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An Algorithm to Analyze of Two-dimensional Function by using Wavelet Coefficients and Relationship between Coefficients

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ABSTRACT

Wavelets are regarded by many as primarily a new subject in pure and applied mathematics. Perhaps one of the most common applications of wavelet is in signal processing. In this study we obtain an algorithm to analyze and synthesize a signal or two-dimensional function s = f(t) by using two-dimensional wavelet method. We consider a sample point $(t_{i,j},s_{i,j})$ includes a value $s_{i,j} = f(t_{i,j})$ at height $s_{i,j}$ and abscissa (time or location) $t_{i,j}$. We propose, obtaining an algorithm of two-dimensional wavelet decomposition to estimate a function by using MATLAB software for computing wavelet coefficients. Some relationships between wavelet coefficients are investigated.

Key words: Haar wavelets, two-dimensional, signal, fast wavelets, estimation function, discreet wavelet, multiresolution analysis

INTRODUCTION

Wavelets are regarded by many as primarily a new subject in pure and applied math-ematics. Indeed, many papers published on wavelets contain esoteric-looking theorems with complicated proofs. Wavelet analysis was led by Daubechies (1988). Many colleagues contributed in different ways: Meyer (1990), Walter (1993), Vidakovic (1999), Cohen *et al.* (1993), Doosti *et al.* 2008, Afshari (2008), Antoniadis *et al.* (1994) and Clyed *et al.* (1998). Perhaps one of the most common applications of wavelets is in signal processing. A signal is a sequence of numerical measurements, typically obtained electronically.

To analyze and synthesize a signal- which can be any array of data- in terms of simple wavelets, we employ shifts and dilation of mathematical function, but do not involve either calculus or linear algebra.

The first step in applying wavelets to any signal consists in representing the signal under consideration by a mathematical function f. For example, a sound, the values s = f(t) measure the sound at each time t at a fixed location.

The first step in the analysis of a one-dimensional signal with wavelets consists in approximating its function by means of sample alone. One of the simplest methods of approximation uses a horizontal stair step extended through each sample point. The resulting steps form a new function denotes by $\tilde{f}$ and called a step function, which approximates the sampled function s = f(t). The analysis of the approximating function f in terms of wavelets requires a precise labeling of each step. By means of shifts and dilations of the basic unit step function, denoted by $\varphi(0.1)$.

If a sample point (t_i,s_j) includes a value $s_j = f(t_j)$ at height s_j and abscissa (time or location) t_j, then the sample point corresponds to the step function-$s_j\varphi_{[t_j,t_{(j+1)})}$ which approximates f at height s_j on the interval $[t_j,t_{(j+1)})$, where $s_j\varphi_{[t_j,t_{(j+1)})}$ denotes the indicator function of set $[t_j,t_{(j+1)})$.

Adding all step functions approximating corresponding to all the points in the sample yields the simple step function below:

$$\tilde{f} = \sum_{j=0}^{n-1} s_j \varphi_{[t_j, t_{(j+1)})}(t) \tag{1}$$

To analyze a signal or function in term of wavelets, the fast Haar wavelet transform begins with initialization of an array with 2^n entries, and then proceeds with n iterations of the basic transform explained in Eq. 1.

For each index j∈{1,2,....,n}, before iteration number j, the array will consist of $2^{n-(j-1)}$ coefficients of $2^{n-(j-1)}$ step function $\varphi_{k,n-(j-1)}$ defined below. After iteration number j, the array will consist of half as many 2^{n-j} coefficient of 2^{n-j} step function $\varphi_{k,n-j}$ and 2^{n-j} coefficient $\psi_{k,n-j}$, such as:

$$\varphi_{k,n-j}(t) = \varphi_{[0,1)}(2^{n-j}[t - k2^{j-n}]) \tag{2}$$

$$\psi_{k,n-j}(t) = \psi_{[0,1)}(2^{n-j}[t - k2^{j-n}]) \tag{3}$$

MAIN RESULTS

Two dimensional wavelets algorithm: For any function $f \in L^2(R)$ we can write a formal expansion (Daubechies, 1992):

$$f = \sum \alpha_{m,k} \varphi_{m,k} + \sum_{j=m}^{\infty} \sum_{k \in z} c_{j,k} \psi_{j,k} \tag{4}$$

