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## **Apparent Power Ratio of the Shunt Active Power Filter Under Balanced Power System Voltages**

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**Abstract:** The present study is focusing mainly on the study of the shunt active power filter compensations apparent power for different perturbations in AC power system caused by the load under balanced power system voltages, such as current unbalance, phase shift current and generated undesired harmonics in case of non linear load. The evaluation of the apparent power maximum rate which can be delivered by the shunt active power filter determines its compensations capabilities. This study is based on the definition of the effective apparent power as defined by IEEE group which was proved to be the suitable amount to be concerned in the design process of different devices.

**Key words:** Effective apparent power, shunt active power filter, unbalanced currents, harmonics

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### **INTRODUCTION**

Due to the large demands and requirements of different industrial consumers plants which are based mainly on power electronics converters, the power quality in AC power system has been intensively degraded with a drastically manner. The proliferation of industrial power electronics converters equipments, unbalanced load, large single phase loads and large unbalanced inductive load of one or more phases can frequently occur (Lee *et al.*, 1997). These kind of loads contribute in generating of undesired current quality, it was proved that the current distortion may cause undesirable effects on the power system operation, where the normal operations of the sensitive consumers load is strongly jeopardized such as relays protection and measure instruments (Jindal *et al.*, 2005; Chang and Yeh, 2005; George and Agarwal, 2007; Montero *et al.*, 2007; Green and Marks, 2005; Bina and Pashajavid, 2009). The unbalance linear/non-linear loads may cause poor power factor, large unbalance current, excessive neutral currents, greater eddy-current losses generated in the cables and in its pipes, unbalanced currents on the primary side of supply transformer, furthermore cables, transformer and other equipment suffer deeply of overheating, de-rating and low operating efficiency. On the other side, the resultant negative sequence causes the loss of electric energy. This currents quality seriously threatens the security and economy of the industrial consumers' plants and the power system. In the same time upstream power system is a non-stiff power supply, where the unbalance currents cause the voltage of the feeding voltage point where, the consumer plant is connected in the PCC

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to be fluctuated and unbalanced. Several studies proved that an unbalanced power system voltage can worsen drastically the power quality, practically with power electronics converters, Ac machines and other equipments. In industrial application, where three phase rectifiers are intensively used, the unbalance currents due to unbalanced power supply voltage causes harmful effects leading to an uneven current distribution over the rectifier bridge legs which increases the conduction loss and may cause failure of rectifying devices, increased RMS ripple current in the smoothing capacitor, increased total RMS line current and harmonics, in particular, noncharacteristic triplen harmonics that do not appear under balanced condition (Conway and Jones, 1993; Grötzbach and Xu, 1993; Bauta and Götzbach, 2000; Jeong and Choi, 2002). Thus, the power ratings of filters and switches are increased due to the power supplied by the source. In the other side an unbalanced voltage system supplied to an AC machine generates large negative-sequence which can increase the machine losses and reduces the machine use qualification (Souto *et al.*, 1998; Svensson and Sannino, 2001). The shunt APFs are presently the powerful tools and the most versatile and effective solution to face up to the challenge of reducing or eliminating the undesired current disturbances, protecting electrical equipment which could be affected by poor power quality and avoiding the propagation of generated disturbances to be followed toward the source or power supply. On the other side, these devices can achieve the compensation of reactive power and unbalance of nonlinear and fluctuating loads.

A perfect power quantity and quality supply would be one that is always available, always within voltage and frequency tolerances and has a pure sinusoidal wave shape; the deviation value from perfection which can be accepted depends on the user's application and their requirements. Users are faced extremely with the exact need for making design investment decisions about the quantity of the compensation power of the shunt APF required to achieve the quality of power delivered from the power system source. This study will give an approach for the evaluation of the power compensation in a way to allow for the manufacturer to dimension the shunt APF devices and to the users to get an optimum technical economical choice. Here the new definition of the apparent power is used to avoid the errors which were made in the last years when the apparent power was evaluated using classical definitions (Akagi *et al.*, 1983, 1984; Moreno *et al.*, 2004; Longhui *et al.*, 2007). In this study, it is supposed that the power system voltage is unbalanced under practical tolerances.

### **SHUNT ACTIVE POWER FILTER**

Active Power Filter (APF) is a power electronics device based on the use of power electronics inverters (Fig. 1). The shunt active power filter is connected in a common point connection between the source of power system and the load system which present the source of the polluting currents circulating in the power system lines (Akagi *et al.*, 1983, 1984; Watanabe and Aredes, 1993; Svensson and Sannino, 2001; Alali *et al.*, 2000a, b). This insertion is realized via low pass filter such as, L, LC or LCL filters (Bina and Pashajavid, 2009).

The shunt APF is proposed in this study, according to the characteristics of the consumer load plant in utilization, it injects current components in the power system in a small amount of power by ratio of the power delivered from source to load (Fig. 2). The compensating power can dynamically suppress the distorted current component, eliminate the components contributing in the current unbalance and make the currents circulating toward the power source to be in phase with the direct voltage sequence of the power system voltage. The result of this is that the utility currents after the compensation become

