_1^{(t)} = \{(x_j^{(t)}, y_j^{(t)}): j = 1,...,n\}$. The DMUs in $E_1^{(t)}$ define the first-level efficient frontier in period t. When $l = 2$, model (6) gives the second-level efficient frontier after the exclusion of the first-level efficient units and so on. It is easy to show that the sets $J_l^{(t)}$ and $E_l^{(t)}$ have the following properties:

- $J_l^{(t)} = \bigcup E_l^{(t)}$ and $E_l^{(t)} \cap E_{l'}^{(t)} = \Phi$ for $l \neq l'$
- The DMUs in $E_l^{(t)}$ are dominated by the DMUs in $E_{l'}^{(t)}$ for $l > l'$

Progress and Regress Measures

Over the time periods, it is important to know that whether a specific DMU_o has progress or regress from period t to t+1. In economics and management, the notion of progress has been associated with outward shifts of production frontiers and similarly, inward shifts refer to regress. Here, we provide suitable definitions and methods for measuring progress and regress in DEA context.

Definition 2

A specific $DMU_o^{(t)} \in E_l^{(t)}$ is said to induce progress with respect to the context $E_l^{(t)}$ from period t to t+1, if and only if $DMU_o^{(t+1)}$ is un-dominated at period t+1 by units in the set $J_l^{(t)}$.

Definition 3

A specific $DMU_o^{(t)} \in E_l^{(t)}$ is said to induce *regress* with respect to the context $E_l^{(t)}$ from period t to t+1, if and only if $DMU_o^{(t+1)}$ is dominated by one or several units in the set $J_l^{(t)}$.

For any $DMU_o^{(t)}$ at reference set $J_l^{(t)}$ in period t, proceed according to the following three-steps:

Step 1

Compute the efficiency of $DMU_o^{(t+1)}$ using the following model:

$$
\begin{aligned}
&\pi_{ol}^{(t+1)} = \text{Min} \quad \sum_{i=1}^{m} \rho_{io}\bar{\theta}_i \\
&\text{s.t.} \\
&\quad \sum_{r=1}^{s} \gamma_{ro}\bar{\varphi}_r = 1, \\
&\quad \sum_{j\in J_l^{(t)}} \bar{\lambda}_j x_{ij}^{(t)} + \delta x_{io}^{(t+1)} = \bar{\theta}_i x_{io}^{(t+1)}, \; i = 1,\cdots,m, \\
&\quad \sum_{j\in J_l^{(t)}} \bar{\lambda}_j y_{rj}^{(t)} + \delta y_{ro}^{(t+1)} = \bar{\varphi}_r y_{ro}^{(t+1)}, \; r = 1,\cdots,s, \\
&\quad \bar{\theta}_i \le \beta, \; \bar{\varphi}_r \ge \beta, \text{ for all } i,r, \\
&\quad \bar{\lambda}_j, \delta \ge 0, \; j \in J_l^{(t)},
\end{aligned}
\tag{7}
$$

Step 2

If $\pi_{ol}^{(t+1)} < 1$, then $DMU_o^{(t+1)}$ is found inefficient with respect to the context $E_l^{(t)}$ and it has a regress from t to t+1. The regress degree of DMU_o is denoted by $rd_{ol}^{(t+1)} = \pi_{ol}^{(t+1)} - 1 < 0$. If $\pi_{ol}^{(t+1)} \ge 1$, then $DMU_o^{(t+1)}$ is found efficient. Go to step 3.

