ISSN 1996-3343

Asian Journal of Applied Sciences

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Asian Journal of Applied Sciences 1 (4): 304-315, 2008
ISSN 1996-3343

An Enhanced Numerical Approach in Entrance Region of Annular Passages

S.F. Ranjbar
Faculty of Mechanical Engineering, Tabriz University, Tabriz, Iran

Abstract: In this study, a numerical method is presented to solve the laminar flow forced convection in combined entry length of annular duct. The fluid viscosity has been taken as a function of temperature. The governing equations which have the elliptic nature are solved in coupled form by successive over-relaxation finite difference method. A feature of this scheme is the easy implementation of solid boundary conditions. A FORTRAN code is written for all the simulation processes. The fluid having Prantdl numbers ranging from 0.01 to 10 are considered. Comprehensive comparisons were made between the results of present method and available data, in which good agreement would exist.

Key words: Finite difference, stream function, vorticity, point successive over-relaxation, annular passage, laminar flow, forced convection

INTRODUCTION

Heat exchangers with double-pipe configuration have tremendous applications in industry. To calculate the heat transfer coefficient in double-pipe heat exchangers, empirical correlations are used which is applicable only for one-side annulus.

In three-fluid heat exchangers, in which one of the fluids flows in annular passage, for calculation of the heat transfer surface, the convection coefficient is required. At the entrance region, local Nusselt number starts to decrease intensively and reaches its minimum value at the end of thermal entry length.

As reported in the references, heat exchangers are designed by using fully-developed flow equations. Therefore, for all exchangers, especially for multi-pass ones, in which fluid flows through entry region of each pass, calculated heat transfer surface becomes over design.

Also, when properties of fluids intensively depend on temperature, heat transfer coefficient varies nonlinearly through the passage and usually wall heat flux is not constant. Therefore, it is necessary to calculate local Nusselt number as a function of passage length and other characteristics. So, the best approach is to solve equations continuously and consecutively on the longitudinal elements.

Heaton *et al*. (1964) and Kays and London (1984) have studied the heat transfer problem in annular duct for laminar flow with constant properties in combined entry length. By Langhaar's (1951) method, the momentum integral equation is solved for the hydrodynamic entry length.

This method was applicable for constant wall heat flux and the results have been obtained by assuming one wall with heat exchange and the other as insulated. For heat exchange at both walls the results of linearized momentum and energy equations by use of superposition principle was applied.

Sellars *et al*. (1956) solved the differential energy equation by separation of variables and changing the constant wall temperature to constant wall temperature gradient the flow in smooth circular tubes with varying heat flux along the tube axis and considered Re.Pr>100 in which the effect of axial heat conduction is ignorable. So, strum-liouville type differential equation has been obtained, so divided whole surface to longitudinal elements with constant heat flux and for each element came up by Stieltjes integral, then entered the flux continuous variation part to the Reimann integral and converted the discontinuous variation part to a summation because this integral was not solvable analytically.

This summation and integral can be solved analytically just on surfaces with simple temperature distribution. It is clear that this simplifying method is complicated and according to several assumptions the results can not be reliable.

Lundberg *et al*. (1972) and Kays *et al*. (2004) presented an analytical method for laminar flow with constant properties in annular duct with unequal heating conditions and solved the problem for fully developed flows separately. By using correlated parameters with diameters ratio in simplified momentum equation to one-dimensional laminar and developed flow from circular tubes and applying congruous limits to integration equation, the velocity profile in annular duct was gotten and by putting it in the energy equation for developed flow and according to this principle that the appointed heat flux from each surface, the temperature gradient on surface was specified and energy equation was integrated directly. In order to provide the heat exchanging boundary conditions from the surface in energy equation two special forms was necessary. Heating from outer surface of annulus as the inner one is insulated and the other one is: Heating from inner surface of annulus as the outer one is insulated.

The linearized energy equation lead to calculation of integral of both cases and for unequal heating via walls superposition principle was applied.

It is obvious that previous solutions used some simplifying assumptions so the results were not accurate enough. This study presents a full numerical method for forced convection heat transfer of laminar flow which uses elliptical differential equations.