Here φ(x) and ψ(x) are the scale function and the orthogonal wavelet, respectively as the following:

$$\varphi_{m,k}(x) = 2^{\frac{m}{2}} \varphi(2^m x - k), \qquad \psi_{m,k}(x) = 2^{\frac{j}{2}} \psi(2^j x - k) \tag{5}$$

Constitute an (inhomogeneous) orthonormal basis of $L^2(R)$. It is clear that for Haar wavelet:

$$\varphi(x) = \frac{1}{\sqrt{2}} \varphi_{1,0}(x) + \frac{1}{\sqrt{2}} \varphi_{1,1}(x), \qquad \psi(x) = \frac{1}{\sqrt{2}} \varphi_{1,0}(x) - \frac{1}{\sqrt{2}} \varphi_{1,1}(x) \tag{6}$$

So, we can write:

$$\varphi_{m,k}(x) = \frac{\varphi_{j+1,2k}(x) + \varphi_{j+1,2k+1}(x)}{\sqrt{2}}, \; \psi_{j,k}(x) = \frac{\varphi_{j+1,2k}(x) - \varphi_{j+1,2k+1}(x)}{\sqrt{2}} \tag{7}$$

Not that $\varphi \in V_0$ therefore $\varphi \in V_1$, because $V_0 \subset V_1$.

Since {$\varphi_{1,k}$,(x), k∈Z is an orthonorml basis for V_1, there exists a sequence b_k such that:

$$\varphi(x)=\sum_{k\in Z} b_k \varphi_{1,k}(x) \tag{8}$$

Know we consider a two-dimensional function $f(x,y)\epsilon L^2(R^2)$. For approximating this function we using two dimensional of wavelets and obtaining an algorithm for estimation of function f with computation of wavelet coefficients. For two dimensional wavelets, encodings can consist of matrices, indexed by rows and columns. To expand this argument, we need some definition as the following:

- **Definition 1:** Let $f(x,y)\epsilon L^2(R^2)$ denote the two dimensional signal. We said $\psi_{j1,j2,k1,k2} = \psi_{j1,k1}(x).\psi_{j2,k2}(x)$ is orthogonal wavelet basis
- **Definition 2:** For each pair functions f and g, the tensor product of f and g is the function denoted by $f\otimes g$ as follows:

$$(f\otimes g)(x,y) = f(x).g(x) \tag{9}$$

- **Definition 3:** Suppose that function space, $V_j,\epsilon z$ to be $\{V_j = f,g\epsilon L^2(R^2)$:f,g are piecewise constant on $[k2^{-j},(k+1)2^{-j}],k\epsilon z\} = V_j\otimes V_j$
  If this sequence of subspaces possesses has the following properties:

- $-...\subset V_{-2}\subset\subset V_{-1}\subset V_{-0}\subset V_{-1}\subset V...$
- $-\bigcap_{j\epsilon z} V_j=(0,1),\overline{\bigcup_{j\epsilon z} V_j}=L^2(R^2)$
- $-f(x,y)\epsilon V_j \Leftrightarrow f(2x,2y)\epsilon V_{j+1}$
- $f(x,y)\epsilon V_0 \Rightarrow f(x-k_1,y-k_2)\epsilon\ V_0 \forall k_1,k_2\epsilon Z$
- There exists a function $\varphi(x,y)=\varphi(x).\varphi(y)\epsilon V_0$ such that the set $\{\varphi_{j,k1,k2}(x,y)=2^j\varphi(2^jx-k_1,2^jy-k_2),j,k_1,k_2\epsilon z\}$ constitutes an orthonormal basis for V_0, then we said $(V_j)_{j\epsilon Z}$ to be form a multiresolution analysis (MRA) of $L^2(R^2)$.