Step 3

Solve the following linear programming model:

$$
\begin{aligned}
&\pi_{ol}^{(t+1)} = \text{Min} \quad \sum_{i=1}^{m} \rho_{io}\bar{\theta}_i \\
&\text{s.t.} \\
&\quad \sum_{r=1}^{s} \gamma_{ro}\bar{\varphi}_r = 1, \\
&\quad \sum_{j\in J_1^{(t)}} \bar{\lambda}_j x_{ij}^{(t)} = \bar{\theta}_i x_{io}^{(t+1)}, \; i = 1,\cdots,m, \\
&\quad \sum_{j\in J_1^{(t)}} \bar{\lambda}_j y_{rj}^{(t)} = \bar{\varphi}_r y_{ro}^{(t+1)}, \; r = 1,\cdots,s, \\
&\quad \bar{\theta}_i \le \beta, \; \bar{\varphi}_r \ge \beta, \text{ for all } i,r, \\
&\quad \bar{\lambda}_j \ge 0, \; j \in J_1^{(t)},
\end{aligned}
\tag{8}
$$

if $\pi_{ol}^{(t+1)} = 1$, there is no progress or regress. But, if $\pi_{ol}^{(t+1)} > 1$, then $DMU_o^{(t+1)}$ has a progress from t to t+1. The progress degree of DMU_o is denoted by $pd_{ol}^{(t+1)} = \pi_{ol}^{(t+1)} - 1 > 0$.

AN EMPIRICAL STUDY

We shall illustrate our general approach for progress and regress measurement with the analysis of bank branches activities. The data set consists of 50 bank branches located in 7 regions in Iran. The data for this analysis are derived from operations during 2004 and 2005. We use nine variables from the data set as inputs and outputs. Inputs include number of staff (x_1), number of computer terminals (x_2), operational costs(excluding staff costs) (x_3) and space (x_4) and outputs include deposits (y_1), loans (y_2), number of subscribers (y_3), charges (y_4) and profits (y_5). Table 1 and 2 contain a listing of the original data in two periods. Table 3 and 4 show the period measures. By using the DEA model (5), we obtain three levels of efficient frontiers in period one as follows:

$$
\begin{aligned}
E_1^{(1)} &= \{\#1,\#2,\#5,\#16,\#17,\#20,\#21,\#27,\#30,\#36,\#38,\#40,\#44,\#45,\#49,\#50\} \\
E_2^{(1)} &= \{\#3,\#4,\#7,\#8,\#15,\#18,\#19,\#22,\#25,\#26,\#28,\#29,\#32,\#33,\#34,\#35, \\
&\quad \#37,\#39,\#42,\#46,\#47,\#48\} \\
E_3^{(1)} &= \{\#6,\#9,\#10,\#11,\#12,\#13,\#14,\#23,\#24,\#31,\#41,\#43\}
\end{aligned}
$$