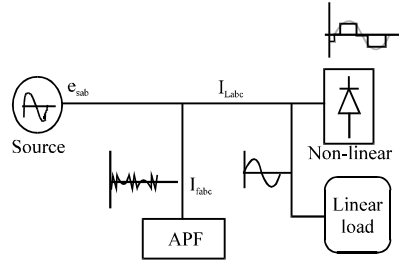


Fig. 1: Shunt active power filter principle schematics

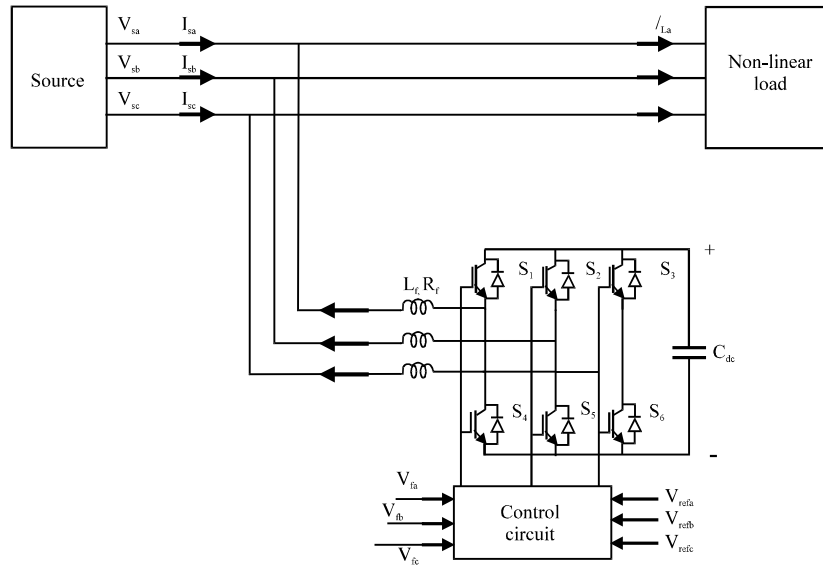


Fig. 2: Three wire schematics of the shunt APF

sinusoidal, balanced and with the desired amplitudes and shift phase. The fundamental equation representing the principle of the shunt APF compensation is given by:

$$I_{Labc} = I_{fabc} + I_{Sabc} \tag{1}$$

Where:

$$I_{Labc} = \begin{bmatrix} I_{La} \\ I_{Lb} \\ I_{Lc} \end{bmatrix}, I_{fabc} = \begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix}, I_{Sabc} = \begin{bmatrix} I_{Sa} \\ I_{Sb} \\ I_{Sc} \end{bmatrix} \tag{2}$$

### SHUNT ACTIVE POWER FILTER APPARENT POWER

To clarify this study a general case was studied theoretically for three phase three wire systems and then special cases which can be occur in industrial loads were derived, such as current harmonics, unbalance and/or distorted currents. Practically each case has its

calculation to achieve exactly the compensation needed to improve the power quality from the source power system. Moreover, lots of studies have been pursued on SAPF. But in most studies, the supply voltage is considered as a sinusoidal variable with constant amplitude (Akagi *et al.*, 1986; Akagi, 1994; Omeiri *et al.*, 2006; Longhui *et al.*, 2007; Saad and Zellouma, 2009). In the present study, as the supply-voltage unbalance is very serious problem for the load, especially due to the appearance of the negative sequence (Singh *et al.*, 1999; Zanchetta *et al.*, 2009), the unbalance of line voltage must be taken into account as a design factor in the shunt APF. Therefore, the power system voltage is expressed by:

$$V_i = V_m \sin\left(\omega t + \varepsilon_i \frac{2\pi}{3}\right) \quad (3)$$

where,  $i = a, b, c$  presenting the three phases,  $\varepsilon_a = 0, \varepsilon_b = -1, \varepsilon_c = 1$ .

The  $h$  component of the load currents are defined as follow:

$$i_{ih} = k_i \cdot I_{mh} \cdot \sin\left(\omega t + \varepsilon_i \frac{2\pi}{3} + \gamma_{ih}\right) \quad (4)$$

where,  $k_a, k_b, k_c$  are the magnitude currents unbalance factors,  $\gamma_{ah}, \gamma_{bh}, \gamma_{ch}$  are the phase shift unbalance for the phases a b and c load currents.  $h$  presents the harmonics order  $h = 1, 2, 3, \dots, I_{mh}$ , the current magnitude of the harmonics order  $h$ .

The necessary apparent power which responds to the load requirement following to the effective apparent definition is expressed as (Watanabe and Aredes, 1993; Alali *et al.*, 2000a, b; Willems *et al.*, 2000, 2005; Emanuel, 2003, 2004; Pajic and Emanuel, 2006a, b, 2009; Kouzou *et al.*, 2008a-d; Basu and George, 2008; IEEE Power Engineering Society, 2000):

$$S_e = 3V_e I_e \quad (5)$$

where,  $V_e$  and  $I_e$  are the corresponding effective voltage and effective current of the power supplied applied to the load which are calculated as follow:

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}} = \sqrt{I_{e1}^2 + I_{eh}^2} \quad (6)$$

Where:

$$I_{e1} = \sqrt{\frac{I_{a1}^2 + I_{b1}^2 + I_{c1}^2}{3}}, I_{eh} = \sqrt{\frac{I_{ah}^2 + I_{bh}^2 + I_{ch}^2}{3}} \quad (7)$$

where,  $I_{e1}$  is the fundamental component effective current, from Eq. 4 and 7 it can be expressed as:

$$I_{e1} = \frac{I_M}{\sqrt{3}} \sqrt{k_a^2 + k_b^2 + k_c^2} \quad (8)$$

The effective voltage of the three-wire power system is expressed as Eq. 37-39:

$$V_e = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{3}} \quad (9)$$

From Eq. 3 and 9 the effective voltage of the power supply can be presented as follow:

$$V_e = V \quad (10)$$

From Eq. 5 and 6 the effective apparent power can be presented by:

$$S_e^2 = 9V_e^2 I_e^2 = 9V^2 \cdot I_{e1}^2 + 9V^2 \cdot I_{eh}^2 \quad (11)$$

$$S_e^2 = S_{e1}^2 + S_{eh}^2 \quad (12)$$

where,  $S_{eh}$  is the apparent power responsible of different harmonics contained in the load current. On the other side the effective apparent power due to the fundamental component of the current is calculated as follow:

$$S_{e1}^2 = 9V^2 \cdot I_{e1}^2 \quad (13)$$

This power contains two parts: A component due to the fundamental positive component of current, it is the one generated by the power system to the load. This power is given by:

$$S_{e1}^+ = 3V \cdot I_{e1}^+ \quad (14)$$

A component due to the negative and zero components of the current, it is the one responsible of the unbalance in the load side. The shunt APF must produce and inject this power to eliminate the unbalance of the current absorbed from the source of the power system. This power is given by:

$$S_{unb}^2 = S_{e1}^2 - S_{e1}^{+2} \quad (15)$$

The effective fundamental positive component of the effective current is given by:

$$I_{e1}^+ = \frac{I_M}{3} \sqrt{k_a^2 + k_b^2 + k_c^2 + \Delta I} \quad (16)$$

Where:

$$\Delta I = \sum_{\substack{i,j=a,b,c \\ i \neq j}} k_i k_j \cdot \cos(\gamma_i - \gamma_j) \quad (17)$$

Figure 3 and 4 present the ratio  $I_{e1}^+ / I_M$  for two case, one phase magnitude and phase shift unbalances and two phases magnitude unbalances. It is obvious that this ratio is equal to one in Fig. 3 for  $\gamma_a = 0$  and  $k_a = 1$  and in Fig. 4 for  $k_a = 1$  and  $k_b = 1$ . On the other side, it is well shown that for an increased unbalance this ratio is decreased.

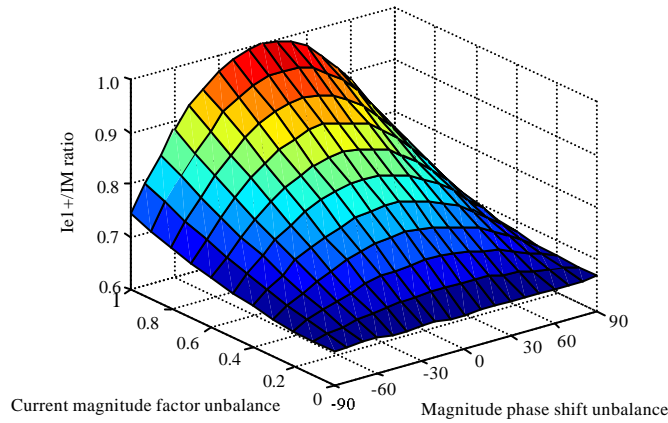


Fig. 3: The fundamental positive sequence effective current for one phase current unbalance

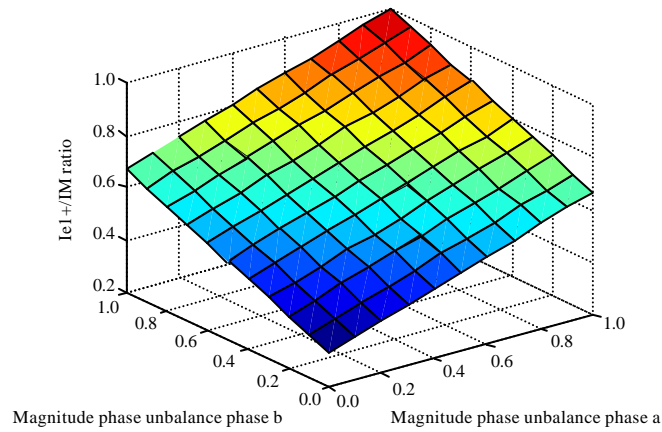


Fig. 4: The fundamental positive sequence effective current under current magnitude unbalance of two different phases

The effective apparent power responsible of the unbalance in the load currents is expressed by:

$$S_{\text{unb}} = 3V \cdot \sqrt{I_{e1}^2 - I_{e1}^{+2}} \quad (18)$$

It can be written as:

$$S_{\text{unb}} = V \cdot I_M \cdot \sqrt{2 \cdot \Delta k - \Delta I} \quad (19)$$

Where:

$$\Delta k = k_a^2 + k_b^2 + k_c^2 \quad (20)$$

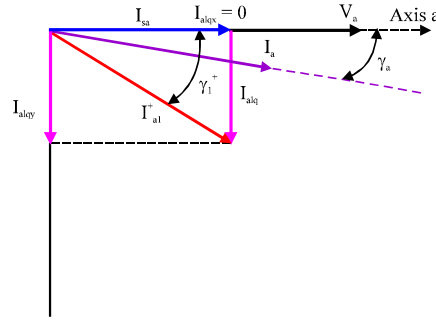


Fig. 5: Principle of canceling the shift phase between voltage and current of phase a