GOVERNING EQUATIONS

For incompressible steady state laminar flow in annular duct with heat transfer because of duct symmetry Navier-Stokes equations in cylindrical coordinate, introducing stream function (ψ) which satisfy continuity equation, vorticity has been obtained as below:

$$u = \frac{1}{r} \cdot \frac{\partial \psi}{\partial r}, \quad \upsilon = -\frac{1}{r} \cdot \frac{\partial \psi}{\partial x} \tag{1}$$

$$\omega = \frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial r} = -\frac{1}{r}\left(\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{r} \cdot \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial r^2}\right) \tag{2}$$

Introducing hydraulic radius as:

$$r_h = \frac{D_h}{4} = (r_o - r_i)/2$$

and velocity (V), which is equal to inlet uniform velocity the variables and vorticity would be non-dimensionalized as below:

$$r^* = \frac{r}{r_h}, \quad x^* = \frac{x}{r_h}, \quad u^* = \frac{u}{V}, \quad \upsilon^* = \frac{\upsilon}{V}, \quad \psi^* = \frac{\psi}{V r_h^2}, \quad \omega^* = \frac{r_h \omega}{V} \tag{3}$$

From now on the asterisk sign will be dropped. So non-dimensional vorticity equation will be as:

$$\frac{\partial (u\omega)}{\partial x} + \frac{\partial (\upsilon\omega)}{\partial r} = \frac{4}{Re}\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{1}{r} \cdot \frac{\partial \omega}{\partial r} - \frac{\omega}{r^2} + \frac{\partial^2 \omega}{\partial r^2}\right) \tag{4}$$

And so on for temperature:

$$T^* = \frac{T - T_w}{T_o - T_w}$$

where, T_o and T_w are inlet and wall temperature, respectively. So, non-dimensional energy equation is:

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial r} = \frac{4}{Pe}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right) + \frac{4Ec}{Re}\cdot\Phi_V \quad (5)$$

$$\Phi_V = 4\left(\frac{1}{r}\cdot\frac{\partial^2\psi}{\partial r\partial x}\right)^2 + \left(\frac{1}{r}\cdot\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r^2}\cdot\frac{\partial\psi}{\partial r} - \frac{1}{r}\cdot\frac{\partial^2\psi}{\partial x^2}\right) \quad (6)$$

Assumptions

- The fluid properties as density, specific heat and thermal conductivity are temperature independent:

$$\rho = \text{Const.} \qquad C_p = \text{Cont.} \qquad K = \text{Const.}$$

- There is no energy source:

$$U_V''' = 0$$

- For probability of high viscosity fluids, dissipation (Φ_V) is used

$$\frac{\partial}{\partial\theta}\left(K\frac{\partial T}{\partial\theta}\right) = 0$$

- Temperature and velocity distribution are axisymmetric

$$\mu = \mu(T)$$

To avoid complexity of nonlinear partial differential equations, at first viscosity considered constant and then obtained temperature distribution is used to define grid viscosity value in order to correct velocity and temperature values, iteratively.

Used non-dimensional groups and temperature difference are:

$$Re = \frac{\rho V D_h}{\mu}, \quad Pr = \frac{\mu . C_P}{K}, \quad Ec = \frac{V^2}{C_P(T_o - T_w)}, \quad Pe = \frac{4Vr_h}{\alpha} = Re.Pr \quad (7)$$

DISCRETIZED EQUATIONS

Central difference is used for interior nodes, hence discretized form of Eq. (2, 5) are:

$$u_{i,j} = (\psi_{i,j+1} - \psi_{i,j-1})/2r_j.\Delta r \quad (8)$$

$$\nu_{i,j} = -(\psi_{i+1,j} - \psi_{i-1,j})/2r_j.\Delta x \quad (9)$$

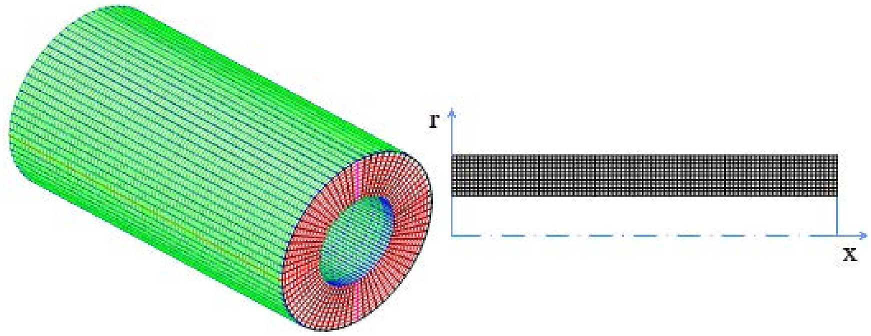

Fig. 1: A part of grid

$$\psi_{i,j} = [r_j(\Delta r)^2.\omega_{i,j} + (1-\Delta r/2r_j).\psi_{i,j+1} + (1+\Delta r/2r_j).\psi_{i,j-1} + (\psi_{i+1,j} + \psi_{i-1,j})]/4 \quad (10)$$

$$\omega_{i,j} = [\omega_{i+1,j}(2-Re_c.u_{i,j}) + \omega_{i-1,j}(2+Re_c.u_{i,j}) + \omega_{i,j+1}(2+\Delta r/r_j - Re_c.v_{i,j}) + \omega_{i,j-1}(2-\Delta r/r_j + Re_c.v_{i,j})]/\{(u_{i+1,j} - u_{i-1,j} + v_{i,j+1} - v_{i,j-1}).Re_c + [8+2(\Delta r)^2/r_j^2]\} \quad (11)$$

$$T_{i,j} = \{T_{i,j+1}[2+(\Delta r/r_j) - Pe_c.v_{i,j}] + T_{i,j-1}.[2-(\Delta r/r_j) + Pe_c.v_{i,j}] + T_{i+1,j}(2-Pe_c.u_{i,j} + T_{i-1,j}(2+Pe_c.u_{i,j})\ \}/8 + Ec.Pr(\Delta r/2)^2.\Phi_V \quad (12)$$

$$\Phi_V = [(\psi_{i+1,j+1} - \psi_{i+1,j-1} - \psi_{i-1,j+1} + \psi_{i-1,j-1})/2r_j(\Delta r)^2]^2 + \{\psi_{i,j+1}[1-(\Delta r/2r_j)] + \psi_{i,j-1}[1+(\Delta r/2r_j)] - (\psi_{i+1,j} + \psi_{i-1,j})\ \}/[r_j(\Delta r)^2] \quad (13)$$

According to Fig. 1, the longitudinal and radial increments are assumed equal ($\Delta x = \Delta r$) and cell Reynolds and Peclet numbers (Arpaci and Larsen, 1984, Minkowycz *et al*., 1988) are defined as:

$$Re_c = \frac{\Delta r}{4}.Re, \quad Pe_c = Re_c.Pr$$

Results independency of grid is related to cell Reynolds number. For example for 30 nodes in radius direction and Re = 780, cell Reynolds number will be $Re_c = 0.05$, thereafter increasing number of nodes has no effect on results.

BOUNDARY CONDITIONS

Non-dimensionalized boundary conditions are:

Inflow Conditions (x = 0):

$$u = 1, \quad \upsilon = 0, \quad \frac{\partial u}{\partial r} = 0 \quad (14)$$

$$\omega_{1,j} = -\frac{1}{r_j} \cdot \frac{\psi_{1,j} - 2\psi_{2,j} + \psi_{3,j}}{(\Delta x)^2} \quad (15)$$

$$\psi_{1,j} = \frac{r_j^2}{2} \quad (16)$$

$$T_{i,j} = \frac{T - T_w}{T_o - T_w} = 1 \tag{17}$$

Outflow Conditions (x = 1):