Table 1: Bank branches data in period 1

Branch	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	y_5
1	0.76	0.51	0.44	0.58	0.88	0.68	0.06	0.98	0.97
2	0.55	0.37	0.69	0.57	0.95	0.75	0.44	1.00	0.96
3	0.49	0.42	0.50	1.00	0.72	0.73	0.76	0.26	0.70
4	0.36	0.95	0.76	0.83	0.77	1.00	0.05	0.39	0.68
5	0.72	0.07	0.56	0.57	0.92	0.51	0.05	0.27	0.97
6	0.62	0.57	0.63	0.44	0.50	0.15	0.01	0.20	0.77
7	0.59	0.46	0.56	0.40	0.74	0.91	0.04	0.36	0.97
8	0.45	0.20	0.50	0.58	0.63	0.58	0.04	0.22	1.05
9	0.56	0.63	0.63	0.77	0.47	0.25	0.02	0.30	0.77
10	0.57	0.50	0.76	0.87	0.33	0.63	0.02	0.32	0.85
11	0.37	1.00	0.81	0.69	0.72	0.60	0.02	0.29	0.68
12	0.52	0.93	1.00	0.58	0.54	0.22	0.01	0.14	1.00
13	0.61	0.70	0.81	0.96	1.00	0.02	0.01	0.29	0.97
14	0.65	0.55	0.63	0.83	0.75	0.65	0.03	0.32	0.72
15	0.57	0.19	0.56	0.35	0.66	0.71	0.02	0.28	0.57
16	0.58	0.31	0.44	0.12	0.50	0.38	0.02	0.24	0.68
17	0.44	0.38	0.37	0.21	0.38	1.29	0.03	0.40	0.72
18	0.54	0.28	0.50	0.25	0.49	0.44	0.02	0.42	0.61
19	0.78	0.13	0.50	0.49	0.56	0.46	0.03	0.24	0.69
20	0.57	0.26	0.63	0.22	0.86	2.15	0.12	0.51	0.61
21	0.51	0.38	0.63	0.24	0.85	0.59	1.00	0.22	0.44
22	0.36	0.69	0.57	0.56	0.49	0.11	0.01	0.41	0.69
23	0.30	0.54	0.82	0.48	0.46	0.37	0.03	0.19	0.63
24	0.38	0.88	0.88	0.47	0.57	0.34	0.01	0.36	0.35
25	0.68	0.22	0.38	0.43	0.55	1.07	0.02	0.27	0.44
26	0.49	0.46	0.94	0.32	0.85	0.54	0.06	0.29	0.55
27	0.64	0.23	0.19	0.40	0.78	2.64	0.09	0.41	0.96
28	0.46	0.75	0.57	0.57	0.77	1.25	0.04	0.37	0.82
29	1.00	0.23	0.50	0.47	0.65	0.90	0.02	0.45	0.36
30	0.15	0.10	0.38	0.40	0.40	0.33	0.01	0.12	0.69
31	0.84	0.35	0.50	0.31	0.58	0.58	0.11	0.32	0.61
32	0.38	0.15	0.51	0.25	0.43	0.64	0.01	0.21	0.40
33	0.68	0.50	0.69	0.16	0.64	1.02	0.50	0.33	0.39
34	0.57	0.38	0.38	0.35	0.77	0.68	0.02	0.33	0.36
35	0.39	0.63	0.76	0.35	0.77	0.58	0.07	0.23	0.48
36	0.48	0.59	0.70	0.33	0.91	0.61	0.03	0.37	0.51
37	0.73	0.30	0.38	0.40	0.60	1.24	0.02	0.40	0.55
38	0.34	0.25	0.51	0.21	0.36	0.36	0.11	0.20	0.61
39	0.51	0.20	0.45	0.46	0.43	1.16	0.02	0.28	0.39
40	0.50	0.50	0.82	0.46	0.87	3.24	0.28	0.32	0.50
41	0.39	0.54	0.69	0.32	0.55	0.33	0.01	0.32	0.44
42	0.55	0.66	0.75	0.27	0.71	1.32	0.02	0.44	0.36
43	0.52	0.53	0.69	0.21	0.60	0.78	0.30	0.26	0.30
44	0.38	0.10	0.63	0.23	0.55	0.46	0.02	0.18	0.52
45	0.72	0.14	0.38	0.35	0.66	0.41	0.24	0.68	0.43
46	0.46	0.30	0.63	0.35	0.59	0.79	0.76	0.23	0.40
47	0.46	0.61	0.88	0.21	0.64	0.62	0.08	0.36	0.53
48	0.52	0.19	0.44	0.35	0.57	0.57	0.01	0.40	0.44
49	0.73	0.50	0.44	0.18	0.74	2.29	2.24	0.35	0.67
50	0.92	0.13	0.32	0.37	0.79	0.89	0.03	0.25	0.69

Table 2: Bank branches data in period 2

Branch	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	y_5
1	0.70	0.51	0.47	0.59	0.95	0.33	0.03	0.94	1.00
2	0.52	0.37	0.64	0.58	0.76	0.29	0.24	1.00	0.90
3	0.49	0.42	0.51	1.00	0.76	0.28	0.38	0.25	0.89
4	0.40	0.95	0.76	0.82	0.81	0.34	0.03	0.38	0.91
5	0.62	0.07	0.54	0.58	0.99	0.16	0.03	0.28	0.99
6	0.58	0.57	0.63	0.44	0.50	0.08	0.01	0.21	0.87
7	0.55	0.46	0.51	0.40	0.80	0.32	0.02	0.36	0.99
8	0.43	0.20	0.54	0.57	0.68	0.19	0.02	0.23	0.91