The power responsible of different harmonics contained in the load current is given by:

$$S_{eh} = 3V \cdot I_{eh} \quad (21)$$

Where the effective harmonic current is:

$$I_{eh} = I_h \sqrt{\frac{k_a^2 + k_b^2 + k_c^2}{3}} \quad (22)$$

$$I_{eh} = I_h \sqrt{\frac{\Delta k}{3}} \quad (23)$$

From Eq. 8, 21 and 24 can be written as:

$$I_{eh} = I_h \cdot \frac{I_{e1}}{I_M} = THD_e^1 \cdot I_{e1} \quad (24)$$

where,  $THD_e^1$  is the total harmonic distortion of the load current, it is presented by  $\sigma$  so:

$$S_{eh} = 3V_e \cdot \sigma \cdot I_{e1} \quad (25)$$

$$S_{eh} = \sqrt{3}V_e I_M \cdot \sigma \cdot \sqrt{\Delta k} \quad (26)$$

Finally in order to achieve a unite power factor in the source, the reactive power needed by the load have to be canceled from the fundamental components of voltage and current. Thus, the shunt APF has to generate the apparent power needed so that the voltages in the three phases have the same shift phase angles as the currents absorbed from the source by the load in the corresponding three phases. In Fig. 5, phase a is presented to show clearly the principle of the reactive power compensation. Hence, the required phase shift between the power system voltage and the source current is obtained.

The magnitude of the positive sequence of the current is the same as the magnitude of the effective positive sequence:

$$\begin{bmatrix} I_{a1}^+ \\ I_{b1}^+ \\ I_{c1}^+ \end{bmatrix} = \begin{bmatrix} I_{e1}^+ \\ I_{e1}^+ \\ I_{e1}^+ \end{bmatrix} \quad (27)$$



The currents needed to achieve the elimination of the reactive power to be absorbed from the power system are  $I_{alq}^+$ ,  $I_{blq}^+$  and  $I_{clq}^+$ . To obtain the minimum magnitude of these components they must be perpendicular on the source currents of the corresponding phases as it is shown in Fig .5, the magnitude of these currents are then:

$$\begin{bmatrix} I_{alq}^+ \\ I_{blq}^+ \\ I_{clq}^+ \end{bmatrix} = \begin{bmatrix} I_{alqy}^+ \\ I_{blqy}^+ \\ I_{clqy}^+ \end{bmatrix} = I_{el}^+ \cdot \begin{bmatrix} \sqrt{1 - \cos^2(\gamma_1^+)} \\ \sqrt{1 - \cos^2(\gamma_1^+)} \\ \sqrt{1 - \cos^2(\gamma_1^+)} \end{bmatrix} \quad (28)$$

Where, the phase shift of the positive components is give by:

$$\gamma_1^+ = \arctan \left[ \frac{\sum_i^{a,b,c} k_i \cdot \sin(\gamma_i)}{\sum_i^{a,b,c} k_i \cdot \cos(\gamma_i)} \right]$$

The effective current of these components can be evaluated as:

$$I_{elq}^+ = I_{el}^+ \cdot \sqrt{1 - \cos^2(\gamma_1^+)} \quad (29)$$

Or:

$$I_{elq}^+ = I_{el}^+ \cdot \Delta q \quad (30)$$

Where:

$$\Delta q = \sqrt{1 - \cos^2(\gamma_1^+)} \quad (31)$$

The corresponding effective apparent power responsible of the phase shift between the power system voltage and the load current is expressed as:

$$S_{elq}^+ = 3 \cdot V \cdot I_{elq}^+ \quad (32)$$

This leads to the following expression:

$$S_{elq}^+ = S_{el}^+ \cdot \sqrt{1 - \cos^2(\gamma_1^+)} \quad (33)$$

Finally it can be written as:

$$S_{elq}^+ = V \cdot I_M \cdot \sqrt{\Delta k + \Delta l} \cdot \Delta q \quad (34)$$

The total apparent power necessary to achieve a good compensation for the unbalances, harmonics and reactive power is deduced from Eq. 20, 27 and 35. It is presented by the following expression:

$$S_{\text{comp}} = \sqrt{S_{\text{mb}}^2 + S_{\text{eh}}^2 + S_{\text{elq}}^2} \quad (35)$$

So:

$$S_{\text{comp}} = V \cdot I_M \sqrt{S_{\text{comp1}} + S_{\text{comp2}}} \quad (36)$$

Where:

$$S_{\text{comp1}} = \Delta k \cdot (2 + 3 \cdot \text{THD}_e^{I^2} + \Delta q^2) \quad (37)$$

$$S_{\text{comp2}} = (\Delta q^2 - 1) \cdot \Delta I \quad (38)$$

The positive apparent power ratio is supposed as:

$$R_p = \frac{S_{\text{el}}^+}{S_{\text{el}}} \quad (39)$$

This can be written as:

$$R_p = \frac{I_{\text{el}}^+}{I_{\text{el}}} \quad (40)$$

It leads to:

$$R_p = \frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{\Delta I}{\Delta k}} \quad (41)$$

Where:

$$0 < R_p \leq 1 \quad (42)$$

But practically values of  $R^+$  are not far from 1.

The main objective described in this study is to obtain the apparent power ratio of the shunt active power filter which characterizes its capability for achieving the main aim of compensation. This ratio is presented as follow:

$$R = \frac{S_{\text{comp}}}{S_s} \quad (43)$$

Where:

$$S_s = 3 \cdot V \cdot I_{\text{se}} \quad (44)$$

Presents the apparent power delivered by the power system (source) to the load with an optimized cost.  $I_{\text{se}}$  is the effective current circulating from the source to the PCC, it can be calculated by:

$$I_{se} = \sqrt{\frac{I_{sa}^2 + I_{sb}^2 + I_{sc}^2}{3}} \quad (45)$$

Where:

$$\begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \end{bmatrix} = I_{ei}^+ \cdot \begin{bmatrix} \cos(\gamma_1^+) \\ \cos(\gamma_1^+) \\ \cos(\gamma_1^+) \end{bmatrix} \quad (46)$$

