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Blood Flow in Uniform Planar Channel

A.M. Sobh and H.A. Abu Oda
Department of Mathematics, Faculty of Science, Al-Aqsa University, Gaza, Palestine

Abstract: In this research, we investigate the peristaltic flow of blood through a planar channel of uniform thickness. Blood is represented by a micropolar fluid (a fluid can support coupled stresses, body couples and exhibit microrotational and microinertial effects). The problem is formulated and solved without any restrictions on the wave ratio and the Reynolds number. Perturbation method in terms of wave number (δ) as a parameter is used to obtain analytic form for the axial velocity, the microrotation velocity and the pressure gradient to the first order. Moreover, the pressure rise and friction force have been computed numerically and the results are studied for various parameters of interest.

Key words: Peristalsis, micropolar fluid, uniform channel

INTRODUCTION

The flow of non-Newtonion fluids has been of an increasing importance. This is due to their applications in polymer processing industries and biofluid dynamics. There are many models of constitutive equations that describe the motion of non-Newtonion fluids. These models are proposed in such a way that they are suitable to describe the fluids flow according to its rheological characteristics.

The analysis of peristaltic flow has great practical importance in many biological and biomedical systems. Such systems include the flow of urine through the ureter, the swallowing process through the oesophagus, the movement of spermatozoa in the ductus efferentes of the male reproductive tract, transport of lymph in the lymphatic vessels and in the vasomotion of small blood vessels such as arterioles, venules and capillaries. The literature on peristalsis of viscous fluid is by now quite extensive. Some investigations dealing with the peristaltic flow of Newtonian and non-Newtonian fluids have been presented. Such recent investigations include the studies of Elshehawey *et al*. (2000), Elshehawey and Sobh (2001), Misra and Pandey (2001), El Misery *et al*. (2003), Mishra and Ramachandra (2003), Abd El Naby *et al*. (2004), Mekheimer (2005) and Sobh and Mady (2008).

Recently, some mathematical studies have been done to understand the blood flow in arteries and small blood vessels. Stud *et al*. (1977) studied the effect of moving magnetic field on blood flow. They observed that the effect of suitable moving magnetic field accelerates the speed of blood. A two-fluid model of non-Newtonian blood flow induced by peristaltic waves has been studied by Srivastava and Saxena (1995). Agrawal and Anwaruddin (1984) also studied the effect of magnetic field on blood flow through an equally branched channel with flexible walls executing peristaltic waves using long wavelength approximation. They observed, for the blood in arteries with arterial disease like arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as a blood pump in carrying out cardiac operations.

Most of the theoretical investigations on the blood have been carried out by assuming the blood to behave like a Newtonian fluid, though it has been accepted that most of the physiological fluids behave like a non-Newtonian fluids. However a few recent studies considered the blood as a non-

Corresponding Author: A.M. Sobh, Department of Mathematics, Faculty of Science, Al-Aqsa University, P. O. Box 4051, Gaza, Palestine Tel: +972-8-2826809 Fax: +972-8-2845488

Newtonian fluid. Some of these studies were done by Charm and Kurland (1974), Valanis and Sun (1969), Popel *et al*. (1974), Srivastava (1985) and Mekheimer (2004).

The study of peristaltic flow of micropolar fluids is of increasing interest. The micropolar fluids are usually defined as isotropic, polar fluids in which deformation of molecules is neglected. Physically, a micropolar model can represent fluids whose molecules can rotate independently of the fluid stream flow and its local vorticity. Such a fluid can support coupled stresses, body couples and exhibit microrotational and microinertial effects (Eringen, 1966). The theory of micropolar fluids is a special case of the theory of simple microfluids introduced by Erigen. This theory includes the effects of local rotary inertia and couple stresses and is expected to provide a mathematical model for the rheological behavior in certain man-made liquids such as polymers and also liquids such as blood which contains red cells, white cells and platelets.

In their research, Muthu *et al*. (2003) studied the influence of viscoplastic wall properties in the peristalic motion of micropolar fluid, using the same technique used by Fung and Yih (1968). Peristaltic pumping of a micropolar fluid in a tube was studied by Srinivasacharya *et al*. (2003). That study was done under the assumption of low Reynolds number and long wave length approximation.

Keeping the above facts in mind, we intend to study the peristaltic flow of blood, as a micropolar fluid, in a planer channel without any restrictions on Reynolds number and amplitude ratio. For this purpose, the perturbation method in terms of wave number is used to obtain explicit form for the fluid velocity and the pressure gradient. This analytic solution takes into account all physical parameters of the problem and gives the solutions for each point within the domain of interest, unlike the numerical solution, which is available only for a set of discrete points in the domain. Moreover, the pressure rise and friction force are computed numerically and are explained graphically.

FORMULATION AND ANALYSIS

Consider the peristaltic flow of unsteady incompressible micropolar fluid through an infinite channel of uniform thickness 2H (Fig. 1). We assume an infinite wave train traveling with velocity c along the walls. Taking $(\bar{X},\bar{Y})$ as rectangular coordinates, the equation of the wall surface is:

$$\bar{H}(\bar{X},\bar{t}) = a + b\sin\frac{2\pi}{\lambda}(\bar{X} - c\bar{t}), \tag{1}$$

where, a is the channel width, b the is the wave amplitude, λ is the wavelength, c is propagation velocity of the wave, $\bar{t}$ is the time.

In the moving coordinates $(\bar{x},\bar{y})$ which travel in the $\bar{x}$ direction with the same speed as the wave, the flow can be treated as steady (Shapiro *et al*., 1969). The coordinate frames are related through:

$$\bar{x} = \bar{X} - c\bar{t}, \qquad \bar{y} = \bar{Y}, \tag{2}$$

$$\bar{u} = \bar{U} - c, \qquad \bar{v} = \bar{V}, \tag{3}$$

$$\bar{w} = \bar{W} \tag{4}$$

where, $(\bar{U},\bar{V},\bar{W})$ and $(\bar{u},\bar{v},\bar{w})$ are the velocity components in the fixed and the moving frames respectively, $\bar{U}$ is the axial velocity, $\bar{V}$ is the perpendicular velocity and $\bar{W}$ is the microrotation velocity.