Remark 1: Suppose that function space, $w_j,j\epsilon z$ to be
$W_j=\{f,g\epsilon V_{j+1}|f,g\perp V_j\} = V_{j+1}\bigcap_j^T$, then we can write as the following:

- $V_j=\text{span}\{\phi_{j,k_1,k_2}(x,y)\}, \qquad W_j=\text{span}\{\psi_{j,k_1,k_2}(x,y)\}$
- $V_{j+1} = V_{j+1}\otimes V_{j+1} = (V_j\oplus W_j)\otimes(V_j\oplus W_j)$
  $=(V_j\otimes V_j)\oplus(V_j\otimes W_j)\oplus(W_j\otimes V_j)\oplus(W_j\otimes W_j) = (V_j\oplus W_j)$

Theorem 1: In Definition 1, The wavelet space $\{W_j,\ j\epsilon Z\}$ and scale space $\{(V_j,\ j\epsilon Z\}$ are mutually orthogonal.

Proof: First we prove that the scaling function and mother wavelet are orthogonal.

$$<\psi,\varphi>=\int\psi(x)\varphi(x)dx=\int(\sum_k(-1)^k b_{-k+1}\varphi_{1,k}(x)\varphi(x)dx$$

$$= \sum_k (-1)^k b_{-k+1} \int \varphi_{1,k}(x)\varphi(x)dx = \sum_k (-1)^k b_{-k+1} b_k = 0$$

The last step follows since the summand for k is the opposite of the summand for k-1-2*l*, so each term is negated.

It can be seen similarly that each integer translation of the mother wavelet ψ is also orthogonal to φ as the following:

$$< \psi_{0,1}, \varphi > = \int \psi(x-1)\varphi(x)dx = \int (\sum_k (-1)^k b_{-k+1} \varphi_{1,k}(x-1)\varphi(x)dx$$

$$= \sum_k (-1)^k b_{-k+1} \int \varphi_{1,2l+k}(x)\varphi(x)dx = \sum_k (-1)^k b_{-k+1} b_{2l} + k = 0$$

The last step follows, since the summand for k is the opposite of the summand k-1-2*l*, so each term is negated, because of the square sumability of the sequence b_k.

Straightforward extension of this argument will show that $\psi_{j,k} \perp \varphi_{j,k}$ for all k,*l*∈Z and complete the proof.

Remark 2: Each basic square-step function has value 1 in a selected square and 0 everywhere as the following:

$$\varphi^{(0)}_{0,0} = \varphi_{[0,1)} \otimes \varphi_{[0,1)}(x,y) = \varphi_{[0,1)}(x).\varphi_{[0,1)}(y) = \begin{cases} 1, & 0 \le x < 1, 0 \le y < 1 \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Similarly we can write:

$$\varphi^{(1)}_{0,0}(x,y) = \varphi^{(0)}_{0,0}(2x,2y), \quad \varphi^{(1)}_{0,1}(x,y) = \varphi^{(1)}_{0,0}(2x,2y-1) \quad (11)$$

$$\varphi^{(1)}_{1,1}(x,y) = \varphi^{(0)}_{0,0}(2x-1,2y-1), \quad \varphi^{(1)}_{1,0}(x,y) = \varphi^{(0)}_{0,0}(2x-1,2y) \quad (12)$$

Lemmal:

$$\varphi^{d,0}_{0,0}(x,y) = \varphi^{(1)}_{0,0} - \varphi^{(1)}_{0,1} - \varphi^{(1)}_{1,0} + \varphi^{(1)}_{1,1}$$

Proof: It is easy to see that ,

$$\varphi^{(0)}_{0,0} = \varphi^{(1)}_{0,0} + \varphi^{(1)}_{0,1} + \varphi^{(1)}_{1,0} + \varphi^{(1)}_{1,1} \quad (13)$$

By substituting $\varphi_{(0,1)}$ and $\psi_{(0,1)}$ instead of f and g in Eq. 9 and changing the place of them, we can write the tensor product of $\psi^{h,0}_{0,0}(x,y)$, $\psi^{v,0}_{0,0}(x,y)$, $\psi^{d,0}_{0,0}(x,y)$ as follows:

$$\psi^{h,0}_{0,0}(x,y)=\varphi_{[0,1)}\otimes\psi_{[0,1)}(x,y)=\varphi_{[0,1)}(x).\psi_{[0,1)}(y)=\begin{cases}1 & 0\le x<1, 0\le y<1\\ -1 & 0\le x<1, \frac{1}{2}\le y<1\\ 0 & \text{otherwise}\end{cases}$$

$$=\varphi^{(1)}_{0,0}+\varphi^{(1)}_{0,1}-\varphi^{(1)}_{0,0}-\varphi^{(1)}_{0,0} \tag{14}$$