According to White (2005):

$$x = \frac{L_H}{r_h} \; (L_H \text{ is hydrodynamic entry length}) \tag{18}$$

$$u = \frac{2}{M}[1 - (Ar)^2 + BLn(Ar)] \tag{19}$$

Where:

$$A = \frac{1 - r^2}{2}$$
$$B = (r^2 - 1)/Ln(r)$$
$$M = 1 + r^2 - B$$
$$r = \frac{r_i}{r_o}$$

$$\omega_{M+1,j} = -\frac{\partial u}{\partial r} = \frac{4}{M}A^2 r_j - \frac{2B}{M} \cdot \frac{1}{r_j} \tag{20}$$

$$\psi_{M+1,j} = \frac{r_j^2}{2M}[2 - (Ar_j)^2 + 2BLn(Ar_j) - B] \tag{21}$$

$$T_{M+1,j} = \frac{T_e - T_w}{T_o - T_w} \qquad j = 2,3,...,N \tag{22}$$

$$T_e = T_o + \frac{1}{\dot{m}C_P}\sum q''_w \cdot dA \tag{23}$$

Solid Boundary Conditions

$$u = 0, \;\; \upsilon = 0, \;\; \frac{\partial}{\partial x} = 0 (\text{No slip condition}) \tag{24}$$

For $r = r_o$:

$$\omega_{i,N+1} = \frac{15\psi_{i,N+1} - 16\psi_{i,N} + \psi_{i,N-1}}{6(\Delta r)^2 - 4(\Delta r)^3}, \; \psi_{i,N+1} = \frac{2}{(1-r)^2} = \text{Const.}, \; T_{i,N+1} = \frac{T - T_w}{T_o - T_w} = 0 \tag{25}$$

For $r = r_i$:

$$\omega_{i,j} = \frac{15\psi_{i,1} - 16\psi_{i,2} + \psi_{i,3}}{6(\Delta r)^2 - 4(\Delta r)^3}, \; \psi_{i,1} = \frac{2r^2}{(1-r)^2} = \text{const.}, \; T_{i,1} = \frac{T - T_w}{T_o - T_w} = 0 \tag{26}$$

Table 1: Initial conditions for water, air and mercury flows

Initial cindition	Water	Air	Mercury
T_i (k)	280.85	350	534
P_r	10	0.7	0.01
V(m sec^{-1})	5×10^{-3}	0.5	5×10^{-3}
Re	57.1043	379.4216	1052.22

Table 2: Viscosity as a function of temperature for water, air and mercury

Water	$\mu = -0.741\times10^{-9}T^3+7.0332\times10^{-6}T^2-2.2375\times10^{-3}T+0.239179$
Air	$\mu = 1.183\times10^{-14}T^3-3.808\times10^{-11}T^2+6.738\times10^{-8}T+1.3554\times10^{-6}$
Mercury	$\mu = (14278.567-2.396T)\times10^{-(6.796+6.337E\text{-}4T}$

NUMERICAL SCHEME

To solve system of elliptic equations in finite difference form, the point successive over relaxation method is chosen because of its fast convergence (Chung, 2002; Parshant, 2000). Over relaxation parameter (α) and also relaxation value $u^r_{i,j}$ is used instead of explicitly calculated values to control the convergence (Apaci and Larsen, 1984; Roache, 1976).

$$u^r_{i,j} = \bar{u}_{i,j} + \alpha_{opt}(u_{i,j} - \bar{u}_{i,j}) \tag{27}$$

where, $\bar{u}_{i,j}$ is obtained from previous iteration.

To speed up the convergence the over relaxation parameter lies in the range of $1\le\alpha_{opt}\le2$. Its optimized value is calculated as (Bejan, 1984; Nakamura, 1991):

$$\alpha_{opt} = \frac{2(1-\sqrt{1-\alpha^2})}{\alpha^2},\ \alpha = \frac{Cos(\pi/N)+bSin(\pi/M)}{1+b},\ b = \left(\frac{\Delta r}{\Delta x}\right)^2 \tag{28}$$

INITIAL CONDITIONS

Initial conditions are shown in Table 1.

The viscosity (kg m sec^{-1}) is given by Reid *et al*. (1988) and Schmidt *et al*. (1984) as shown in Table 2.