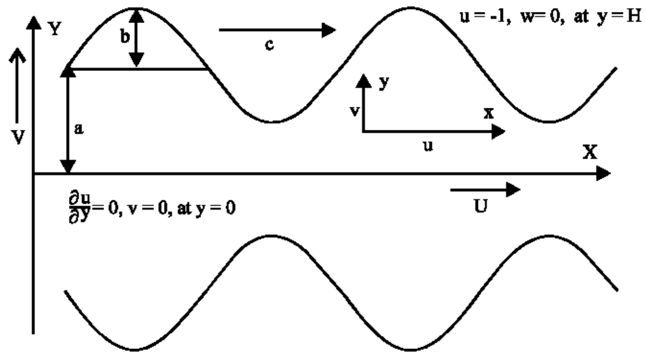

Fig. 1: Geometry of the problem

Using the following non-dimensional parameters

$$x=\frac{\bar{x}}{\lambda}, y=\frac{\bar{y}}{a}, t=\frac{c\bar{t}}{\lambda}, P=\frac{a^2\bar{P}}{c\lambda\mu}, u=\frac{\bar{u}}{c}, v=\frac{\lambda\bar{v}}{ac}, w=\frac{a}{c}\bar{w}, Re=\frac{\rho ca}{\mu}, \delta=\frac{a}{\lambda},$$
$$J=\frac{\bar{J}}{a^2}, n=\frac{k}{\mu+k}, m^2=\frac{a^2k(2\mu+k)}{\sigma(\mu+k)}, \varphi=\frac{b}{a}, H=\frac{\bar{H}}{a}=1+\varphi\sin 2\pi x \tag{5}$$

where, μ, k and σ are material constants, δ is the wave number, Re is the Reynolds number and φ is the amplitude ratio, the non-dimensional equations of motion in the absence of body forces and body couple are (Srinivasacharya *et al*., 2003)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$Re\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{1}{1-n}\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\left(\frac{n}{1-n}\right)\frac{\partial w}{\partial y}, \tag{7}$$

$$Re\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\delta^2}{1-n}\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{n}{1-n}\delta^2\frac{\partial w}{\partial x}, \tag{8}$$

$$Re\delta J\frac{1-n}{n}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}\right)=-2w+\frac{2-n}{m^2}\left(\delta^2\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+\delta^2\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}, \tag{9}$$

where, n is the coupling number ($0\le n\le 1$), m is the micropolar parameter and J is the microgyration parameter.

The nom-dimensional boundary conditions are

$$\frac{\partial u}{\partial y}=0, \quad v=0 \quad \text{for} \quad y=0, \tag{10a}$$

$$u=-1, \quad v=-\frac{dH}{dx} \quad \text{for} \quad y=H. \tag{10b}$$

$$w=0 \quad \text{for} \quad y=\pm H \tag{10c}$$

RATE OF VOLUME FLOW

The instantaneous volume flow rate in the fixed frame is given by:

$$Q = \int_0^{\bar{H}} \bar{U}(\bar{X}, \bar{Y}, \bar{t})d\bar{Y} \tag{11}$$

where, $\bar{H}$ is a function of $\bar{X}$ and $\bar{t}$.

The rate of volume flow in the moving frame (wave frame) is given by:

$$\bar{q} = \int_0^{\bar{H}} \bar{u}(\bar{x}, \bar{y})d\bar{y} \tag{12}$$

where, $\bar{H}$ is a function of $\bar{x}$.

Using Eq. 12, one finds that the two rates of volume flow are related by:

$$Q = \bar{q} + c\bar{H} \tag{13}$$

The time-mean flow over a period $T = \lambda/c$ at a fixed position $\bar{X}$ is defined as:

$$\bar{Q} = \frac{1}{T}\int_0^T Q\, d\bar{t} \tag{14}$$

which can be written, using 1 and 13, as:

$$\bar{Q} = \bar{q} + ac \tag{15}$$

Defining the dimensionless time-mean flows Θ and F in the fixed and the wave frame respectively as:

$$\Theta = \frac{\bar{Q}}{ac} \quad \text{and} \quad F = \frac{\bar{q}}{ac} \tag{16}$$

then making use of 16, Eq. 15 can be rewritten as:

$$\Theta = F + 1 \tag{17}$$

where,

$$F = \int_0^{H(x)} u\, dy \tag{18}$$

PERTURBATION SOLUTION

We expand the following quantities in a power series of the small parameter δ as follows:

$$\begin{aligned} u &= u_0 + \delta u_1 + O(\delta^2), \\ v &= v_0 + \delta v_1 + O(\delta^2), \\ w &= w_0 + \delta w_1 + O(\delta^2) \end{aligned}$$

$$\frac{\partial p}{\partial x} = \frac{\partial p_0}{\partial x} + \delta \frac{\partial p_1}{\partial x} + O(\delta^2)$$

$$F = F_0 + \delta F_1 + O(\delta^2) \tag{19}$$

Substituting the expansions 19 into Eq. 6-10 and equating the coefficients of like powers of δ on both sides, we obtain the following systems.