The resulting effective source current is:

$$I_{se} = I_{ei}^+ \cdot \cos(\gamma_1^+) \quad (47)$$

And the apparent power becomes as follow:

$$S_s = 3 \cdot V \cdot I_{ei}^+ \cdot \cos(\gamma_1^+) = S_{ei}^+ \cdot \cos(\gamma_1^+) \quad (48)$$

The compensation apparent power produces by the active power filter is presented as:

$$S_{comp} = \frac{S_{ei}^+}{R_p} \sqrt{1 + \sigma^2 + R_p^2 \cdot (\Delta q^2 - 1)} \quad (49)$$

The apparent power ratio of the shunt APF can then be written by the following expression:

$$R = \frac{1}{R_p \cdot \cos(\gamma_1^+)} \cdot R_0 \quad (50)$$

Where:

$$R_0 = \sqrt{1 + \sigma^2 + R_p^2 \cdot (\Delta q^2 - 1)} \quad (51)$$

where, R gives a clear idea about the shunt active power filter dimension to fulfill the desired compensations, it can also be used in the process design of the devices used in this compensators. In this study the loses due to the devices operations such as the switching lose of static switches were not taken into account, as it is neglected beyond the apparent power needed for the compensation.

### EVALUATION OF THE APPARENT POWER RATIO OF THE SHUNT APF FOR DIFFERENT COMPENSATION CASES

The flowchart of Fig. 6 gives the calculation algorithm of the Apparent power ratio of the shunt active filter R, where in this study, it is supposed that the power system voltages is balanced and have sinusoidal forms, even though directly or through a compensating system. This algorithm is based on the information given by the user or consumer about the disturbances limits of the load currents to be compensated which is connected to three phase

three wire AC power system. Table 1-4 give the calculation of the intermediary parameters for different unbalance possibilities, which can be occurred in practical cases for the load current such as magnitude unbalances, phase shift unbalances and harmonics distortion. In this study the THD is presented by the variable  $\sigma$  which is supposed to be the same for the three phases.

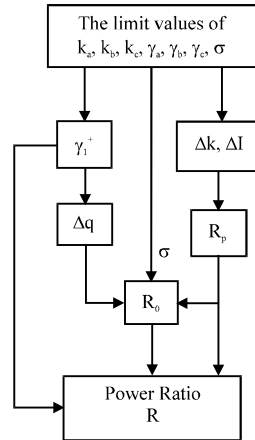


Fig. 6: The apparent power ratio calculation of the shunt APF

Table 1: Calculation of the intermediary parameters of the shunt APF for different cases of one phase unbalance

Unbalance nature	$\Delta l$	$\Delta k$	$\gamma_1^+$	$\Delta q$
One phase (k, $\gamma$ )	$2+4k \cos(\gamma)$	$2+k^2$	$\frac{k \sin(\gamma)}{k \cos(\gamma) + 2}$	$\sqrt{1 - \cos^2(\gamma_1^+)}$
One phase (k, $\gamma$ , $\sigma$ )	$2+4k \cos(\gamma)$	$2+k^2$	$\frac{k \sin(\gamma)}{k \cos(\gamma) + 2}$	$\sqrt{1 - \cos^2(\gamma_1^+)}$
One phase (k, $\sigma$ )	$4k+2$	$k^2+k^2$	0	0
One phase ( $\gamma$ , $\sigma$ )	$2+4 \cos(\gamma)$	3	$\frac{\sin(\gamma)}{\cos(\gamma) + 2}$	$\sqrt{1 - \cos^2(\gamma_1^+)}$
One phase (k)	$4k+2$	$k^2+2$	0	0
One phase ( $\gamma$ )	$2+4 \cos(\gamma)$	3	$\frac{\sin(\gamma)}{\cos(\gamma) + 2}$	$\sqrt{1 - \cos^2(\gamma_1^+)}$

Table 2: Shunt APF apparent power ratios  $R_p$  and  $R$  of one phase unbalance

Unbalance nature	$R_p$	$R_0$	$R$
One phase (k, $\gamma$ )	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{2+4k \cos(\gamma)}{2+k^2}}$	$\sqrt{1 - R_p^2 \cdot \cos^2(\gamma_1^+)}$	$\frac{R_0}{R_p \cdot \cos(\gamma_1^+)}$
One phase (k, $\gamma$ , $\sigma$ )	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{2+4k \cos(\gamma)}{2+k^2}}$	$\sqrt{1 + \sigma^2 - R_p^2 \cdot \cos^2(\gamma_1^+)}$	$\frac{R_0}{R_p \cdot \cos(\gamma_1^+)}$
One phase (k, $\sigma$ )	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{4k+2}{k^2+2}}$	$\sqrt{1 + \sigma^2 - R_p^2}$	$\frac{R_0}{R_p}$
One phase ( $\gamma$ , $\sigma$ )	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{2+4 \cos(\gamma)}{3}}$	$\sqrt{1 + \sigma^2 - R_p^2 \cdot \cos^2(\gamma_1^+)}$	$\frac{R_0}{R_p \cdot \cos(\gamma_1^+)}$
One phase (k)	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{4k+2}{k^2+2}}$	$\sqrt{1 - R_p^2}$	$\frac{R_0}{R_p}$
One phase ( $\gamma$ )	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{2+4 \cos(\gamma)}{3}}$	$\sqrt{1 - R_p^2 \cdot \cos^2(\gamma_1^+)}$	$\frac{R_0}{R_p \cdot \cos(\gamma_1^+)}$