In order to compare results of present study with others, eight non-dimensional lengths are chosen:

$$x^+ = x/(D_h \cdot Re.Pr) = 0.001, 0.002, 0.01, 0.02, 0.05, 0.1, 0.2, 0.25 \tag{29}$$

RESULTS AND DISCUSSION

The results of present study and Kays *et al*. (2004) are shown in diagrams. Nu_o and Nu_i by Kays's *et al*. (2004) method are given:

$$Nu_i = \frac{Nu_{ii}}{1-(q''_o/q''_i)Z_i},\ \ Nu_o = \frac{Nu_{\infty}}{1-(q''_i/q''_o)Z_o} \tag{30}$$

Nu_{ii} ≡ Nusselt number when outer surface is insulated

Nu_∞ ≡ Nusselt number when inner surface is insulated

Z_0 and Z_1 are correction factors related to Prandtl number and non-dimensional length. Also to match the curves the following coordinate conversion is used:

$$Nu = 100\log(Nu_x),\ X = 50\log(1000x^+) \tag{31}$$

Regarding Fig. 2, from the Kays's *et al.* (2004) data the Nu_0 curves for Pr = 1,0.7 after x = 0.2 approaches to a constant and same value, but for Pr = 0.01 with considerable difference reaches a higher value. Because the molten metal have great conductivity and during the thermal entry length in which temperature gradient is considerable along the flow direction the longitudinal heat conduction effect, especially in the laminar flow and for the Pe<50, is important and effective (Özisik, 1988).

Here according to the input temperature Pe ≅ 10.5, so it will decrease the local Nusselt number along the entry length. In Kays's *et al.* (2004) method because of linearization of conservation equations this effect hasn't been considered so Kays's *et al.* (2004) data show higher Nusselt number.

Figure 5 for Kays's *et al.* (2004) data shows that when, Pr = 0.01 Nusselt number is higher than two other fluids, because air and water flows have Pe>50, so longitudinal heat transfer is ignorable. But in this study according to Fig. 3 and 4 for Nu_0 and Fig. 6 and 7 for Nu, the curves are converged to one specific value for all three Prantdl numbers because conservation equations are solved considering longitudinal heat transfer.