Zero Order System

$$\frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} = 0, \tag{20}$$

$$\frac{1}{1-n}\frac{\partial^2 u_0}{\partial y^2} + \frac{n}{1-n}\frac{\partial w_0}{\partial y} = \frac{\partial p_0}{\partial x}, \tag{21}$$

$$\frac{\partial p_0}{\partial y} = 0, \tag{22}$$

$$-2w_0 + \left(\frac{2-n}{m^2}\right)\frac{\partial^2 w_0}{\partial y^2} - \frac{\partial u_0}{\partial y} = 0. \tag{23}$$

The boundary conditions corresponding to the zero order system are:

$$\frac{\partial u_0}{\partial y} = 0, \qquad v_0 = 0 \qquad \text{at} \qquad y = 0, \tag{24}$$

$$u_0 = -1, \qquad v_0 = -\frac{dH}{dx}, \qquad \text{at} \qquad y = H, \tag{25}$$

$$w_0 = 0 \qquad \text{at} \qquad y = \pm H. \tag{26}$$

On solving the system 20-23, with the boundary conditions 24-26, the zero order solution can be obtained in the form:

$$u_0 = c_0(y^2 - H^2) - n\,c_0\frac{H}{m}(\cosh my - \cosh mH) - 1, \tag{27}$$

$$\begin{aligned} v_0 = &-c_0'(\frac{y^3}{3} - H^2 y - \frac{2}{3}H^3) + \frac{n}{m}(c_0'H + c_0 H')\left(\frac{1}{m}(\sinh my - \sinh mH) - (y-H)\cosh mH\right) \\ &-c_0 HH'(n\sinh mH - 2)(y-H) - H', \end{aligned} \tag{28}$$

$$w_0 = c_0(\sinh my - 1), \tag{29}$$

where,

$$c_0 = 3m^2(F_0 + H)\Big/\Big[-2m^2H^3\sinh mH - 3nH(\sinh mH - H\cosh mH)\Big] \tag{30}$$

$$F_0 = \int_0^H u_0\,dy = -\frac{(1-n)}{(2-n)}\left(\frac{dp_0}{dx}\right)\left[\frac{2}{3}H^3 + \frac{nH}{m^2}(1-mH\coth mH)\right] - H \tag{31}$$

First Order System

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \tag{32}$$

$$\mathrm{Re}\left(u_0\frac{\partial u_0}{\partial x} + v_0\frac{\partial u_0}{\partial y}\right) = -\frac{\partial p_0}{\partial x} + \frac{1}{1-n}\frac{\partial^2 u_1}{\partial y^2} + \frac{n}{1-n}\frac{\partial w_1}{\partial y}, \tag{33}$$

$$\frac{\partial p_1}{\partial y} = 0, \tag{34}$$

$$\mathrm{Re}J\left(\frac{1-n}{n}\right)\left(u_0\frac{\partial w_0}{\partial x} + v_0\frac{\partial w_0}{\partial y}\right) = -2w_1 + \left(\frac{2-n}{m^2}\right)\frac{\partial^2 w_1}{\partial y^2} - \frac{\partial u_1}{\partial y}. \tag{35}$$

with the boundary conditions

$$\frac{\partial u_1}{\partial y} = 0, \qquad v_1 = 0 \qquad \text{at} \qquad y = 0, \tag{36}$$

$$u_1 = 0, \qquad v_1 = 0 \qquad \text{at} \qquad y = H, \tag{37}$$

$$w_1 = 0 \qquad \text{at} \qquad y = \pm H. \tag{38}$$

On substituting the zero-order solution into equations 32-35 and using corresponding boundary conditions 36-38, the first-order components of the axial and the microrotation velocities, respectively, can be obtained in the form:

$$\begin{aligned} u_1 &= D_1 y^2\cosh my + D_2 y\sinh my + D_3\cosh my + \frac{B_7}{m}y\cosh my - \left(\frac{B_6}{m} + \frac{B_7}{m^2}\right)\sinh my \\ &+ \left(\frac{B_4}{m} - \frac{4B_8}{m^2}\right)y^3\sinh my + \frac{B_5}{2m}\sinh 2my + \frac{B_8}{m}y^4\cosh my + \frac{B_9}{2m}\cosh 2my + \frac{B_{10}}{2}\sinh^2 my \\ &+ \frac{B_{11}}{2}y^2 + \frac{B_{12}}{3}y^3 + \frac{B_{13}}{4}y^4 + \frac{B_{14}}{6}y^6 + B_{15}y - T - 1 \\ &+ \frac{dp_1}{dx}\left(\frac{1-n}{2-n}\right)\left((y^2 - H^2) - \frac{nH}{m\sinh mH}(\cosh my - \cosh mH)\right), \end{aligned} \tag{39}$$

$$\begin{aligned} w_1 &= \left(L_1 + L_2 y + L_3 y^3 + L_4 y^4\right)\sinh my + \left(L_5 + L_6 y + L_7 y^3\right)\cosh my + L_8\sinh 2my \\ &+ L_9\cosh 2my + L_{10}y + L_{11}y^2 + L_{12}y^3 + L_{13}y^5 + L_{14} \end{aligned} \tag{40}$$

where,

$$D_1 = \frac{B_1}{m} - \frac{3B_4}{m^3} + \frac{12B_8}{m^4}, D_2 = -\frac{2B_1}{m^2} + \frac{6B_4}{m^3} - \frac{24B_8}{m^4} + \frac{B_3}{m}, D_3 = \frac{2B_1}{m^3} - \frac{B_3}{m^2} + \frac{B_2}{m} - \frac{6B_4}{m^4} + \frac{24B_7}{m^5},$$

$$B_1 = -\mathrm{Re}J\frac{n-1}{n}c_0 c_0' + 2m^2\frac{1-n}{2-m}\left(\frac{3A_5}{4m^3} - \frac{A_8}{4m^2} - \frac{9A_9}{4m^4}\right) + (1-n)\left(\frac{A_5}{4m} + \frac{A_8}{4} - \frac{3A_9}{4m^2}\right)$$

$$B_2 = \mathrm{Re}\,J\frac{n-1}{n}\left(c_0 c_0' H^2 - \frac{n}{m}c_0 c_0' H\cosh mH + c_0' + nc_0(c_0'H + c_0 H')\right) - nc_1$$
$$+\frac{1-n}{2-n}\left(\frac{A_4}{m} + \frac{7A_5}{2m^3} - \frac{3A_8}{2m^2} - \frac{39A_9}{4m^4}\right) + (1-n)\left(\frac{A_4}{2m} + \frac{A_5}{4m^3} - \frac{A_8}{4m^2} - \frac{3A_9}{8m^4}\right)$$