Table 3: Calculation of the intermediary parameters of the shunt APF for different cases of two phases and three phases unbalance

Unbalance nature	$\Delta I$	$\Delta k$	$\gamma_1^+$	$\Delta q$
Three phases (k, $\gamma$ ), ( $\gamma$ )	$6k^2$	$3k^2$	$\gamma$	$\sqrt{1 - \cos^2(\gamma)}$
Three phases (k, $\gamma$ , $\sigma$ ), ( $\gamma$ , $\sigma$ )	$6k^2$	$3k^2$	$\gamma$	$\sqrt{1 - \cos^2(\gamma)}$
Three phases (k, $\sigma$ )	$6k^2$	$3k^2$	0	0
Two phases (k)	$2+4k^2$	$1+2k^2$	0	0
Two phases ( $\gamma$ )	$2+4 \cos(\gamma)$	3	$\frac{2 \sin(\gamma)}{2 \cos(\gamma) + 1}$	$\sqrt{1 - \cos^2(\gamma_1^+)}$
Two phases (k, $\sigma$ )	$2+4k^2$	$1+2k^2$	0	0

Table 4: Apparent power ratios  $R_p$  and  $R$  of the shunt APF for different cases of two phases and three phases unbalance

Unbalance nature	$R_p$	$R_0$	$R$
Three phases (k, $\gamma$ ), ( $\gamma$ )	1	$\sqrt{1 - \cos^2(\gamma)}$	$\sqrt{\frac{1}{\cos^2(\gamma)} - 1}$
Three phases (k, $\gamma$ , $\sigma$ ), ( $\gamma$ , $\sigma$ )	1	$\sqrt{1 + \sigma^2 - \cos^2(\gamma)}$	$\sqrt{\frac{1 + \sigma^2}{\cos^2(\gamma)} - 1}$
Three phases (k, $\sigma$ )	1	$\sigma$	$\sigma$
Two phases (k)	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{4k + 2k^2}{1 + 2k^2}}$	$\sqrt{1 - R_p^2}$	$\frac{R_0}{R_p}$
Two phases ( $\gamma$ )	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{2 + 4 \cos(\gamma)}{3}}$	$\sqrt{1 - R_p^2 \cdot \cos^2(\gamma_1^+)}$	$\frac{R_0}{R_p \cdot \cos(\gamma_1^+)}$
Two phases (k, $\sigma$ )	$\frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{4k + 2k^2}{1 + 2k^2}}$	$\sqrt{1 + \sigma^2 - R_p^2}$	$\frac{R_0}{R_p}$

Four different unbalance cases are presented in this study to show the variation of the apparent power ratio  $R$  versus the variation of the limits or constraints of the parameters given in the flowchart of Fig. 6.

**One Phase Unbalanced Load**

The following values are taken to calculate the different parameters used in the evaluation of the power ratio:

$$k_b = k_c = 1, k_a = k, \gamma_b = \gamma_c = 0, \gamma_a = \gamma, \sigma = 0 \tag{52}$$

where,  $k$  and  $\gamma$  are presenting the limits of the magnitude unbalance and phase shift unbalance respectively of one phase. According to the flowchart the resulting parameters are:

$$\Delta I = 2 + 4k \cos(\gamma), \Delta k = 2 + k^2, \gamma_1^+ = \arctan\left(\frac{k \cdot \sin(\gamma)}{k \cdot \cos(\gamma) + 2}\right) \text{ and } \Delta q = \sqrt{1 - \cos^2(\gamma_1^+)} \tag{53}$$

$$R_p = \frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{2 + 4k \cos(\gamma)}{2 + k^2}}, R_0 = \sqrt{1 + R_p^2 \cdot \cos^2(\gamma_1^+)} \tag{54}$$

The apparent power ratio is:

$$R = \frac{1}{R_p \cdot \cos(\gamma_1^+)} \cdot R_0 \tag{55}$$

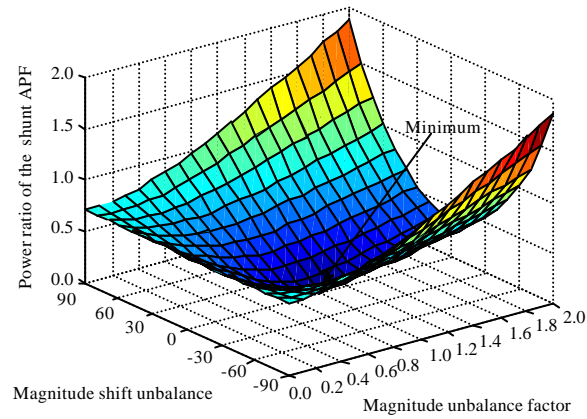


Fig. 7: Apparent power ratio R of the shunt APF

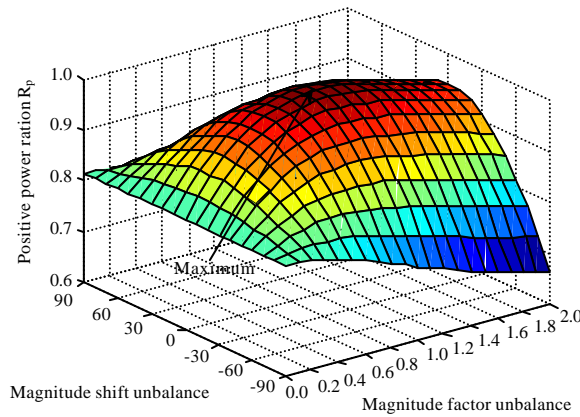


Fig. 8: Positive apparent power ratio Rp of the shunt APF

Figure 7 and 8 give the values of the apparent power ratio R of the shunt APF and the positive sequence apparent power ratio  $R_p$ , respectively. It is clear from Fig. 7 that the power ratio is equal to 0, when there is no unbalance in phase a, this means that the compensation power needed from the shunt APF is nil.

