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Simulation of Paris-Erdogan Crack Propagation Model with Power Value, M = 3: The Impact of Applying Discrete Values of Stress Range on the Behaviour of Damage and Lifetime of Structure

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Abstract: The aim of this study is to investigate the impact of applying stress sequence in the form of discrete values of ascending and descending order of the same range on the behaviour of damage. The behaviour of the crack propagation as well as the lifetime of structure is investigated when M, which is the power value in the crack propagation Paris-Erdogan model is taken to be equal to three (M = 3). It is found that bigger stress or loadings imposed on the structure at the beginning of its life will result in a bigger damage. As a result, the lifetime of the structures is shorter as compared to the one which starts with lower stress.

Key words: Crack length, crack propagation, deterministic stress range, behaviour of damage, lifetime of structure

INTRODUCTION

Fatigue failure due to crack propagation in structures is very important in the field of material engineering. The random nature in fatigue crack growth process contributes a significant problem in the area of structural design and engineering. This is due to the fact that it is difficult to model accurately. Part of this difficulty is due to the presence of uncertainty in the values of parameters and variables associated with fatigue models (Ahmad-Shariff *et al*., 1998). The stochastic nature of fatigue crack growth is clearly evident in experimental results and field data which indicate that such variability may depends on many factors, including material properties, types of loading, environmental conditions, etc. The random nature of fatigue is also, of course, to be expected in structures subjected to randomly varying loads (Sobczyk and Spencer, 1992).

In most studies the stochastic models proposed for fatigue crack growth have assumed constant amplitude cyclic loading or program loading: The models are essentially randomised versions of deterministic models obtained by the introduction of random coefficients in the well-known Paris-Erdogan equation (Ahmad-Shariff *et al*., 1998). In contrast, in this study, the effect of applying discrete values of stress range is studied. This is done by incorporating the discrete form of stress range in the simulation model derived from Paris-Erdogan law (Paris and Erdogan, 1963).

PARIS-ERDOGAN MODEL OF CRACK GROWTH

One of the empirical models which is widely used under conditions of linear-elastic fracture is that proposed by Paris and Erdogan (1963). The model relates the increment of crack growth per cycle, da/dn, to the parameters of stress-range Δs and instantaneous crack length a.

Paris and Erdogan derived the model based on large range of data and arrived at the expression of the form:

$$\frac{da}{dN} = C(\Delta K)^M \tag{1}$$

where, C > 0, is an empirical crack-growth constant determined by material properties (i.e., elasticity, yield stress and fracture strength). A non-negative material constant, M is a coefficient of model influence and ΔK is the fluctuation range of the crack tip stress-intensity factor K ($\Delta K = K_{max} - K_{min}$) which depends upon the size and type of the crack (Clauss *et al*., 1994). A simple model for K is given by:

$$K = B\sqrt{\pi a}\, s \tag{2}$$

where, s is the stress at the crack tip and B is the geometry correction factor which depends on the crack shape, crack length a and the shape of the component.

Based on (2) we can have:

$$\Delta K = B\sqrt{\pi a}\ \Delta s \tag{3}$$

where, Δs denotes the fixed stress range per load cycle. The value B is taken to be independent of a, so that it relates only to the crack geometry and the configuration of the structural component, even though this is only practical for small crack growth. Since the problem of estimating crack length is in practice important for relatively large crack growth, the immediate engineering application of this paper is limited. Substituting (3) in (1), the expression can be written as:

$$\frac{da}{dN} = \lambda(\sqrt{a}\ \Delta s)^M \tag{4}$$

where, $\lambda = C(B(a))^M\ \pi^{M/2}$. From Eq. 4, a backward difference approximation is obtained as:

$$a_{N+1} - a_N = \lambda a_N^{m/2}(\Delta s_N)^M \tag{5}$$

in which crack growth (damage) accumulates relatively slowly and continuously with the load cycles N. On the basis of experimental results, it is generally accepted that the value of M lies between 2 and 4.

In this study, M is chosen to be 3 and therefore (5) becomes:

$$a_{N+1} = a_N + \lambda a_N^{3/2}(\Delta s_N)^3 \tag{6}$$

Since most load-bearing structural elements have relatively small initial cracks or fine notches, the initial crack length a_0 must begin with a very small number. Different types of model for the stress range can be used to investigate their effects on crack growth. For each model, an appropriate constant λ need to be chosen, so as to get 'readable' results. What is meant by 'readable' is that the results obtained should be meaningful and reasonable, that is, the damages obtained should increase relatively slowly and continuously. The rates of crack growth do not grow too rapidly or abruptly, except after failure occurs. Conventionally, failure is assumed to occur when the damage reaches some critical value. With suitable normalization this value can be taken to be 1.