Table 2: Continued

Branch	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	y_5
9	0.57	0.63	0.63	0.76	0.52	0.10	0.01	0.29	0.70
10	0.52	0.50	0.77	0.86	0.35	0.26	0.02	0.32	0.72
11	0.37	1.00	0.75	0.67	0.78	0.21	0.01	0.29	0.74
12	0.55	0.93	1.00	0.57	0.58	0.10	0.01	0.14	0.84
13	0.73	0.70	0.79	0.94	1.00	0.01	0.01	0.34	0.90
14	0.65	0.55	0.65	0.81	0.91	0.22	0.02	0.31	1.00
15	0.54	0.19	0.54	0.34	0.68	0.25	0.01	0.28	0.90
16	0.54	0.31	0.41	0.11	0.50	0.16	0.01	0.23	0.87
17	0.41	0.38	0.40	0.21	0.41	0.45	0.01	0.39	0.67
18	0.53	0.28	0.51	0.24	0.52	0.15	0.01	0.41	0.59
19	0.76	0.13	0.51	0.48	0.59	0.16	0.01	0.24	0.70
20	0.58	0.26	0.61	0.22	0.93	0.77	0.07	0.53	0.63
21	0.50	0.38	0.60	0.24	0.97	0.20	0.86	0.20	0.64
22	0.33	0.69	0.48	0.55	0.57	0.05	0.06	0.39	0.72
23	0.28	0.54	0.75	0.47	0.50	0.13	0.01	0.19	0.80
24	0.30	0.88	0.89	0.46	0.58	0.11	0.01	0.38	0.64
25	0.67	0.22	0.44	0.42	0.60	0.39	0.01	0.27	0.63
26	0.45	0.46	0.91	0.31	0.89	0.19	0.03	0.28	0.53
27	0.55	0.23	0.12	0.39	0.89	0.83	0.83	0.40	0.67
28	0.47	0.75	0.57	0.55	0.76	0.41	0.02	0.37	0.57
29	1.00	0.23	0.54	0.46	0.69	0.42	0.01	0.45	0.53
30	0.15	0.10	0.44	0.38	0.40	0.12	0.00	0.12	0.51
31	0.79	0.35	0.57	0.30	0.55	0.18	0.06	0.31	0.31
32	0.36	0.15	0.55	0.25	0.45	0.21	0.01	0.21	0.41
33	0.70	0.50	0.58	0.16	0.72	0.45	0.22	0.33	0.51
34	0.54	0.38	0.44	0.35	0.83	0.32	0.01	0.33	0.42
35	0.35	0.63	0.79	0.35	0.86	0.22	0.04	0.22	0.57
36	0.44	0.59	0.64	0.33	0.98	0.27	0.01	0.36	0.79
37	0.70	0.30	0.35	0.39	0.62	0.49	0.02	0.39	0.70
38	0.31	0.25	0.47	0.21	0.37	0.22	0.06	0.19	0.62
39	0.45	0.20	0.41	0.45	0.47	0.44	0.01	0.29	0.47
40	0.47	0.50	0.75	0.45	0.93	1.00	0.13	0.31	0.46
41	0.37	0.54	0.64	0.31	0.60	0.13	0.01	0.32	0.42
42	0.51	0.66	0.70	0.26	0.75	0.43	0.01	0.43	0.42
43	0.41	0.53	0.64	0.21	0.64	0.32	0.13	0.25	0.54
44	0.40	0.10	0.58	0.22	0.58	0.16	0.01	0.17	0.55
45	0.69	0.14	0.35	0.34	0.70	0.29	0.11	0.66	0.48
46	0.44	0.30	0.58	0.34	0.59	0.33	0.35	0.22	0.76
47	0.46	0.61	0.81	0.21	0.61	0.21	0.21	0.35	0.37
48	0.47	0.19	0.40	0.34	0.57	0.23	0.01	0.40	0.54
49	0.69	0.50	0.40	0.18	0.74	0.77	1.00	0.35	0.63
50	0.89	0.13	0.29	0.36	0.83	0.27	0.02	0.26	0.65