Figure 8 gives a clear idea about the effect of the unbalance in the load current specially the positive component, when this ratio is near to unit, the compensation apparent power of the shunt APF is near to zero and the compensation needed is minimal.

### Two Phase Unbalanced Load

The magnitude factor unbalance and the phase shift unbalance are in this case presented as follows:

$$k_c = 1, k_a = k_b = k, \gamma_c = 0, \gamma_a = \gamma_b = \gamma, \sigma = 0 \tag{56}$$

Hence, the intermediary parameters are:

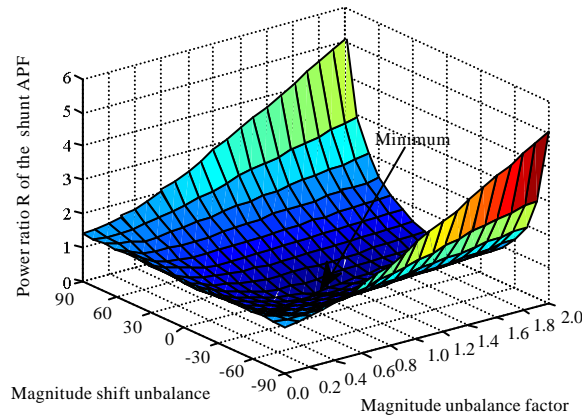


Fig. 9: Apparent power ratio R of the shunt APF

$$\Delta I = 2k^2 + 4k \cos(\gamma), \Delta k = 1 + 2k^2, \gamma_1^+ = a \tan \left( \frac{2 \cdot k \cdot \sin(\gamma)}{2 \cdot k \cdot \cos(\gamma) + 1} \right) \text{ and } \Delta q = \sqrt{1 - \cos^2(\gamma_1^+)} \quad (57)$$

$$R_p = \frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{2k^2 + 4k \cos(\gamma)}{1 + 2k^2}}, R_0 = \sqrt{1 - R_p^2 \cdot \cos^2(\gamma_1^+)} \quad (58)$$

The apparent power ratio is:

$$R = \sqrt{\frac{1}{R_p^2 \cdot \cos^2(\gamma_1^+)} - 1} \quad (59)$$

It is clear in Fig. 9 that the power ratio equals 0 for  $k_b = k_a = 1$ , this means that no power compensation is needed for balanced power system voltages and balanced linear load currents. This value is maximal when the currents of phases b and c are nil, in this case the power compensation needed from the shunt APF is nearly twice the power produced from the power system to force the power system currents to be balanced, these results are given with linear loads, but such constraints are so far from practical cases and leads the shunt APF to be useless.

From Eq. 58 and 59 for  $k = 1$  the following values are obtained:  $R_0 = 0$ ,  $R_p = 1$  and  $R = 0$  no compensation is needed.

These results can be shown clearly in Fig. 9 and 10 where,  $R_p$  has a maxima value equals to one, this means that no magnitude unbalance is occurred.

### One Phase Unbalanced with non Linear Load

Under this condition of unbalance the different magnitude factor unbalance, the phase shift unbalance and  $\sigma$  can be presented as follow:

$$k_a = k, k_b = k_c = 1, \gamma_a = \gamma_b = \gamma_c = 0, \sigma > 0 \quad (60)$$

The resulting intermediary parameters are:

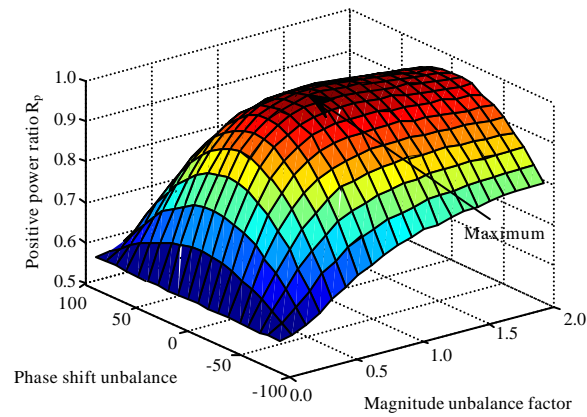


Fig. 10: Positive apparent power ratio  $R_p$  of the shunt APF

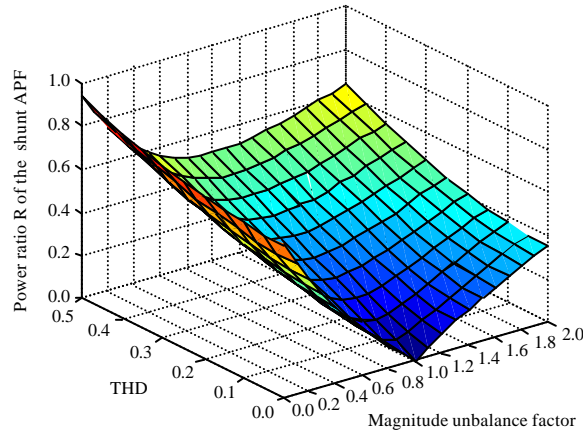


Fig. 11: Apparent power ratio  $R$  of the shunt APF

$$\Delta I = 4k + 2, \Delta k = k^2 + 2, \gamma_1^+ = 0 \text{ and } \Delta q = 0 \quad (61)$$

$$R_p = \frac{1}{\sqrt{3}} \cdot \sqrt{1 + \frac{4k + 2}{k^2 + 2}}, R_0 = \sqrt{1 + \sigma^2 - R_p^2} \quad (62)$$