$$B_3 = \mathrm{Re}\frac{1-n}{n}J\left(\frac{n}{m}c_0 c_0' H + c_0 c_0' m H^2 - nc_0(c_0'H + c_0 H')\cosh mH - mc_0 HH'(n\sinh mH - 2)\right)$$
$$-\frac{1-n}{2-n}\left(A_4 - \frac{7A_5}{2m^2} - \frac{3A_8}{2m} - \frac{39A_9}{4m^3}\right) + (1-n)\left(\frac{A_4}{2} - \frac{A_5}{4m^2} + \frac{A_8}{4m} - \frac{9A_9}{8m^3}\right)$$

$$B_4 = \mathrm{Re}\,J\frac{n-1}{3n}mc_0 c_0' - \frac{1-n}{2-m}\left(\frac{A_5}{3} - \frac{5A_9}{4m}\right) + (1-n)\left(\frac{A_5}{6} - \frac{3A_9}{8m}\right), B_5 = \frac{(n-1)A_6}{12(2-n)} + \frac{(1-n)A_6}{6}$$

$$B_6 = \mathrm{Re}\frac{1-n}{n}J\left(-\frac{2}{3}mc_0 c_0' H^3 + nc_0 H\cosh mH + c_0 H'H^2(n\sinh mH - 2) - mH'\right.$$
$$\left.-\frac{n}{m}c_0(c_0'H + c_0 H')\sinh mH\right) + \frac{1-n}{2-n}\frac{A_7}{m} + \frac{(1-n)A_7}{2m}, B_7 = \frac{(n-1)A_7}{(2-n)} + \frac{(1-n)A_7}{2}$$

$$B_8 = \frac{(n-1)A_9}{4m(2-n)} + \frac{(1-n)A_9}{8m},\ B_9 = \frac{(n-1)A_{10}}{6m(2-n)} + \frac{(1-n)A_{10}}{3m},\ B_{10} = \mathrm{Re}\,J\frac{1-n}{nm}c_0^2 H'$$

$$B_{11} = \mathrm{Re}\frac{1-n}{n}J\left(c_0' + \frac{n}{m}c_0^2 H'\cosh mH + c_0 HH'(n\sinh mH - 2) + \frac{2(1-n)}{(2-n)}A_{11}\right)$$
$$+\frac{(1-n)}{(2-n)}\left(\frac{4A_2}{m^2} + \frac{A_6}{2} + \frac{48A_1}{m^4}\right) - \frac{1-n}{m^2}\left(2A_2 + \frac{24A_1}{m^2}\right), B_{12} = \frac{(1-n)A_3}{(2-n)},$$

$$B_{13} = \mathrm{Re}\,J\frac{2(n-1)}{3n}c_0 c_0' + \frac{2(1-n)}{(2-n)}\left(\frac{A_2}{3} + \frac{4A_1}{m^2}\right) - \frac{4(1-n)A_1}{m^2},\ B_{14} = \frac{2(1-n)A_1}{5(2-n)},$$

$$B_{15} = \mathrm{Re}\frac{1-n}{n}J\left(\frac{2}{3}c_0 c_0' H^3 + \frac{nc_0}{m}(c_0'H + c_0 H')\frac{1}{m}\sinh mH - H\cosh mH) + H' - c_0 H^2 H'(n\sinh mH - 2)\right)$$
$$+2c_1\sinh mH - \frac{(1-n)}{(2-n)}\left(2HA_{11} + \frac{2H^5 A_1}{5} + \frac{2H^3 A_2}{3} + H^2 A_3 + \frac{8H^3 A_1}{m^2} + \frac{4HA_2}{m^2} + \frac{48HA_1}{m^4}\right)$$
$$-\frac{2m^2(1-n)}{(2-n)}\left(\frac{A_{10}H}{2m^2} + \frac{A_6 H}{4m^2} - \frac{A_4 H}{2m^2}(\cosh mH - \frac{\sinh mH}{m}) - A_5\left\{H^3\cosh mH - \frac{3H^2}{4m^3}\sinh mH\right.\right.$$
$$\left.+\frac{7H\cosh mH}{4m^4} - \frac{7\sinh mH}{4m^5}\right\} - \frac{A_6\cosh 2mH}{24m^2} - A_7\left\{\frac{\sinh mH}{2m^2} - \frac{\cosh mH}{2m^3}\right\}$$
$$-A_9\left\{\frac{H^4\sinh mH}{8m^2} - \frac{5H^3\cosh mH}{8m^3} + \frac{9H^2\sinh mH}{4} - \frac{39H\cosh mH}{8m^5} + \frac{39H\sinh mH}{8m^6}\right\}$$
$$\left.-A_8\left\{\frac{H^2}{4m^2} - \frac{3H\cosh mH}{4m^3} + \frac{3\sinh mH}{4m^4}\right\} - \frac{A_{10}\sinh 2mH}{12m^3}\right) - \frac{(1-n)A_3}{m^2}$$

$$T = (D_1 H^2 + D_3 + D_4 H + D_8 H^4)\cosh mH + (D_2 H + D_5 + D_6 H^3 + D_{10}\sinh mH)\sinh mH$$
$$+D_7\sinh 2mH + D_9\cosh 2mH + D_{11}H^2 + D_{12}H^3 + D_{13}H^4 + D_{14}H^6 + D_{15}H$$

$$A_1 = \frac{\mathrm{Re}\,c_0 c_0'}{3}, A_2 = -\mathrm{Re}\left(2c_0 c_0' H^2 - \frac{n}{m}c_0(2c_0'H + c_0 H')\cosh mH - \frac{J}{n}c_0' c_0 + 2c_0' c_0 H^2\right.$$
$$\left.-\frac{2n}{m}c_0(c_0'H + c_0 H')\cosh mH - 2c_0^2 HH'(n\sinh mH - 2) + \frac{Jc_0 c_0'}{n}(2-m)\right)$$

