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A Hybrid Accelerated Random Search-Centrifugal Force Algorithm for Dynamic Finite Element Model Updating

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Abstract: An effective algorithm, which combined an adaptive centrifugal force algorithm with accelerated random search method, is proposed to update numerical models for dynamics problems. The objective is to minimize the difference between measured and simulated vibration data. The related problem is formulated as an optimization problem by considering multi-objective function defined in modal domain. A simulated beam structure is examined to test the applicability of the new approach for FE model updating applying multi-objective criteria. A real beam-like structure model is updated, making use of experimental modal data. The application of the centrifugal force algorithm enables us to obtain results well correlated with experiments in reduced time.

Key words: Centrifugal force, accelerated random search, model updating, finite element, multi-objective optimization

INTRODUCTION

The development of an accurate analytical model for a structural system is a fundamental requirement of engineering analysis. The modelling of dynamic problems often does not generate the dynamic parameters (frequencies, mode shapes, etc.) that adequately go with experimental results and, therefore, the updating of initial Finite element models by an iterative procedure becomes necessary to adjust them with measured results. This situation concerns also the dynamic models obtained by Finite Element Method - a standard tool of engineering analysis.

The FE model updating is becoming a standard methodology applied to enhance the correlation between finite element results and measured data. Several approaches have been proposed to tackle with this problem (Friswell and Mottershead, 1995). On the other hand, a large part of recent methods consider the Finite Element updating as an optimization process and use powerful new updating methods. For instance, Kwon and Lin (2005) applied the Taguchi method to model updating by optimizing a carefully defined objective function in presence of random errors in measured data and systematic errors in the analytical model. Jaishi and Ren (2007) presented a new updating procedure based on eigenfrequency residual and modal strain energy residual and used as two objective functions of the multiobjective optimization.

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In recent years, several novel computational intelligence approaches (Neural network and genetic algorithm) have proposed and applied for several problems (Kumarci *et al*., 2010; Hasangholipour and Khodayar, 2010). Other approaches have also been tried and applied to the practice of model updating. Neural networks (Atalla and Inman, 1998) can quickly achieve accurate updated model once they have been trained. However, updated results are dependent on the training cases used. Other procedures using stochastic search methods, including simulated annealing (Levin and Lieven, 1998) and genetic algorithms (Touat, 2008) have been applied for different model updating optimization problems. The stochastic methods are efficient in finding a global solution of difficult multimodal optimisation problems however they remain slow and time-consuming.

In an attempt to apply stochastic search methods in model updating, Touat *et al*. (2007) has developed a new updating procedure using a novel algorithm inspiring its theory from the functioning principle of a centrifugal device. The preliminary version of the centrifugal force algorithm CFA has been applied for mathematical hard optimization functions and has shown an acceptable aptitude to optimize these functions. For model updating, the method was efficient but slow and time-consuming.

However, the lack of hill-climbing ability, an inherent shortage of CFA, has not been tackled by Touat (2008). Recently, a new variant of stochastic search algorithm called the Accelerated Random Search (ARS) algorithm (Appel *et al*., 2004) has been developed to accelerate the stochastic search process for mathematical optimisation problems. The algorithm has shown a very good ability at hill climbing for optimum solutions and its convergence speed is very fast (Appel *et al*., 2004; Touat *et al*., 2007).

To improve the inherent shortage of CFA and to adopt the advantage of ARS for model updating, this study proposes a novel hybrid algorithm. This algorithm is based on a Centrifugal Force Algorithm (CFA) with novel parametering and accelerating optimization process by ARS algorithm. It is coded with MATLAB and using the performance of MATLAB sparse matrices algorithms.

THE PROPOSED ALGORITHM

The accelerated random search, the standard centrifugal force algorithm and the hybrid algorithm are presented. They can be applied to solve general unconstrained optimization problem. In this study, we considered just the hybrid algorithm and it can be easily adapted to model updating problems. The effectiveness of hybrid approach is tested on two standard functions, difficult to optimize. It will enable us to select the best parameters of the algorithm for model updating.

ARS Algorithm

Let us consider the general unconstrained optimization problem:

$$\begin{aligned}&\text{Find } x \in D\\&\text{Such that } f(x) \rightarrow \max\end{aligned} \tag{1}$$

where, x is the vector of n design variables x_i, f(x) is the objective function and D is the search space.

This problem can be solved using the Pure Random Search (PRS) algorithm (which is the Monte Carlo classical optimization technique) in the following way:

Generate a random vector $\{X_i\}_{i=1}^{n}$ with uniform distribution on D
Compute $M_n = \max\ \{f(X_i): i = 1, ..., n\}$

The convergence of this algorithm depends on several parameters (e.g., the type of problem to be optimized, its dimension, the size of the space search ...) but it is very slow in most cases of optimization.

The Accelerated Random Search (ARS) algorithm is a stochastic search methodology (Appel *et al*., 2004), presented to improve the convergence of the PRS algorithm and to accelerate the random search to obtain the global optimum in a reduced time.