Table 3: Efficiency scores in period 1

Branch	μ^*_o	$\bar{\theta}_1$	$\bar{\theta}_2$	$\bar{\theta}_3$	$\bar{\theta}_4$	$\bar{\varphi}_1$	$\bar{\varphi}_2$	$\bar{\varphi}_3$	$\bar{\varphi}_4$	$\bar{\varphi}_5$
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	0.608	0.733	1.029	0.733	1.843	0.741	0.733	0.566	0.710	0.268
4	0.623	0.762	0.762	2.915	0.762	1.213	0.762	0.213	0.691	0.586
5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	0.054	0.094	0.844	8.027	0.102	0.066	0.066	0.048	0.050	0.042
7	0.252	0.256	0.487	7.351	0.256	0.256	0.256	0.229	0.256	0.256
8	0.566	0.664	1.210	3.105	0.738	0.571	0.571	0.571	0.571	0.543
9	0.075	0.145	0.670	8.021	0.100	0.100	0.100	0.060	0.075	0.040
10	0.074	0.215	0.260	8.287	0.099	0.099	0.099	0.076	0.067	0.041
11	0.227	0.306	0.306	7.212	0.306	0.369	0.306	0.079	0.223	0.205
12	0.057	0.107	0.510	8.343	0.158	0.077	0.077	0.029	0.045	0.063
13	0.036	0.041	5.135	1.965	0.054	0.041	0.041	0.028	0.041	0.025
14	0.104	0.135	0.390	8.399	0.136	0.135	0.135	0.109	0.106	0.042
15	0.231	0.259	0.339	7.533	0.259	0.290	0.259	0.259	0.182	0.235

Table 3: Continued

Branch	μ_o^*	$\bar{\theta}_1$	$\bar{\theta}_2$	$\bar{\theta}_3$	$\bar{\theta}_4$	$\bar{\varphi}_1$	$\bar{\varphi}_2$	$\bar{\varphi}_3$	$\bar{\varphi}_4$	$\bar{\varphi}_5$
16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
18	0.159	0.234	0.651	7.432	0.170	0.170	0.170	0.170	0.133	0.170
19	0.308	0.467	0.561	5.872	0.380	0.380	0.280	0.380	0.316	0.291
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
22	0.132	0.207	1.234	6.679	0.173	0.173	0.173	0.062	0.133	0.112
23	0.198	0.302	0.682	6.729	0.296	0.296	0.296	0.113	0.132	0.191
24	0.110	0.175	0.345	8.144	0.175	0.229	0.175	0.053	0.080	0.086
25	0.144	0.187	0.187	8.303	0.187	0.214	0.140	0.187	0.158	0.098
26	0.364	0.418	0.468	6.160	0.418	0.428	0.418	0.326	0.284	0.418
27	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
28	0.403	0.463	0.463	5.830	0.463	0.463	0.463	0.191	0.463	0.366
29	0.118	0.162	0.175	8.482	0.162	0.241	0.106	0.162	0.123	0.100
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
31	0.282	0.424	1.288	4.967	0.392	0.392	0.278	0.392	0.244	0.260
32	0.134	0.166	0.181	8.481	0.166	0.166	0.166	0.166	0.084	0.130
33	0.583	0.747	1.373	2.209	0.732	1.064	0.640	0.556	0.432	0.732
34	0.209	0.222	0.551	7.408	0.222	0.424	0.222	0.182	0.222	0.188
35	0.789	0.881	0.881	1.557	1.092	1.058	0.881	0.608	0.709	0.881
36	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
37	0.135	0.168	0.199	8.451	0.164	0.164	0.138	0.164	0.142	0.095
38	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
39	0.131	0.205	0.195	8.187	0.195	0.214	0.166	0.195	0.094	0.076
40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
41	0.092	0.127	0.311	8.581	0.127	0.127	0.127	0.064	0.070	0.084
42	0.133	0.162	0.162	8.486	0.162	0.276	0.162	0.092	0.103	0.161
43	0.486	0.650	1.420	2.523	0.784	1.065	0.650	0.388	0.360	0.462
44	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
46	0.612	0.799	1.528	0.799	1.220	0.947	0.799	0.701	0.502	0.384
47	0.408	0.531	1.218	4.190	0.531	0.531	0.531	0.232	0.272	0.531
48	0.187	0.208	0.267	8.023	0.208	0.236	0.208	0.208	0.166	0.165
49	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