$$A_3 = \mathrm{R\,e}\left(-\frac{4}{3}c_0 c_0' H^3 - \frac{2n}{m^2}c_0(c_0'H + c_0 H')\sinh mH + 2c_0^2 H^2 H'(n\sinh mH - 2)\right.$$
$$\left.+\frac{2c_0 nH}{m}(c_0'H + c_0 H')\cosh mH - 2c_0 H'\right),\ A_4 = -\mathrm{Re}\frac{n}{m}c_0(2c_0'H + c_0 H')$$

$$A_5 = \mathrm{Re}\left(\frac{n}{m}c_0 H^2(c_0'H + c_0 H') - \frac{J}{n}c_0' c_0 H^2 m + \frac{n}{m}c_0' c_0 H^3 - \frac{2n^2}{m^2}c_0 H(c_0'H + c_0 H')\right.$$
$$\left.-\frac{n}{m}c_0^2 H^2 H'(n\sinh mH - 2) + Jc_0 c_0' H(\frac{1}{m} + \cosh mH) + \frac{n}{m}(c_0'H + c_0 H') - \frac{J}{n}c_0' m\right)$$

$$+\mathrm{Re}\frac{J}{n}\left(c_0c_0'H^2m-nc_0(c_0'H+c_0H')(\cosh mH+\frac{1}{m})-c_0^2mHH'(n\sinh mH-2)\right)$$

$$A_6=\mathrm{Re}\left(\frac{n^2}{m^2}c_0H(c_0'H+c_0H')+c_0^2JH'\right),\ A_7=\frac{\mathrm{Re}}{3}c_0c_0'\left(nH+\frac{Jm^2}{n})+c_0^2JH'\right)$$

$$A_8=\mathrm{Re}\left(-c_0c_0'nH^3+\frac{n}{m}c_0(c_0'H+c_0H')(\frac{2}{m}+Hn\cosh mH)+c_0^2nH^2H'(n\sinh mH-2)\right.$$
$$\left.-\frac{J}{n}c_0c_0'm^2H^2-Jmc_0(c_0'H+c_0H')\cosh mH-\frac{J}{n}c_0^2m^2HH'(n\sinh mH-2)+\frac{J}{n}(2c_0'c_0+nH)\right)$$

$$A_9=\mathrm{Re}\left(\frac{2}{3}c_0c_0'nH^4+\frac{n^2}{m}c_0H(c_0'H+c_0H')(\frac{1}{m}\sinh mH-H\cosh mH)+c_0^2nH^3H'(n\sinh mH-2)\right.$$
$$-c_0nHH'+\frac{2J}{3n}c_0c_0'm^2H^3-Jc_0(c_0'H+c_0H')(\sinh mH-m\cosh mH)$$
$$\left.+\frac{J}{n}c_0^2m^2H^2H'(n\sinh mH-2)-\frac{J}{n}m^2c_0H'\right)$$

$$A_{10}=\mathrm{Re}\left(c_0(c_0'H+c_0H')(J-\frac{n^2}{m^2}H)-c_0c_0'nH\right)$$

$$A_{11}=\mathrm{Re}\left(c_0c_0'H^4-\frac{n}{m}c_0H^2(2c_0'H+c_0H')\cosh mH-c_0HH'(n\sinh mH-2)(c_0H^2-\frac{n}{m}c_0H-\frac{J}{n}c_0+1)\right.$$
$$\left.+\frac{n}{m}(c_0'H+c_0H')(\frac{n}{m}c_0H\cosh mH+\frac{J}{m}c_0-1)\cosh mH-\frac{J}{m}c_0c_0'H\cosh mH+c_0'H^2\right)$$

$$c_1=\frac{m(1-n)}{2(2-n)\sinh mH}\left(\frac{2A_1H^5}{5}+\frac{2A_2H^3}{3}+\frac{1}{m^2}(8A_1H^3+4A_2H)+\frac{48A_1H}{m^4}\right)$$
$$+\frac{(1-n)}{2(2-n)}\left[-mHA_4\mathrm{Coth}\,mH-A_4+\frac{A_5}{6}\left(\frac{12}{m^2}-\frac{12H\mathrm{Coth}\,mH}{m}-2mH^3\mathrm{Coth}\,mH+6\right)\right.$$
$$-\frac{A_5}{2}\left(\frac{3H\mathrm{Coth}\,mH}{m}-\frac{3}{m^2}-H^2\right)+\frac{mA_6H\mathrm{Cosech}\,mH}{2}+\frac{A_8}{4}\left(6H\mathrm{Coth}\,mH-\frac{6}{m}-2mH^2\right)$$
$$+\frac{A_9}{8}\left(6H^3\mathrm{Coth}\,mH+\frac{36H\mathrm{Coth}\,mH}{m^2}-2mH^4-\frac{18H^2}{m}-\frac{36}{m^3}\right)+\frac{3A_9}{8}\left(\frac{2H\mathrm{Coth}\,mH}{m^2}-\frac{2}{m^3}\right)$$
$$+\frac{A_9}{4}\left(2H^3\mathrm{Coth}\,mH+\frac{12H\mathrm{Coth}\,mH}{m^2}-6mH^2-\frac{12}{m}\right)+\frac{3A_9}{8}\left(\frac{4H\mathrm{Coth}\,mH}{m^2}-\frac{H^2}{m}-\frac{4}{m^3}\right)$$
$$\left.-A_{10}\left(\frac{1}{6}+m^3H\mathrm{Cosech}\,mH\right)\right]$$