In order to solve (1), Appel *et al*. (2004) assumed that D is the d-dimensional unit hyper-cube $[0,1]^d$. Moreover, the closed ball of radius r centered at x, $\{y \in D,\}: \|x-y\| \le r$, is denoted by B(x,r), where $\| . \|$ is the sup-norm in D. Let a contraction factor $c > 1$ and a precision threshold $\rho > 0$ be given. The ARS algorithm is formulated in the following way:

Step 0: Set n = 1 and $r_1 = 1$. Generate X_i from a uniform distribution on D
Step 1: Given $X_n \in D$ and $r_n \in (0,1]$, generate, Y_n from a uniform distribution on $B(X_n, r_n)$
Step 2: If $f(Y_n) > f(X_n)$, then let $X_{n+1} = Y_n$ and $r_{n+1} = 1$

Else if $f(Y_n) \le f(X_n)$, then let $X_{n+1} = Y_n$ and $r_{n+1} = r_n/c$
If $r_{n+1} < \rho$, then rn+1 = 1.
Increment n = n+1 and go to step 1

This version of the algorithm can be referred as finite descent ARS, $\{X_n\}$ as the sequence of record generators and the sequence $\{M_n = f(X_n)\}$ as the record sequence. The ARS algorithm may be readily applied to problems defined over a general metric space, for example, to combinatorial optimization problems and to optimization problems whose domains involve complicated constraints (e.g., the case of Finite Element model updating).

Centrifugal Force Algorithm (CFA)

The Centrifugal Force Algorithm (CFA) has been developed by Touat (2008). It can be used to find a global solution of difficult multimodal optimisation problems. In this section, we give the theoretical outset of the standard CFA algorithm.

The Centrifugal Force Algorithm (CFA) is a stochastic search algorithm derived from an analogy with the functioning principle of simple mechanical machine, presented in Fig. 1. The device is composed of an electrical motor, a rotational axis, a conical tube (with variable angle α) and a set of balls of different masses. After setting the angle α and putting a number of

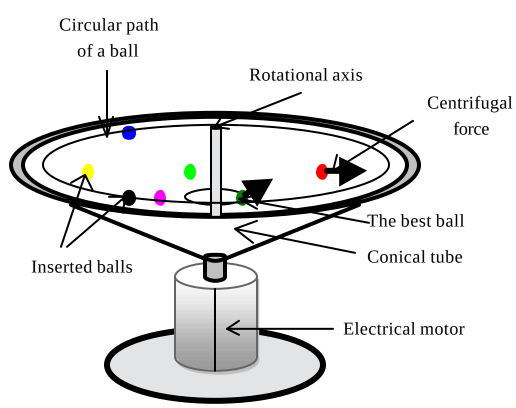

Fig. 1: Centrifugal force machine

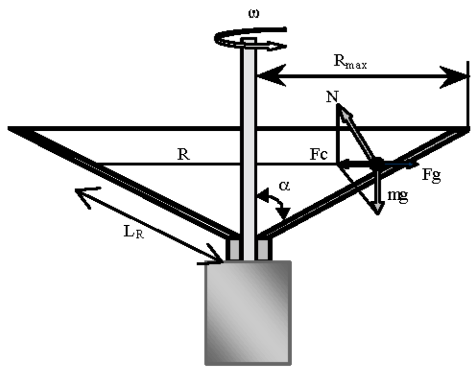

Fig. 2: Representation of forces applied on a ball, R_{max}: Maximum radius, ω: Rotational speed, N: Normal force, Fc: Centripetal force, Fg: Centrifugal force, mg: Weight of ball, α: Conical tube angle, R: Radius of curvature and L_R: Length

different balls into the conical tube, the motor is turned with the angular velocity ω. Under the effect of the angular velocity ω, a centrifugal force (F_g) of each ball is developed and a part of balls tend to go out of the tube. We are looking for the balls that remain in the tube, i.e., those of minimal value of the centrifugal force. This search process is repeated several times, for increasing values of angle α and velocity ω at each iteration. The balls remaining in the cone influence the choice of new balls inserted for further tests. While arriving at the maximal speed of the motor and the maximal angle, a set of balls remain in the tube. The ball corresponding to the minimal value of the centrifugal force is the solution of our search procedure.

The mechanical model of this search process involves a conical tube, characterized by the opening angle α, rotating linearly around the rotational axis with the angular velocity ω, with a set of balls of different masses m_i, put inside. The inward force necessary to maintain uniform circular motion is defined as the centripetal force. Assuming that there is no friction and the balls take the same speed ω, for a ball of mass m, the centripetal force can be expressed as the sum of the weight $m\vec{g}$ and the normal force $\vec{N}$ (Fig. 2):

$$\vec{F}_c = \vec{N} + m\vec{g} \tag{2}$$

According to Newton's third lows of motion, the ball exercises an equal and opposite reaction force called the centrifugal force ($\vec{F}_g$), equals to the product of mass and normal acceleration:

$$\left|\vec{F}_g\right| = ma_c = m\omega^2 R \tag{3}$$

where, R is the radius of trajectory.

From Eq. 2 and 3, the radius R can be expressed as:

$$R = g \big/ \left(\omega^2 * \tan(\alpha)\right) \tag{4}$$

Two parameters have a great effect on the radius of trajectory (Eq. 4): The rotational velocity ω and the angle of the conical tube α. The ball of minimal trajectory radius R_{min} is searched for varied values of parameters ω and α. This value corresponds to the minimal centrifugal force Fg_{min} that is influenced by the variation of ω and α. When the trajectory radius of a ball exceeds the maximal radius R_{max}, corresponding to the maximal centrifugal force Fg_{max}, the ball leaves the conical tube.