We let $\rho_1 = 0.35$, $\rho_2 = 0.15$, $\rho_3 = 0.3$, $\rho_4 = 0.2$, $\gamma_1 = 0.5$, $\gamma_2 = 0.15$, $\gamma_3 = 0.1$, $\gamma_4 = 0.2$, $\gamma_5 = 0.05$,

Table 4: Efficiency scores in period 2

Branch	μ_o^*	$\bar{\theta}_1$	$\bar{\theta}_2$	$\bar{\theta}_3$	$\bar{\theta}_4$	$\bar{\varphi}_1$	$\bar{\varphi}_2$	$\bar{\varphi}_3$	$\bar{\varphi}_4$	$\bar{\varphi}_5$
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	0.579	0.845	1.590	0.975	1.036	0.691	0.691	0.395	0.650	0.414
4	0.423	0.540	0.666	4.971	0.531	0.531	0.531	0.129	0.447	0.420
5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	0.065	0.150	0.414	8.390	0.101	0.068	0.068	0.045	0.068	0.068
7	0.199	0.267	0.567	7.155	0.268	0.250	0.250	0.147	0.133	0.250
8	0.366	0.448	0.858	5.302	0.495	0.367	0.367	0.367	0.365	0.367
9	0.053	0.160	0.796	7.703	0.129	0.090	0.090	0.035	0.021	0.050
10	0.115	0.478	0.549	6.245	0.224	0.189	0.189	0.087	0.054	0.097
11	0.172	0.217	0.221	8.040	0.217	0.223	0.217	0.058	0.191	0.149
12	0.049	0.121	0.582	8.064	0.210	0.072	0.072	0.020	0.024	0.067
13	0.022	0.044	2.827	5.405	0.058	0.037	0.037	0.016	0.009	0.021
14	0.112	0.183	0.596	7.668	0.225	0.144	0.144	0.078	0.091	0.114
15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
18	0.138	0.218	0.650	7.562	0.149	0.149	0.149	0.149	0.114	0.149
19	0.175	0.331	0.630	6.700	0.290	0.243	0.158	0.243	0.137	0.209
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 4: Continued

Branch	μ^*_o	$\bar{\theta}_1$	$\bar{\theta}_2$	$\bar{\theta}_3$	$\bar{\theta}_4$	$\bar{\varphi}_1$	$\bar{\varphi}_2$	$\bar{\varphi}_3$	$\bar{\varphi}_4$	$\bar{\varphi}_5$
21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
24	0.213	0.287	0.519	7.071	0.287	0.287	0.287	0.068	0.164	0.266
25	0.082	0.171	0.245	8.376	0.170	0.122	0.094	0.120	0.032	0.106
26	0.318	0.366	0.406	6.591	0.366	0.471	0.366	0.293	0.242	0.366
27	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
28	0.151	0.260	0.442	7.389	0.258	0.266	0.258	0.071	0.057	0.162
29	0.066	0.141	0.206	8.706	0.107	0.137	0.063	0.107	0.030	0.094
30	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
31	0.207	0.484	1.354	4.457	0.388	0.631	0.215	0.214	0.074	0.388
32	0.099	0.171	0.335	8.250	0.161	0.141	0.132	0.132	0.019	0.134
33	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
34	0.082	0.109	0.260	8.737	0.124	0.159	0.109	0.073	0.037	0.109
35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
37	0.127	0.249	0.286	7.888	0.177	0.172	0.141	0.142	0.072	0.172
38	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
39	0.093	0.206	0.202	8.289	0.150	0.153	0.132	0.125	0.032	0.093
40	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
41	0.084	0.124	0.411	8.448	0.124	0.141	0.124	0.045	0.041	0.109
42	0.160	0.191	0.192	8.249	0.191	0.257	0.191	0.101	0.134	0.191
43	0.528	0.717	1.040	2.977	0.717	0.884	0.717	0.312	0.288	0.717
44	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
45	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
46	0.467	0.840	1.410	1.314	0.999	0.749	0.691	0.425	0.117	0.634
47	0.500	0.812	2.030	0.942	0.742	0.941	0.668	0.222	0.283	0.742
48	0.105	0.159	0.329	8.390	0.129	0.129	0.129	0.129	0.052	0.123
49	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 5: Progress and regress measures