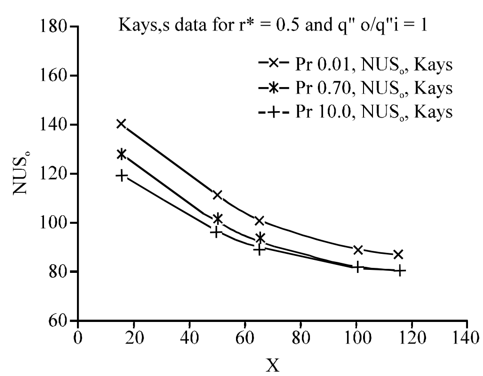

Fig. 2: Local Nu_0 versus x^+ for different Pr from the Kays data

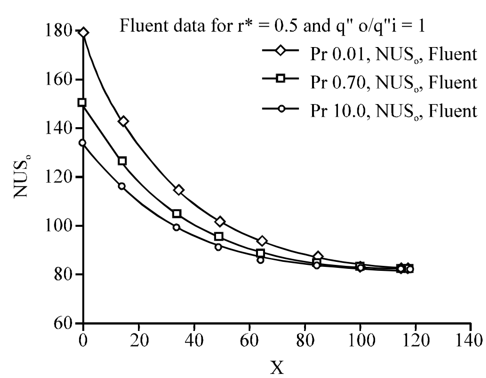

Fig. 3: Local Nu_0 versus x^+ for different Pr from the Fluent data

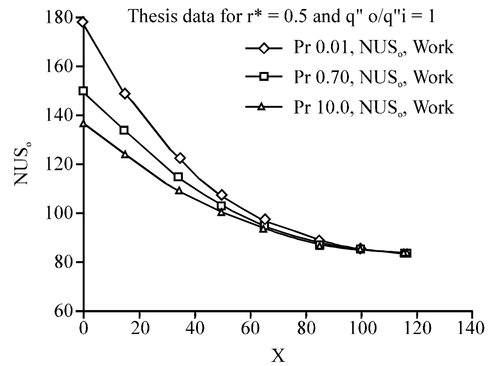

Fig. 4: Local Nu_o versus x^+ for different Pr from the present study data

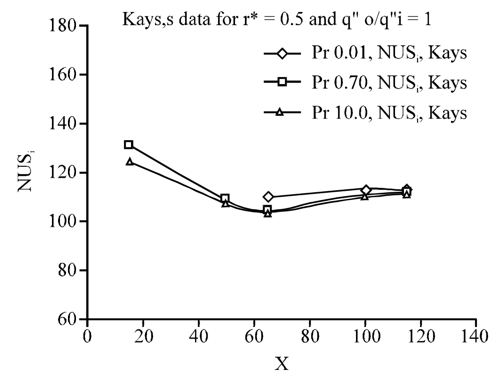

Fig. 5: Local Nu_i versus x^+ for different Pr from the Kays data

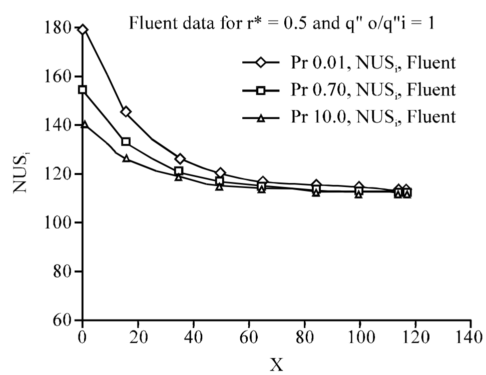

Fig. 6: Local Nu_i versus x^+ for different Pr from the present study data

Figure 8-10 shows that curves of Nu_o, calculated by different methods, are approximately equal for all three prentdl numbers.

From Fig. 11-13, it is concluded that Nu_i curves of present study is only decreasing but by Kays's *et al*. (2004) data, at first it decreases and then begins to increase.

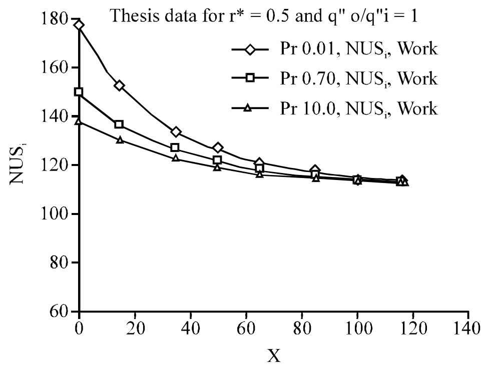

Fig. 7: Local Nu_i versus x^+ for different Pr from the present study data

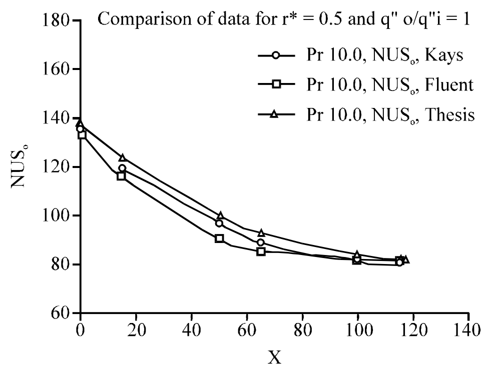

Fig. 8: Nu_o comparison among three method at Pr = 10

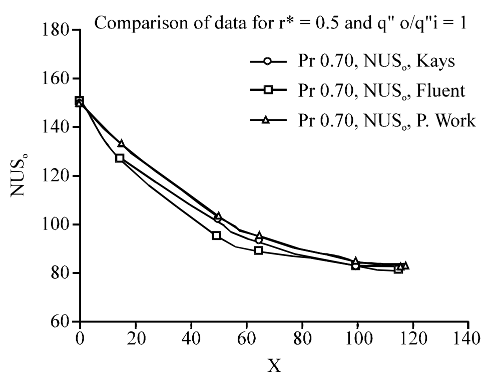

Fig. 9: Nu_o comparison among three method at Pr = 0.70

Kays's *et al*. (2004) method has no results for Pr = 0.01 and x<0.02, because he has used Langhaar's (1951) approximation so truncated series, in addition neglected longitudinal heat conduction that led to considerable error.