The application of the functioning principle of the machine to construct an optimization algorithm can be based on the following analogy. The ball masses play the role of optimization parameters, the centrifugal force becomes the objective function to minimize and the rotational speed and the angle of the conical tube become control parameters of the algorithm. The goal is to find the ball corresponding to the lowest centrifugal force (that corresponds to the lowest trajectory radius) after checking different cases of cone tube geometry and rotation speed.

The exploration of the optimization space is realized by changing the control parameters ω and α of the algorithm in an iterative manner and by examining the behavior of balls which masses are generated randomly for each configuration. One can see that the variation of ω affects the radius and the force in an inverse way (Eq. 3, 4) and the variation of α affects directly the radius and the force. Different modification strategies of ω and α can be applied. The exploitation of promising solutions will use random procedures. It takes place at the moment of selection of new propositions (balls) to be inserted into the cone. New balls are chosen in the vicinity of already found good solutions, depending on values of the centrifugal force of balls remaining in the tube after the previous iteration. A neighborhood function is used to generate in a random way the next state from the previous one (Levin and Lieven, 1998). Since random number functions are implied to create problem solutions, the proposed Centrifugal Force algorithm has stochastic character.

The CFA algorithm is given by the following steps:

Let ω_0, ω_{max}, α_0 and α_{max} the characteristics of the machine be given.

Step 0: Set α_0, n = 1 and m = 1;

- Calculate the extra parameter $\Delta\omega\alpha$
- Insert M balls into the conical tube
- Evaluate and classify the centrifugal force of each ball
- Choose the best N ones

Step 1: Vary α_m by $\Delta\alpha$ to $\Delta\omega\alpha+\alpha_n$
Insert a new M balls

Step 2: Evaluate and classify and choose the best N balls
Increment m = m +1 and go to step 1

Step 3: Vary ω_n by $\Delta\omega$ to ω_{max}
Increment n = n +1 and go to step 1

Select the best solution from remained balls.

Hybrid ARS-CF Algorithm

A novel hybrid algorithm that combines CFA with ARS is proposed in this section and called the centrifugal force accelerated random search algorithm (ARS-CF). This novel hybrid algorithm maintains the merit of CFA by using its optimization parametering as well as parameter variation process and merges the merit of ARS by exploiting its accelerated process and its hill-climbing ability to improve the searching capability for optimum solutions.

Several strategies for the standard CFA, concerning the variation of rotational speeds, the variation of conical tube angles and the manner of generating balls (choice of neighborhood and insertion functions), have been investigated by Touat (2008). These strategies are implemented in the pseudo-code of the hybrid algorithm proposed to solve the model updating problem:

The parameter ω characterizes the variation of the angle for each rotational speed. M is the number of balls inserted at each step, N is the number of selected solutions. The values of M and N, as well as the parameters α_0, α_{max}, $\Delta\alpha$, ω_0, ω_{max}, $\Delta\omega$ have to be chosen individually for each problem.

The value of ω_{max} and α_{max} depend both on the particular function to be optimized, the neighbourhood function used in the algorithm and the size of optimization problem. Generally, there is no intuitive way of deciding what values of ω_{max} and α_{max} (α_{max}<90°) should be applied. Therefore, the approximate initial values ω_{max} = 500 rad sec^{-1} and α_{max} = 80°, proposed by Touat (2008), can be used.

The accuracy of optimization solution depends also on $\Delta\omega$ and $\Delta\alpha$. Usually, smaller values of $\Delta\omega$ and $\Delta\alpha$ give more detailed sampling of the research space (and more chances to reach the global optimum) but at the expense of the number of iterations and the processing time. In an analogous way, the choice of ω_{max} and α_{max} has to be adapted for each optimization problem.

Using the accelerated process of the ARS, the masses of balls inserted into the conical tube are generated in the following way. The initial set of balls is created randomly within a given search interval (or proposed in the function of the problem). The balls for further search are generated randomly by using the neighborhood function applied to the best solution from the previous set. If a new proposition is better than a previous one (i.e., its objective function is smaller), then it is inserted in the cone and the same mass search interval will be applied at the next iteration. If a new proposition is worse, then the previous solution is inserted but the search interval will be diminished using a contraction factor at the next iteration (Appel *et al*., 2004). This part of algorithm is inserted in the standard CFA to obtain the new hybrid algorithm (Algorithm 1).

Algorithm 1: Pseudo-code of ARSCF algorithm

For a set of chosen parameters: α_0, α_{max}, $\Delta\alpha$, ω_0, ω_{max}, $\Delta\omega$,
and the additional step value $\Delta\omega\alpha$
$\Delta\omega\alpha(\alpha max-\alpha_0)/(\omega_{max}\omega_0/\Delta\omega)$
Set $\omega = \omega_0$
  $\alpha = \alpha_0$
  The search interval equal to 1
  A predefined threshold
  While $\omega<\omega_{max}$ then
  $\alpha_{max=}\alpha_{0+}\Delta\omega\alpha$
  While $\alpha<\alpha_{max}$ then
    Generate randomly M balls (neighborhood function) and insert them into the conical tube
    Evaluate and classify the centrifugal force of each ball,
    Keep the N best balls
    If there is convergence then
      Maintain the search interval
    else
      Reduce the search interval by a contraction factor
    End if
    If (search interval < predefined threshold)
      Reinitialize the search interval
      $\alpha_= \alpha + \Delta\alpha$
    End if
  End while
  $\alpha_{0=}\alpha_{max}$
  $\omega = \omega + \Delta\omega$
End while
Select the best solution from remained balls

Mathematical Validation of ARS-CF Algorithm

Two standard mathematical test functions have been performed to investigate the behaviour of the presented algorithm. The results of the two benchmark examples are summarized below to illustrate the efficiency and the quality of solution of standard CFA and the hybrid ARS-CF algorithm applied for different search space dimension and various number of variables.