Branch	$\pi_{oj}^{(2)}$	$pd_{oj}^{(2)}$	$rd_{oj}^{(2)}$
1	1.0254	0.0254	-
2	0.9578	-	0.0422
3	1.2514	0.2514	-
4	1.3102	0.3102	-
5	1.0120	0.0120	-
6	1.0254	0.0254	-
7	0.7414	-	0.2586
8	0.8714	-	0.1286
9	0.9613	-	0.0387
10	1.0202	0.0202	-
11	1.0302	0.0302	-
12	1.7841	0.7841	-
13	1.4441	0.4441	-
14	1.0365	0.0365	-
15	1.0874	0.0874	-
16	1.2111	0.2111	-
17	0.7222	-	0.2778
18	0.8332	-	0.1668
19	0.7241	-	0.2759
20	1.8547	0.8547	-
21	1.3212	0.3212	-
22	1.2587	0.2587	-
23	1.3698	0.3698	-
24	1.2547	0.2547	-
25	1.0231	0.0231	-
26	1.0587	0.0587	-
27	1.0325	0.0325	-

Table 5: Continued

Branch	$\pi_{ol}^{(2)}$	$pd_{ol}^{(2)}$	$rd_{ol}^{(2)}$
28	1.1471	0.1471	-
29	0.9874	-	0.0126
30	0.8547	-	0.1453
31	0.9632	-	0.0368
32	0.7842	-	0.2158
33	0.9741	-	0.0259
34	1.2541	0.2541	-
35	0.5041	-	0.4959
36	1.0987	0.0987	-
37	1.2365	0.2365	-
38	1.1236	0.1236	-
39	1.2228	0.2228	-
40	1.2563	0.2563	-
41	1.9725	0.9725	-
42	0.9214	-	0.0784
43	0.8716	-	0.1284
44	0.9128	-	0.0872
45	0.9749	-	0.0251
46	0.8529	-	0.1471
47	1.0298	0.0298	-
48	1.0214	0.0214	-
49	1.9312	0.9312	-
50	1.0149	0.0149	-

To determine the measure of progress or regress from period 1 to 2, we have used the models (7) and (8). Table 5 displays the measure of progress and regress. As the Table 5 indicates, 32 companies have a progress and other 18 companies have regress from period 1 to period 2. Also, company 41 has the most progress and company 35 has the most regress.

CONCLUSIONS

This study is concerned with the measurement of efficiency and progress or regress of DMUs from DEA perspective. Context-dependent DEA by incorporating value judgment, inspired by the Russell measure of technical efficiency is developed to measurement the efficiency of DMUs with respect to a given evaluation context for T periods. Different strata of efficient frontier are used as evaluation context. Then, the movement of DMUs with respect to their stratum interpreted as progress or regress. The current study proposes a re-formulation of the Russell measure by incorporating value judgment into the inputs and outputs. An illustrative application of the methodology to a sample of bank branches is given.

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