MODELLING THE STRESS RANGE

The stress sequence used in this model to simulate crack growth is in the form of discrete values of ascending order and descending order of the same range where the stress is formulated as a function of days. By doing this, the damage and lifetimes of structures or mechanical components experiencing different orders of stress can be compared. We are interested in studying the effect of having low or high stress at the initial crack propagation phase to the final damage and lifetime. Since we are going to generate the stress in ascending and descending order of the same range, it can be presented as:

$$S_a = i + c\frac{t}{N} \tag{7}$$

and

$$S_d = l + c\frac{t}{N} \tag{8}$$

Equation 7 is for ascending order where, the stress starts at initial value i and ends at final value l and Eq. 8 is of descending order where the stress starts at initial value l and terminates at final value i, t = 0, 1, 2,.... N and N is the total number of days the model is to be run. As a result of the above conditions we have i + c = l. In this simulation we decide to take the range from 5 to 10 for S_a and 10 to 5 for S_d. Therefore, we have i = 5, l = 10 and c = 5 so that Eq. 7 becomes $S_a = 5 + 5\left(\frac{t}{N}\right)$ and Eq. 8 becomes $S_d = 10 - 5\left(\frac{t}{N}\right)$.

The crack model is then simulated for values of t from 0 to N and N can take any possible or suitable number of days. In the results presented here we have taken the parameter values $a_0 = 0.65\times10^{-2}$ and $\lambda = 0.5\times10^{-4}$ for N = 1000. The damages obtained are compared between the two results produced by both stresses and part of the output are presented in Table 1 to show the difference at the beginning and at the end of the simulation. The results are then plotted on graphs of damage against number of days in Fig. 1-3, respectively.

RESULTS AND DISCUSSION

Figure 1 compares the damage obtained from the two types of stress up to N = 1000. From Fig. 1 and Table 1 it is found that even though the stress is decreasing for S_d, the damage increases more rapidly because the present damage depends on the previous ones which in turn are raised to the power of 1.5. Therefore S_d gives higher damage and reached 1.0 faster than the damage produced by S_a. Figure 2 shows the behaviour of the damage at the early part of the simulation. It can be shown that the damage simulated from s_d propagates more rapidly. Figure 3 shows that when as$_d$ reaches 1.0 at

Table 1: Results of damage obtained for ascending order and descending order of stress

No. of days (N)	Dam (asc)	Dam (desc)
0	0.006500	0.006500
1	0.006503	0.006526
2	0.006507	0.006553
918	0.141590	0.987260
919	0.143930	0.995030
920	0.146350	1.002900
990	0.973760	1.778600
991	1.021100	1.793800
992	1.072000	1.809300
999	1.566100	1.921600
1000	1.663900	1.938300

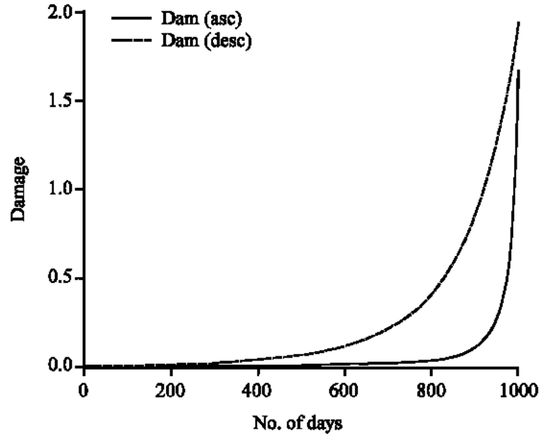

Fig. 1: Damage compared between ascending and descending order of stress

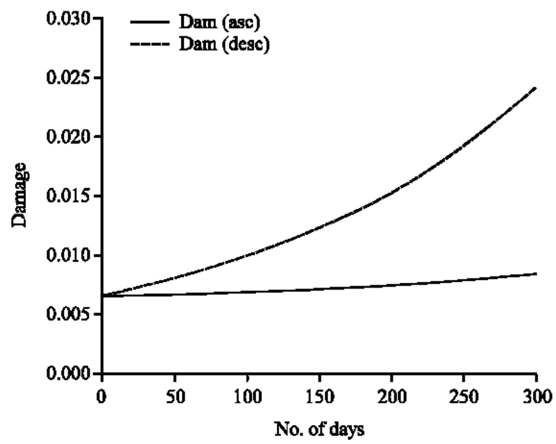

Fig. 2: Damage compared between ascending and descending order of stress at the earlier part of simulation

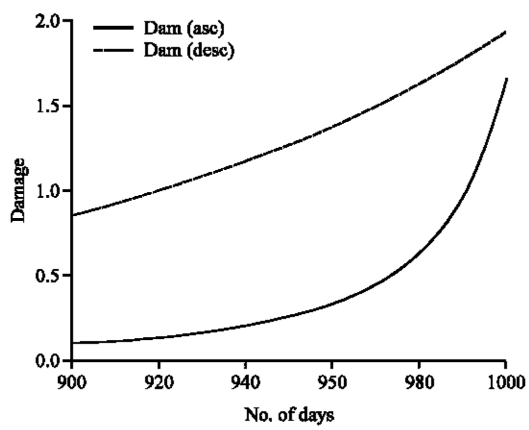

Fig. 3: Damage compared between ascending and descending order of stress at the end of simulation

N = 920, as_a is still catching up and reached 1.0 at N = 991, 70 cycles later. From the results obtained it can be concluded that bigger stress or loadings imposed on the structure at the beginning of its life will result in a bigger crack length or damage since initial damage accumulates more rapidly. As a result, the lifetime of the structures is shorter as compared to the one which starts with the lower stress. It is important to note that, the simulation work is done to study the behaviour of cracks or damage in such condition of stress and the results obtained cannot be compared with the real situation since the data for such behaviour are not available.

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