$$L_1=c_{11}+\frac{(1-n)}{(2-n)}\left(\frac{H}{\sinh mH}\frac{dp_1}{dx}-\frac{A_4}{2m}-\frac{7A_5}{4m^3}+\frac{3A_8}{4m^2}+\frac{39A_9}{8m^4}\right),\ L_2=\frac{A_7}{2}\frac{(1-n)}{(2-n)},$$
$$L_3=m^2\frac{(1-n)}{(2-n)}\left(-\frac{3A_5}{4m^3}+\frac{A_8}{4m^2}+\frac{18A_9}{8m^4}\right),\ L_4=\frac{A_9}{8}\frac{(1-n)}{(2-n)},\ L_5=-\frac{L_2}{m},$$
$$L_6=m^2\frac{(1-n)}{(2-n)}\left(\frac{A_4}{2m}+\frac{7A_5}{4m^4}-\frac{3A_8}{4m^3}-\frac{39A_9}{8m^9}\right),\ L_7=m^2\frac{(1-n)}{(2-n)}\left(\frac{A_5}{6m^2}-\frac{5A_8}{8m^3}\right),$$
$$L_8=\frac{A_{10}}{12m}\frac{(1-n)}{(2-n)},\ L_9=\frac{A_6}{24}\frac{(1-n)}{(2-n)},\ L_{10}=-\frac{(1-n)}{(2-n)}\left(\frac{dp_1}{dx}+\frac{24A_1}{m^4}+2A_2+A_6+A_{10}+A_{11}\right),$$
$$L_{11}=-\frac{A_3}{2}\frac{(1-n)}{(2-n)},\ L_{12}=-\frac{(1-n)}{(2-n)}\left(\frac{A_2}{3}+\frac{9A_1}{m^2}\right),\ L_{13}=-\frac{A_1}{5}\frac{(1-n)}{(2-n)}. \qquad (41)$$

Using zero order and first order solutions, the complete solution for the velocity field to the first order can be written in the following analytical form

$$u = c_0(y^2 - H^2) - n c_0 \frac{H}{m}(\cosh my - \cosh mH) - 1 + \delta\Bigg[\left(D_3 + \frac{B_7}{m}y + D_1 y^2 + \frac{B_8}{m}y^4\right)\cosh my + \left(-\frac{B_6}{m} - \frac{B_7}{m^2} + D_2 y + (\frac{B_4}{m} - \frac{4B_8}{m^2})y^3 + \frac{B_{10}}{2}\sinh my\right)\sinh my + \frac{B_9}{2m}\cosh 2my + \frac{B_5}{2m}\sinh 2my + \frac{B_{14}}{6}y^6 + \frac{B_{13}}{4}y^4 + \frac{B_{12}}{3}y^3 + \frac{B_{11}}{2}y^2 + B_{15}y + T - 1 + - \frac{(1-n)}{(2-n)}\left(\frac{dp_1}{dx}\right)\left(\frac{nH}{m\sinh mH}(\cosh my - \cosh mH) - (y^2 - H^2)\right)\Bigg] \tag{42}$$

$$w = c_0(\sinh my - 1) + \delta\left[\left(L_1 + L_2 y + L_3 y^3 + L_4 y^4\right)\sinh my + \left(L_5 + L_6 y + L_7 y^3\right)\cosh my + L_8 \sinh 2my + L_9 \cosh 2my + L_{10} y + L_{11} y^2 + L_{12} y^3 + L_{13} y^5 + L_{14}\right] \tag{43}$$

PRESSURE GRADIENT

An explicit form for dp_0/sx can be obtained from Eq. 31 in the form

$$\frac{dp_0}{dx} = g(x)(F_0 + H) \tag{44}$$

where

$$g(x) = \frac{-3(2-n)m^2 \sinh mH}{(1-n)\left[2m^2H^3 \sinh mH + 3nH(\sinh mH - mH\cosh mH)\right]} \tag{45}$$

The first order instantaneous volume flow rate F_1 is given by:

$$F_1 = \int_0^H u_1 dy = \Bigg[\left[\left(\frac{H^2}{m} + \frac{2}{m^3}\right)D_1 - \frac{D_2}{m^2} + \frac{D_3}{m} + \left(\frac{6}{m^5} - \frac{3H^2}{m^4}\right)B_4 + \frac{H}{m^2}B_7 + \left(\frac{12H^2}{m^5} + \frac{H^4}{m^2} + \frac{12H^2}{m^4}\right)B_8\right]\sinh mH + \left[-\frac{2H}{m^2}D_1 + \frac{H}{m}D_2 - \frac{B_6}{m^2} - \frac{2B_7}{m^3} + \left(\frac{H^3}{m^2} + \frac{6H}{m^4}\right)B_4 - \left(\frac{8H^3}{m^3} + \frac{48H}{m^5}\right)B_8\right]\cosh mH + \frac{B_9}{2m}\sinh 2mH + \left(\frac{B_5}{4m^2} + \frac{B_{10}}{8m}\right)\cosh 2mH + \frac{H}{4}B_{10} - \frac{H^3}{6}B_{11} + \frac{H^4}{12}B_{12} + \frac{H^5}{20}B_{13} + \frac{H^7}{42}B_{14} + \frac{H^2}{2}B_{15} + (T-1)H - \frac{(1-n)}{(2-n)}\left(\frac{dp_1}{dx}\right)\left[\frac{3nH(\sinh mH - H\cosh mH) + 2m^2H^3\sinh mH}{3m^2\sinh mH}\right] \tag{46}$$