Rosenbrock's Banana Function Test

$$f(x_1,x_2) = 100(x_1 - x_2)^2 + (1 - x_1)^2 \quad (5)$$

This standard test function was designed specifically to be hard to optimize. It has only one minimum x_{opt} = (1,1), $f(x_{opt})$= 0, located in a steep flat-bottomed parabolic valley. The generalization to n dimensions is given by:

$$f(x_1,x_2,...,x_n) = \sum_{i=2}^{n} f(x_{i-1},x_i) \quad (6)$$

The global minimum of the generalized function is at x_{opt} = (1, 1, ...,1) with $f(x_{opt})$= 0. In the test examples, a four-dimensional version is applied. The starting position used for all functions is given either by choosing the opening point (2, 2, 2, 2) or by introducing a point randomly. Each variable is bounded between -2 and 2. A result is said to be acceptable for this function if all optimization variables are within 0.1 of the global minimum and the value of the objective function is within 0.05 of the minimum. The two values 0.1 and 0.05 represent predefined convergence thresholds. They are fixed to calculate the number of iterations required for each algorithm to reach these two limits simultaneously and to test in this way the efficiency of each method. The same considerations are applied also to the following benchmark.

Hemmelblau's Function Test

$$f(x_1,x_2) = 100(x_1^{\,2} + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2 \quad (7)$$

The Hemmelblau's function takes its minimum value of 0 at four points: P_1(3.5844, -1.8481), P2(3,2), P_3(-2.8052,3.1313), P_4(-3.7793,-3.2832), given by the intersection of the two conic sections $x_2 = x_1^2 + 11$ and $x_1 = -x_2^2 + 7$. In this test, the variables are allowed to vary within the range x,y ∊ [-10, 10]. The result is said to be acceptable if all variables are within 0.001 of the global minimum and the values of the objective function are within 10^{-6} of the minimum.

Each of two investigated algorithms has been run independently several times. The CFA and the ARS-CF have applied the contraction factor c = $2^{1/2}$, the precision threshold $\rho = 10^{-4}$.

For the parameters of the centrifugal force device, we consider the following:

$\omega_0,\Delta\omega,\omega_{max}$, $\alpha_0,\Delta\alpha,\alpha_{max}$ are selected at the beginning of the optimization process. They have been set, respectively to 0, 10, 500 (rad/s), 15°, $\Delta\alpha = \Delta\omega\alpha/10$, 80° (Touat, 2008).

The first set of tests investigates the number of obtained optima. The threshold for each function is given directly in the table of the results. The starting point has been generated randomly for all algorithms.

Table 1: No. of successful results found in 50 independent runs

Function/interval	CFA	ARS-CF
Rosenbrock4/[-2,2]	26	50
Himmelblau2/[-10,10]	15	50

Table 2: Average number of iterations needed for satisfactory result (in 20 runs)

Function/interval/threshold	CFA	ARSCF
Rosenbrock4/[-2,2]/< 0.1	75000	20
Himmelblau2/[-10,10]/< 10^{-5}	5501	95

Table 1 gives the number of successful results found in 50 independent runs of the standard CFA and the hybrid ARSCE algorithm, it may be seen from this Table 1 that the ARS-CF outperforms the CFA algorithm. The ARS-CF algorithm combines the strength of the ARS algorithm and the force of parametering process given by CFA algorithm. The dimension of the search interval has a great effect on the convergence of the algorithms, but ARS-CF remains very robust and shrinks the space search very fast and ultimately sample only in neighbourhoods of the global optima. The time-consuming of the ARS-CF algorithm is very short compared to CFA algorithm.

In Table 2 we compare the average number of iterations needed to reach a threshold previously fixed for each function.

For the CFA algorithm, the number of iterations required to reach the threshold of each function is extremely high compared to the ARS-CF algorithm results. On the other hand, the ARS-CF algorithm gives good results, with a number of iterations very small. In conclusion, the ARS-CF algorithm outperforms the CFA approach and it is expected to work well also for model updating, where the parameters tend to be highly coupled.

UPDATING PROBLEM FORMULATION

FEM updating can be formulated as an optimisation problem and the minimization of differences between simulated and experimental data. Different optimization criteria, defined in modal or frequency domain and parameters can be considered. Good quality results can be obtained by taking into account simultaneously several cost functions and solving corresponding multi-objective problem.

Two different objective functions will be considered in the present study. The natural frequency criteron J_{ω}, is inspired from the study of Kwon and Lin (2005), by considering the second order formulation:

$$J_{\omega} = \sum_{k=1}^{n} \left(\frac{\omega_k^m - \omega_k^a}{\omega_k^m} \right)^2 \rightarrow \min \tag{8}$$

where, ω_k are natural frequencies, superscripts m and a represent measured and analytical data and n is the number of first modes used.