Remember that these differences between present study and Kay's *et al*. (2004) method are limited to:

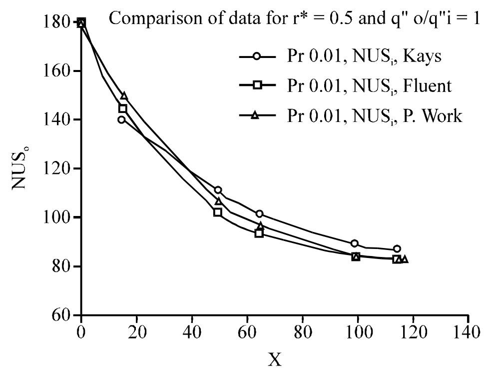

Fig. 10: Nu_o comparison among three method at Pr = 0.01

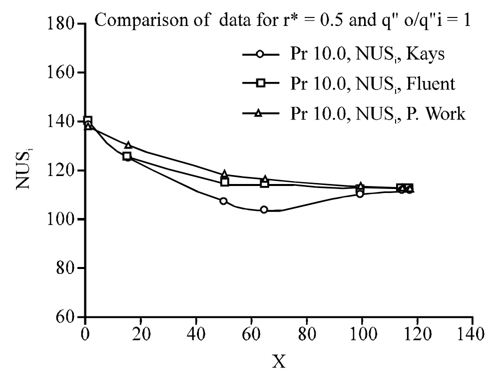

Fig. 11: Nu_o comparison among three method at Pr = 10

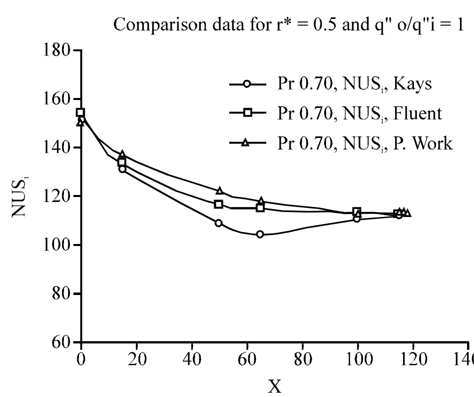

Fig. 12: Nu_o comparison among three method at Pr = 0.7

$$q''_o / q''_i = 1$$

in other cases the results are the same.

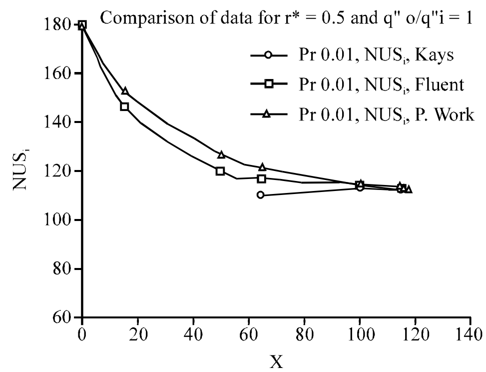

Fig. 13: Nu_o comparison among three method at Pr = 0.01

Two basic assumptions of Kays`s *et al*. (2004) method is considering the viscosity constant and using specified heat fluxes but this study assumes viscosity as a function of temperature and is applicable for all symmetric boundary conditions.

CONCLUSION

Present study has more accurate results in comparison with Kays's *et al*. (2004) data, because:

- Viscosity is assumed as a function of temperature
- Longitudinal heat conduction is considered
- Elliptic differential conservation equations are solved in coupled form.
- It is suitable for all types of boundary conditions, at last it is applicable for varying boundary conditions along the tube
- In spite of Kays's *et al*. (2004) method this study is full numeric and more simple

NOMENCLATURE

Cp = Specific heat at constant pressure
D_h = Hydraulic diameter
Ec = Eckert number
k = Thermal conductivity
L_H = Hydrodynamic entry length
Nu_{oo} = Nusselt number when outer wall is isolated
Nu_{ii} = Nusselt number when inner wall is isolated
Nu_o = Nusselt number at outer wall
Nu_i = Nusselt number at inner wall
Pe = Pecklet number
Pe_c = Cell Pecklet number
Pr = Prantdl number
r_h = Hydraulic radius
Re = Reynolds No.
Re_c = Cell Reynolds No.
r_o = Outer radius
r_i = Inner radius

T = Bulk temperature
T_w = Wall temperature
T_o = Inlet bulk temperature
$x^+ = \frac{x}{r_h \, RePr}$ = Gratz No.
Z_i, Z_o = Influence coefficient for annulus heat transfer

GREEK SYMBOLS

ψ = Stream function
ω = Vorticity
ϕ_v = Dissipation function
ρ = Density
α = Thermal diffusivity

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