On solving Eq. 46 for dp_1/dx, one finds

$$\frac{dp_1}{dx} = g(x)(F_1 + H) - g(x)\Bigg[\left\{\left(\frac{H^2}{m} + \frac{2}{m^3}\right)D_1 - \frac{D_2}{m^2} + \frac{D_3}{m} + \left(\frac{6}{m^5} - \frac{3H^2}{m^4}\right)B_4 + \frac{H}{m^2}B_7 + \left(\frac{12H^2}{m^5} + \frac{H^4}{m^2} + \frac{12H^2}{m^4}\right)B_8\right\}\sinh mH + \left\{-\frac{2H}{m^2}D_1 + \frac{H}{m}D_2 - \frac{B_6}{m^2} - \frac{2B_7}{m^3} + \left(\frac{H^3}{m^2} + \frac{6H}{m^4}\right)B_4 - \left(\frac{8H^3}{m^3} + \frac{48H}{m^5}\right)B_8\right\}\cosh mH + \frac{B_9}{2m}\sinh 2mH + \left(\frac{B_5}{4m^2} + \frac{B_{10}}{8m}\right)\cosh 2mH + \frac{H}{4}B_{10} - \frac{H^3}{6}B_{11} + \frac{H^4}{12}B_{12} + \frac{H^5}{20}B_{13} + \frac{H^7}{42}B_{14} + \frac{H^2}{2}B_{15} + HT\Bigg] \tag{47}$$

Substituting the zero-order and the first order pressure gradient into the expansion 19 we obtain an explicit form for the pressure gradient, to the first order, as:

$$\begin{aligned}\frac{dp}{dx} &= g(x)(F+H) - g(x)\delta\Bigg[\Bigg\{\left(\frac{H^2}{m}+\frac{2}{m^3}\right)D_1 - \frac{D_2}{m^2} + \frac{D_3}{m} + \left(\frac{6}{m^5}-\frac{3H^2}{m^4}\right)B_4 + \frac{H}{m^2}B_7 \\ &+ \left(\frac{12H^2}{m^5}+\frac{H^4}{m^2}+\frac{12H^2}{m^4}\right)B_8\Bigg\}\sinh mH + \Bigg\{-\frac{2H}{m^2}D_1 + \frac{H}{m}D_2 - \frac{B_6}{m^2} - \frac{2B_7}{m^3} + \left(\frac{H^3}{m^2}+\frac{6H}{m^4}\right)B_4 \\ &- \left(\frac{8H^3}{m^3}+\frac{48H}{m^5}\right)B_8\Bigg\}\cosh mH + \frac{B_9}{2m}\sinh 2mH + \left(\frac{B_5}{4m^2}+\frac{B_{10}}{8m}\right)\cosh 2mH + \frac{H}{4}B_{10} \\ &- \frac{H^3}{6}B_{11} + \frac{H^4}{12}B_{12} + \frac{H^5}{20}B_{13} + \frac{H^7}{42}B_{14} + \frac{H^2}{2}B_{15} + (T-1)H\Bigg]\end{aligned} \tag{48}$$

Accordingly, the pressure rise Δp_λ and friction force F_λ can be computed numerically, where

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dx}dx \tag{49}$$

and

$$F_\lambda = \int_0^1 (-H)\frac{dp}{dx}dx \tag{50}$$

RESULTS AND DISCUSSION

In this research, we have obtained an explicit form for the fluid velocity, the microrotation velocity and the pressure gradient. The only restriction we have used that the wave number δ is small so that $\delta<1$. When Re = 0, we obtain the special case of Srinivasacharya *et al*. (2003), which was done for cylindrical case. Moreover when n = 0, we obtain the results for Newtonian fluid.

To discuss present results quantitatively, the integrals 49 and 50 are computed numerically using MATHEMATICA package and then the pressure rise and friction force are explained graphically.

Figure 2 shows the effect of coupling number n on Δp_λ at $\delta = 0.02$, Re = 10, m = 3, J = 1, $\varphi = 0.8$ and (n = 0.5, 0.7, 0.9). As shown the relation between pressure rise and flow rate is linear for micropolar fluid and pressure rise does not depend on coupling number n at a certain value of flow rate. Also, it is noted that the pumping increases with an increase in n. This means that pumping of micropolar fluid is greater than for Newtonian at the same values of physical parameters.

In Fig. 3, the pressure rise is plotted versus flow rate at $\delta = 0.02$, n = 0.5, m = 3, J = 1, $\varphi = 0.8$ and (Re = 0, 5, 10). We note that the pressure rise decreases with increasing Reynolds number Re.

The effect of microrotation parameter m on Δp_λ is shown in Fig. 4 at $\delta = 0.02$, Re = 1, n = 0.5, J = 1, $\varphi = 0.6$ and (m = 3, 4, 5). The figure reveals that the pressure rise decreases as m increases. Again, for Newtonian fluid (when m tends to infinity), the pressure rise is smaller than for micropolar fluid.

Figure 5 shows the variation of friction force F_λ with amplitude ratio φ at $\delta = 0.02$, Re = 1, m = 3, J = 1 and (n = 0.5, 0.7, 0.9). Here we observe that the friction force decreases with increasing φ and increases by increasing coupling number n.

Figure 6 shows the effect of Reynolds number Re on friction force F_λ at $\delta = 0.02$, n = 0.5, m = 3, J = 1 and (Re = 0, 5, 10). It is clear that the friction force increases with increasing Reynolds number.

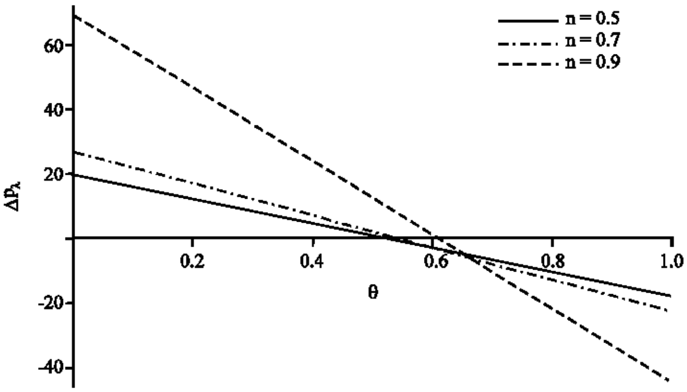

Fig. 2: Pressure rise versus flow rate at $\delta = 0.02$, Re = 10, J = 1, m = 3, $\varphi = 0.8$ and (n = 0.5, 0.7, 0.9)