The apparent power ratio is finally given by:

$$R = \frac{R_0}{R_p} \quad (63)$$

Figure 11 and 12 give the values of the power ratio  $R$  of the shunt APF and the positive sequence apparent power ratio  $R_p$ . It is clear in Fig. 11 that the power ratio equals 0 for  $k = 1$  and  $\sigma = 1$ , in this case no power compensation is needed, while the power system voltage is balanced and the load is linear and balanced. This value is maximal when the current of phases is nil and  $\sigma$  equals to unit, in this case the power compensation needed



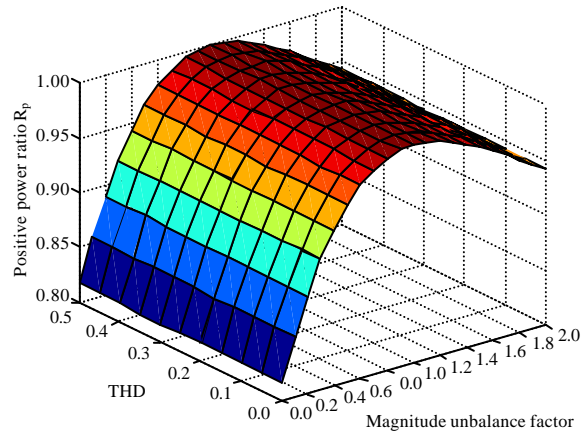


Fig. 12: Positive apparent power ratio  $R_p$  of the shunt APF

from the shunt APF is greater than the power produced from the power system to improve the quality of the currents circulating toward the power system to be balanced. It is clear that these results are given with non linear loads with the same high level harmonics distortion in the three phases, but such constraints are so far from practical cases (where  $\sigma < 1$ ) this leads the shunt APF to be useless.

From Eq. 62 and 63 if  $k = 1$  (balanced load currents) then:

$$R_p = 1 \text{ and } R = R_0 = \sigma \tag{64}$$

Hence, the compensating power needed is:

$$S_{\text{comp}} = \sigma \cdot S_s \tag{65}$$

It is depending on the quality of the non linear load. This presents the curve where, the factor unbalance is equal to unit.

One phase unbalanced with non linear load

In this case  $\sigma > 0$  where:

$$k_a = k_b = k_c = k, \gamma_a = \gamma_b = \gamma_c = \gamma, \sigma > 0 \tag{66}$$

The resulting intermediary parameters are:

$$\Delta I = 6k^2, \Delta k = 3k^2, \gamma_1^+ = \gamma \text{ and } \Delta q = \sqrt{1 - \cos^2(\gamma)} \tag{67}$$

$$R_p = 1, R_0 = \sqrt{1 + \sigma^2 - \cos^2(\gamma)} \tag{68}$$

Finally the apparent power ratio is presented by:

$$R = \frac{R_0}{\cos(\gamma)} = \sqrt{\frac{1 + \sigma^2}{\cos^2(\gamma)}} - 1 \tag{69}$$

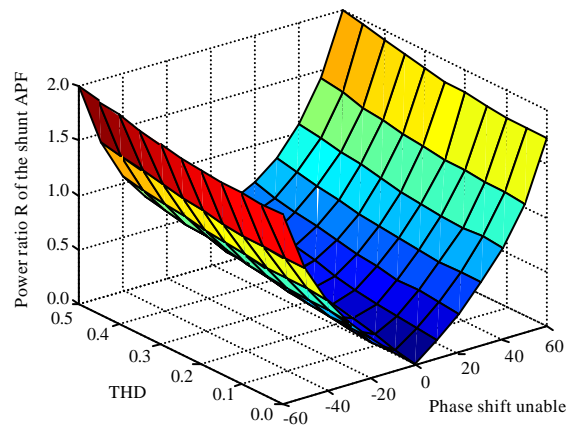


Fig. 13: Apparent power ratio R of the shunt APF

From Fig. 13, it is clear that for increased phase shift unbalance, the shunt APF suffers to compensate this kind of disturbance in the load currents. The phase shift gives the level of the reactive power absorbed by the load, where it is impossible to be greater than the active components, thus for practical cases it is around  $30^\circ$ . Figure 13 gives a clear idea about the apparent power ratio of the shunt APF for this kind of disturbances in the load side.

### CONCLUSION

This study deals with the evaluation of the shunt active power filter apparent power ratio for different current disturbances such as harmonics, reactive power and unbalances. The evaluation of this value contributes directly in the process design optimization for the determination of the used devices dimensions. The calculated apparent power defines the needed compensating apparent power subject to special loads or special consumers' needs, where the consumers' equipments are well known previously. The objective of this optimization calculation is to avoid over or/and under dimensions evaluations of the shunt active power filter. This is important for the manufacturers and users to minimize economically the burdens of production and the use of such equipments. On the other side, This study is based on new definition of the apparent power, which is proved to be the suitable amount to be considered in the process design, it shows the errors which were made for the evaluation of the apparent power to dimension these equipments using the classical definitions, where these definitions are correct only for sinusoidal balanced systems of voltages and currents. It is important to clarify that the evaluation of the apparent power of the shunt active power filter needed by the users is determined by the constraints of the loads to be fed and also by the constraints of the power system source. The approach given in this study can achieve this aim.

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