The Modal Assurance Criterion (MAC) is verified by the following expression given in (Kim and Park, 2004).

$$J_{MAC} = \sum_{k=1}^{n} \left| \Gamma(I) - \Gamma\left(MAC\left(\phi_k^m, \phi_k^a\right)\right) \right| \rightarrow \min \tag{9}$$

where, $\Gamma(I)$ is the trace of the identity matrix and, the MAC is defined as:

$$MAC = \frac{|(\phi^m)^T \phi^a|^2}{((\phi^m)^T \phi^m)((\phi^a)^T \phi^a)} \quad (10)$$

ϕ^m and ϕ^a are measured and analytical mode shape vectors, T stands for the transpose of a vector.

The weighting objectives method is applied to transform the problem into a scalar optimization formulation. The objective function Ψ to be minimized takes the form:

$$\Psi = W_\omega J_\omega + W_\varphi J_\varphi \quad (11)$$

where, W_ω and W_{MAC} are weighting coefficients, representing the relative importance of the criteria J_ω and J_{MAC}. The selection of the weighting factors is difficult since the relative importance among the objective terms is not obvious. An appropriate choice of these parameters can improve results significantly. Therefore, relative weights of natural frequencies and mode shapes should be chosen carefully (Kwon and Lin, 2005). In this study, the weighting coefficients have been selected after several tests and several runs of the updating process, starting with weighting proposition given in (Kwon and Lin, 2005) and adjusting it in order to obtain the best value of the objective function.

The optimization parameters of the updating problem are model correction factors (p-values), affecting the simulation results ω_k^a and ϕ_k^a.

NUMERICAL EXAMPLES

Simulated System of Beam Model

In order to judge the efficiency of employing the hybrid CFA-ARS algorithm for dynamic FE model updating, a simulated FE model is investigated. The model is an undamped ten element free-free beam of one meter length (Fig. 3); it is used to illustrate the performance of the CFA algorithm to update FE models. This model is characterized by 20 p-values to be updated because each of 10 elements has the stiffness and the mass to update. The beam has a square section of $4*10^{-4}$ m^2 and its material is characterized by the Young's modulus of $70*10^9$ N m^{-2} and the density of 2700 kg m^{-3}.

For this example, the simulated (target) experimental data consists of six natural frequencies and six mode shapes. The first six natural frequencies of the analytical (initial) and real (target) models and the diagonal values of the MAC are shown in Table 3.

The mass and stiffness matrices are used to determine the system eigendynamics. The updating centrifugal force algorithm tends to determine the values of the target system.

In this example, the mass (Eq. 3) corresponds directly to objective function Ψ evaluated for the set of updating parameters (p-values) that are grouped in the updating parameters vector of 20 elements. In order to create a neighborhood proposition of solution, the initial p-values are set to 1 and the p-values of the updated vector are multiplied by random

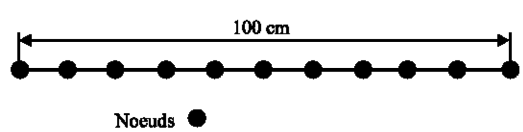

Fig. 3: Beam model for updating tests

Table 3: Modal properties of simulated beam model

Target frequencies (Hz)	Analytical frequencies (Hz)	Diagonal of MAC	Frequency error (%)
105.0	101.6	1.0722	3.2955
283.6	274.4	1.0537	3.2528
541.8	527.8	0.9828	2.5793
884.7	856.9	0.9687	3.1387
1281.8	1257.6	0.9149	1.8863
1710.1	1721.8	0.9177	0.6841

Table 4: Target p-values for simulated beam model

Stiffness	Mass
0.8	0.8
1.4	1.4
1.0	1.0
1.0	1.0
1.0	1.0
1.0	1.0
1.3	1.3
1.0	1.0
1.0	1.0
1.0	1.0

numbers generated between 0 and 2, where the value equals to 1 corresponds to no change of the parameter. New propositions of solution are evaluated using the objective function (Eq. 11). The target p-values (factors) to be obtained are given in Table 4. These factors are fixed at the beginning of the updating process. To validate the accuracy of obtained results, the updated p-values must match very well the target p-values.

The updating process starts by evaluating the objective function of initial FE model. The values of weighting factors W_ω and W_{MAC} have been selected and set after several runs to 50 and 1, respectively; depending on the effect of each part of the objective function on the convergence of updating process. The contraction factor has been set to $\sqrt{2}$ (Appel *et al*., 2004).

The values of CFA parameters α_0, α_{max}, $\Delta\alpha$, ω_0, ω_{max}, $\Delta\omega$ are selected at the beginning of the optimization process. They have been set respectively to 0, 10, 500 (rad sec^{-1}), 15°°, $\Delta\alpha = \Delta\omega\alpha/10$, 80°. The centrifugal force corresponds to the product of objective function (Eq. 11) and the control parameters ω and α (Eq. 3 and 4). The start value of the objective function Ψ is equal to 15.65. This value must be reduced to small value by applying the minimizing problem procedure.

The illustration of MAC matrix elements before updating is given on Fig. 4 and 5.