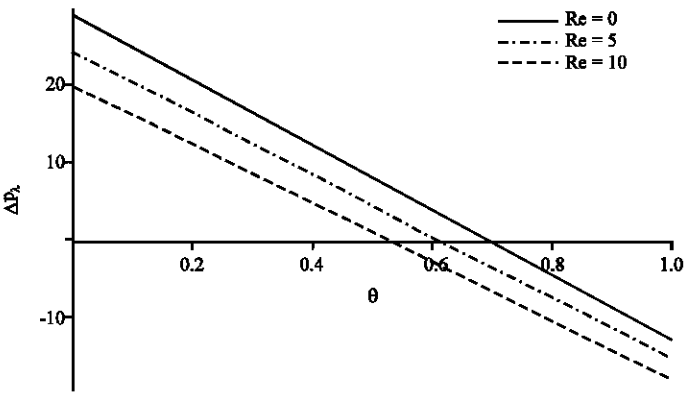

Fig. 3: Pressure rise versus flow rate at $\delta = 0.02$, n = 0.5, J = 1, m = 3, $\varphi = 0.8$ and (Re = 0, 5, 10)

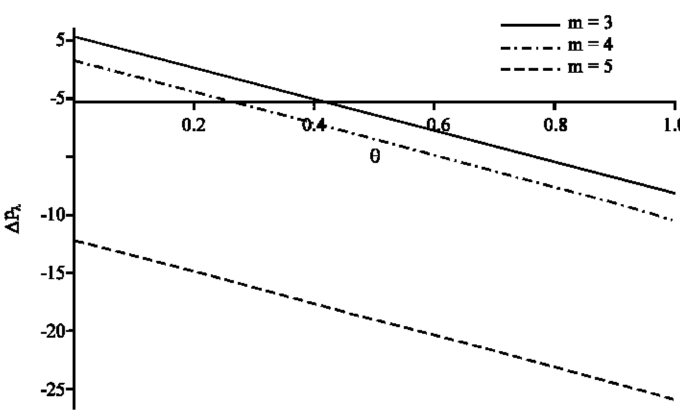

Fig. 4: Pressure rise versus flow rate at $\delta = 0.02$, Re = 1, J = 1, n = 0.5, $\varphi = 0.6$ and (m = 3, 4, 5)

Finally, in Fig. 7 the friction force is graphed versus the amplitude ratio at $\delta = 0.02$, n = 0.5, Re = 1, J = 1 and (m = 3, 4, 5). It is noted that the friction force increases as the micropolar parameter m increases.

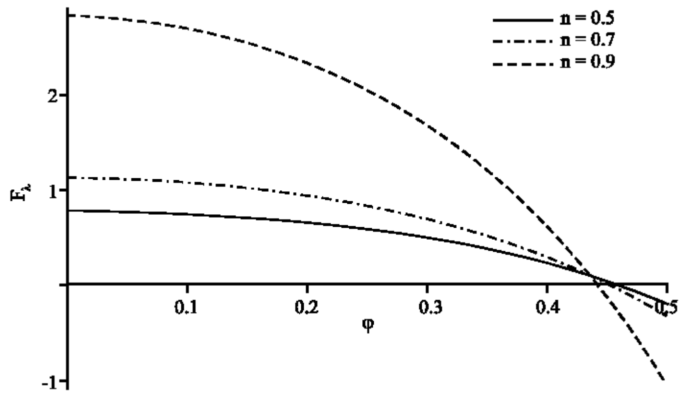

Fig. 5: Friction force versus φ at δ = 0.02, Re = 1, J = 1, m = 3 and (n = 0.5, 0.7, 0.9)

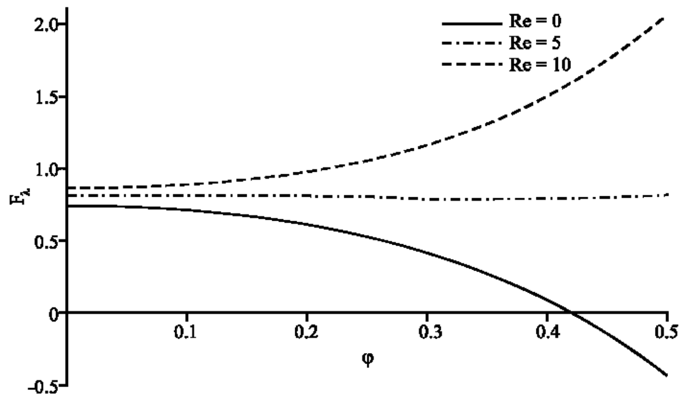

Fig. 6: Friction force versus φ at δ = 0.02, n = 0.5, J = 1, m = 3and (Re = 0, 5, 10)

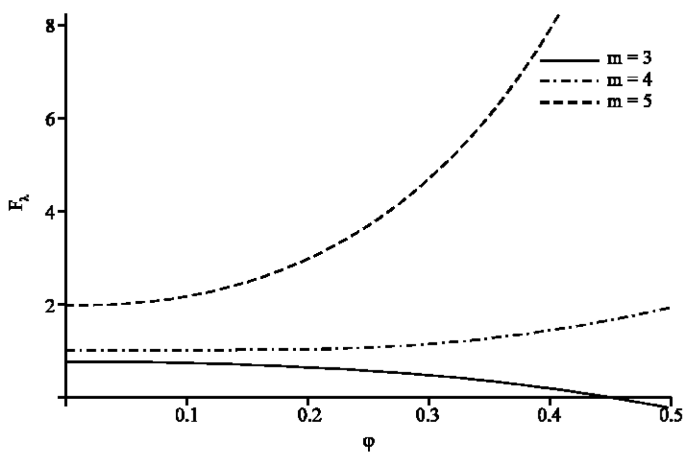

Fig. 7: Friction force versus φ at δ = 0.02, Re = 1, J = 1, n = 0.5 and (m = 3, 4, 5)

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