The MAC values and the relative error given in Table 3 and Fig. 5 show that the initial FE model needs updating and requires to correct its initial p-values to match the target ones. Therefore and by considering the stochastic process, the CFA algorithm has been run 20 times independently. The best results of model updating are presented in Table 5, Fig. 5 and 6.

The illustration of MAC matrix elements after updating is given on Fig. 6.

It can be seen from Table 5 that the CFA algorithm has improved significantly the start analytical model and the initial errors of eigenvalues and eigenvectors are reduced to very acceptable levels. As seen in Table 4 and Fig. 5, the diagonal values of updated MACs are more close to 1 than those of initial MACs. The MAC value close to 1 means eigenvector of an analytical model is close to that of reference (experimental) model at a specific mode.

For example, 4th and 6th updated MACs, which represent correlation between updated and reference eigenvectors, become 0.9989 and 1.000, respectively as seen in Table 5. Also,

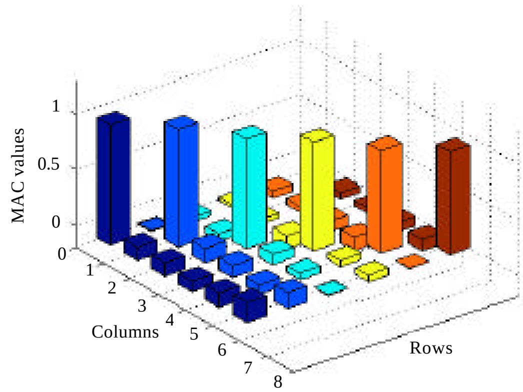

Fig. 4: MAC values before updating for simulated example

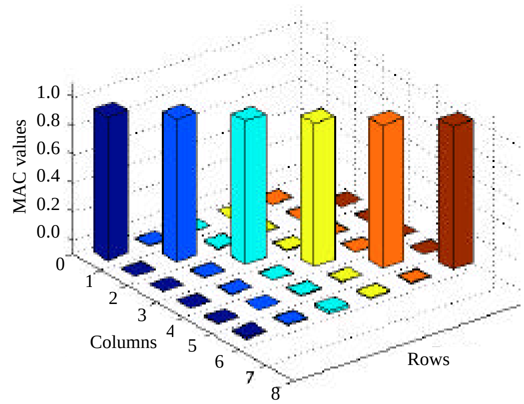

Fig. 5: MAC values after updating for simulated example

Table 5: Results of CFA updating for simulated beam structure

	Initial modal				ARSCF		
Mode	Target frequencies (Hz)	Analytical frequencies (Hz)	Diagonal of MAC	Frequency error (%)	Frequencies (Hz)	Error (%)	MAC
1	105.0	101.6	1.0722	3.2955	0.1051	0.1098	0.9983
2	283.6	274.4	1.0537	3.2528	0.2828	0.2698	0.9995
3	541.8	527.8	0.9828	2.5793	0.5421	0.0713	0.9999
4	884.7	856.9	0.9687	3.1387	0.8817	0.3399	0.9989
5	1281.8	1257.6	0.9149	1.8863	1.2817	0.0067	1.0000
6	1710.1	1721.8	0.9177	0.6841	1.7096	0.0292	0.9984

the maximal value of relative error has been reduced from 3.2955 before updating to 0.3399 after updating.

In the other hand, the target p-values are used as mathematical constraint to test the degree of correlation between the analytical model and experiment. Figure 6 shows that the p-values of updating model matches very well the target p-values reducing thus the objective function from 15.65 (initial model) to 0.33 (updating model) for an elapsed time of 54.7650 second. The updating results confirm that the ARSCF procedure is efficient and faster than the original CFA and other stochastic algorithms (Levin and Lieven, 1998) and can be a useful tool for dynamic FE model updating and can be applied for updating real structure with experimental references.

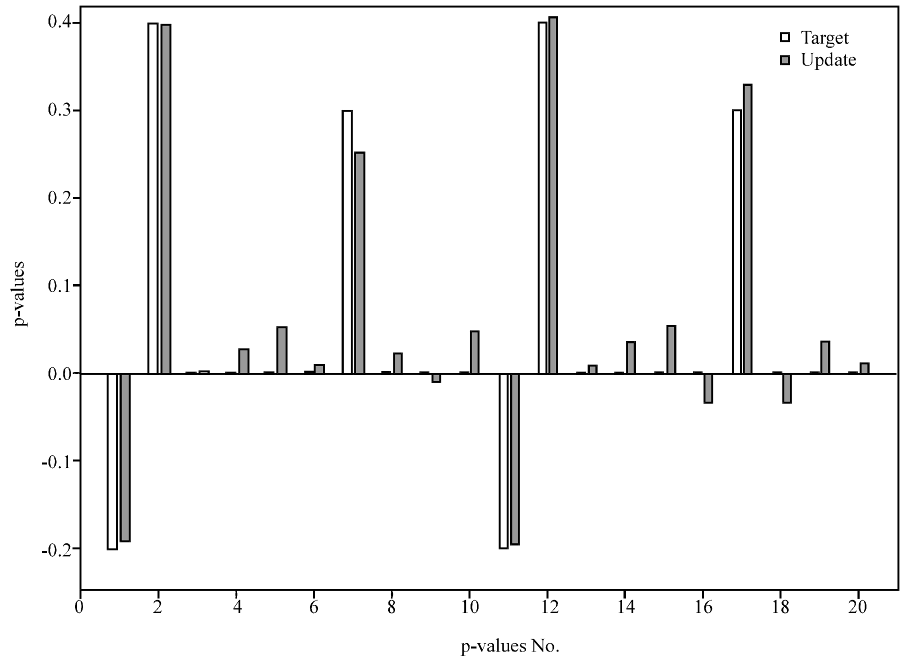

Fig. 6: Updated and target p-values using the ARSCF

Fig. 7: Experimental setup

Real Overlaid Beam Structure

In this part, a real physical structure composed of two interconnected beams, is updated using experimental data. Figure 7 represents the experimental setup to obtain experimental results; the modal test was performed on two overlaid beams of 26.5 mm width and 8 mm thickness, connected with bolts. The total structural length is 471 mm and the material is characterized by the Young's modulus of $71*10^9$ N m^{-2} and the density of 2660 kg m^{-3}. The modeled structure is divided into 23 equidistant elements of 20 mm length and the 24th element of 11 mm (i.e., 25 nodes). The beam assembly is complex enough to provide a practical test for the updating algorithm. Figure 8 represents the real bolted beam structure and FE meshing.

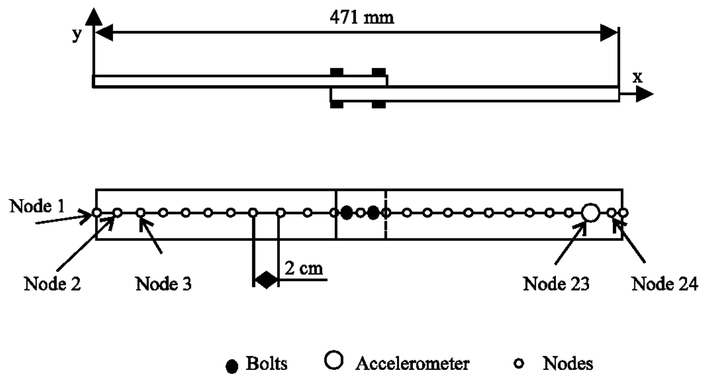

Fig. 8: Real bolted beam structure and FE meshing

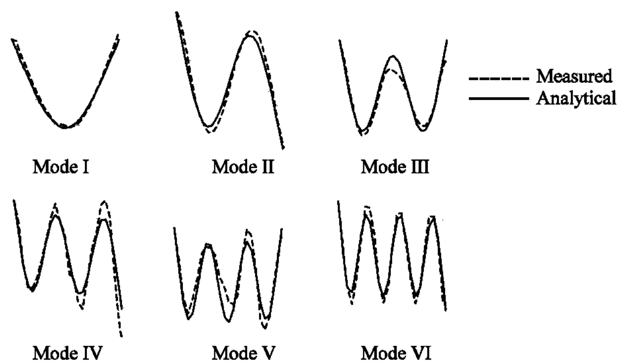

Fig. 9: Experimental and analytical mode shapes

The roving hammer test was performed. In the experiment, the accelerometer was fixed at the 23 st point in the x-direction (Fig. 8) and the beam was impacted at 25 points to define the mode shapes of the structure. Using a 4-channel FFT analyzer, 25 Frequency Response Functions (FRFs) were computed one at a time, corresponding to each impact Degree of Freedom (DOF). The structural response was measured within the frequency range of 0 to 6 kHz. The first six modal shapes and the first seven frequencies were extracted. Each data set consisted of 25 FRFs. Post-processing modal software enabled us to extract mode shapes and natural frequencies (Fig. 9, 10).

Figure 9 gives a comparison between experimental and analytical mode shapes and Fig. 10 represents the experimental Frequency Response Function and phase.

The FE model of the real structure was created by taking 24 two-dimensional beam elements with an inter-node of 20 mm. The model will is characterized by 48 updating parameters: two p-values for each element, corresponding to its mass and stiffness matrix. Similarly to the previous example, the ball mass corresponds the objective function Ψ, p-values are allowed to vary between 0 and 2, with 1 indicating no change of the parameter.

Table 6 gives the experimental natural frequencies and those calculated using the FE model after updating. The illustration of MAC matrix elements before updating is given on Fig. 11 and the mode shapes of the FE analysis and experimental results are compared using the MAC values. The relative error between experimental and analytical frequencies and the diagonal MAC values are given as well.

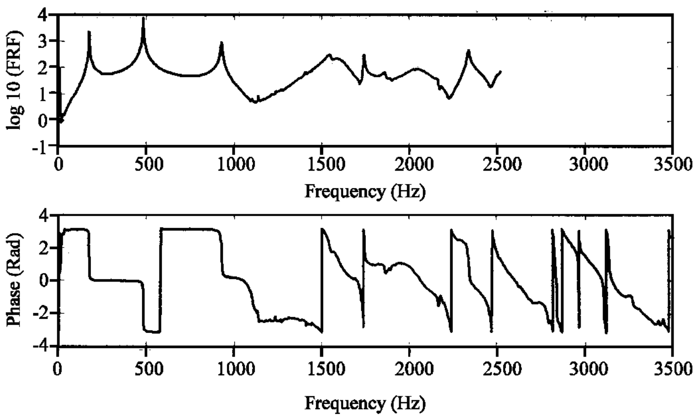

Fig. 10: Experimental frequency response function and phase

Table 6: Results of ARSCF updating for overlaid beam structure

	Initial modal				ARSCF		
Mode	Experimental frequencies (Hz)	Analytical frequencies (Hz)	Relative error (%)	MAC	Frequencies (Hz)	Error (%)	MAC
1	174.6	173.65	0.5457	0.9976	174.6	0.0221	0.9947
2	480.7	477.03	0.7640	1.0494	480.3	0.0881	0.9987
3	927.8	932.24	0.4789	0.9239	927.9	0.0116	0.9997
4	1564.3	1536.55	1.7740	0.8689	1564.3	0.0004	0.9365
5	2344.5	2289.12	2.3622	0.8214	2346.8	0.0966	0.9547
6	3110.5	3189.13	2.5279	0.9511	3105.2	0.1706	0.9816
7	4280.0	4235.81	1.0325	--	4294.5	0.3399	--

The CFA updating procedure has been carried out for the same set of parameters α_0, α_{max}, $\Delta\alpha$, ω_0, ω_{max}, $\Delta\omega$, as in the previous numerical example. The values of weighting factors W_ω and W_{MAC} have been set to 50 and 1, respectively and have been selected after several test runs of the CFA procedure.

Table 6 gives the diagonal of MAC matrix and frequency errors for each mode applying the ARSCF updating for the overlaid beam structure. The illustration of updating MAC matrix elements is given on Fig. 12.

We can see clearly that the CFA algorithm has enhanced significantly the modal properties of the initial model. The diagonal terms of the MAC are greater than 0.93 (0.82 before) and the relative error has been reduced considerably (the maximal values has been decreased from 2.52 to 0.297). Satisfactory results have been obtained for reduced processing time compared to the original CFA and other stochastic algorithms (Levin and Lieven, 1998). The CFA algorithm has successfully updated 48 parameters of the initial model and can be an efficient and a useful tool for model updating.

The extraction of modal parameters from measured FRFs can cause possible errors that affect measured mode shapes first and then the MAC part of the objective function. To improve the accuracy of the procedure (MAC values), one can consider an objective function based on frequency data.

The evaluation of the modal objective function for each iteration involves the determination of eigendynamic solutions (corresponding to the variation of α and ω). Inevitably, in the case of large systems, the processing time increases considerably. The application of sparse matrice algorithms (MathWorks Inc., 2007) may be used to reduce the

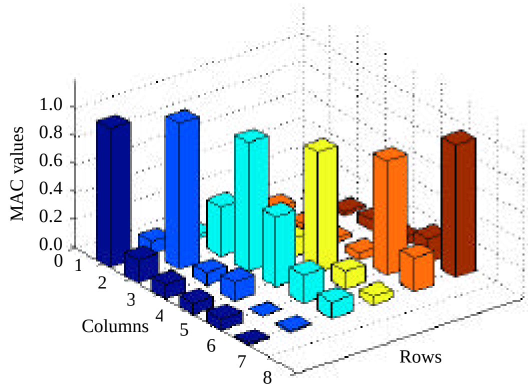

Fig. 11: MAC values before updating for real example

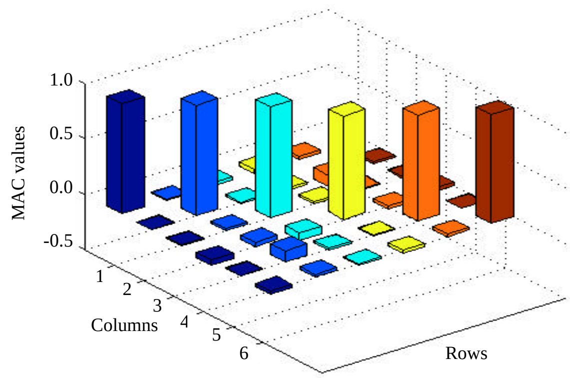

Fig. 12: MAC values after updating for real example

CPU time needed by model updating methodology. They have been applied in the last example and the ARSCF algorithm required less than 3 min processing on a PC computer (for the proposed set of parameters).

CONCLUSION

The dynamic FE model updating formulated as optimization problem has been solved by the random search methodology. The hybrid accelerated random search centrifugal force algorithm has been applied. The numerically simulated test example has confirmed a good capacity of the ARSCF algorithm to deal with this kind of problems. The ARSCF algorithm has successfully updated the beam structure model (characterized by 20 p-values) and significantly improved the correlation between the analytical parameters and the simulated experimental data.

The ARSCF algorithm is not very sensitive to the initial model parameters and in addition to that, no modal sensitivities have to be calculated. On the other hand, the use of the accelerating technique and the generation of steps decreasing the search interval at each iteration permit to reduce considerably the number of iterations needed to reach acceptable results. Also, the algorithm has improved the inherent shortage of CFA and has shown a very good ability at hill climbing for optimum solutions and its convergence speed is very fast.

The results of the ARSCF algorithm have shown its high aptitude to update large models in reduced execution time and to converge to satisfactory results. It enables us to control the number of iterations and can be certainly a useful tool for more complex engineering applications of a moderate size. Further studies, devoted to the application of this procedure for the dynamics model updating of more complex 2D and 3D structures and the consideration of objective function based on frequency data, are intended in the continuation of this study. The proposed algorithm is very simple and can be easily adapted to solve other mathematical and physical optimization problems.

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