$$\psi^{v,0}_{0,0}(x,y)=\psi_{[0,1)}\otimes\varphi_{[0,1)}(x,y)=\psi_{[0,1)}(x).\varphi_{[0,1)}(y)=\begin{cases}1 & 0\le x<\frac{1}{2}, 0\le y<\frac{1}{2}\\ -1 & \frac{1}{2}\le x<1, 0\le y<1\\ 0 & \text{otherwise}\end{cases}$$

$$=\varphi^{(1)}_{0,0}-\varphi^{(1)}_{0,0}+\varphi^{(1)}_{0,0}-\varphi^{(1)}_{0,0} \tag{15}$$

$$\psi^{d,0}_{0,0}(x,y)=\psi_{[0,1)}\otimes\psi_{[0,1)}(x,y)=\psi_{[0,1)}(x).\psi_{[0,1)}(y)=\begin{cases}1 & 0\le x<\frac{1}{2}, 0\le y<\frac{1}{2}\\ 1 & \frac{1}{2}\le x<1, \frac{1}{2}\le y<\\ -1 & \frac{1}{2}\le x<1, 0\le y<\frac{1}{2}\\ -1 & 0\le x<\frac{1}{2}, 0\le y<\frac{1}{2}\\ 0, & \text{otherwise}\end{cases}$$

$$=\varphi^{(1)}_{0,0}-\varphi^{(1)}_{0,0}-\varphi^{(1)}_{0,0}+\varphi^{(1)}_{0,0} \tag{16}$$

Remark 3: The subscript h and v and d indicates the correspondence of such wavelets with horizontal and vertical, and diagonal changes in the data, since the detail spaces $W^{(h)}_j, W^{(v)}_j, W^{(d)}_j$ tend to emphasize coefficient-cliques describing horizontal and vertical and diagonal futures the image. The spaces $W^{(h)}_j, W^{(v)}_j, W^{(d)}_j$ are spanned by translation of:

$$\varphi^{(j)}_{k,l}=\varphi^{(j)}_k\otimes\varphi^{(j)}_l, \psi^{h,(j)}_{k,l}=\varphi^{(j)}_k\otimes\psi^{(j)}_l \tag{17}$$

$$\psi^{v,(j)}_{k,l}=\psi^{(j)}_k\otimes\phi^{(j)}_l, \psi^{d,(j)}_{k,l}=\psi^{(j)}_k\otimes\psi^{(j)}_l \tag{18}$$

For nonnegative integers j which is denoting the frequency and k,l which are denoting the location.

Algorithm: We consider a function f with sample values $f(0,0), f(0,\frac{1}{2}), f(\frac{1}{2},0), f(\frac{1}{2}0,\frac{1}{2})$ approximated by a square-function $\tilde{f}$ and denote by a matrix:

$$\tilde{f}=\begin{pmatrix} f(0,0) & f\left(0,\frac{1}{2}\right) \\ f\left(\frac{1}{2},0\right) & f\left(\frac{1}{2},\frac{1}{2}\right) \end{pmatrix}=\left[\frac{s_{0,0}}{s_{1,0}}\;\frac{s_{0,0}}{s_{1,1}}\right]$$

We can write as follows:

$$\tilde{f}=s_{0,0}\phi_{0,0}^{(1)}+s_{0,1}\phi_{0,1}^{(1)}+s_{1,0}\phi_{1,0}^{(1)}+s_{1,1}\phi_{1,1}^{(1)} \tag{19}$$

Algorithm begins with one dimensional wavelet transform as described in algorithm, for each row as follows:

$$\left[\frac{s_{0,0}}{s_{1,0}}\;\frac{s_{0,0}}{s_{1,1}}\right]\Rightarrow\begin{bmatrix} \frac{s_{0,0}+s_{0,1}}{2} & \frac{s_{0,0}-s_{0,1}}{2} \\ \frac{s_{1,0}+s_{1,1}}{2} & \frac{s_{1,0}-s_{1,1}}{2} \end{bmatrix}$$

$$\Rightarrow\begin{bmatrix} \frac{(s_{0,0}+s_{0,1})+(s_{1,0}+s_{1,1})}{4} & \frac{(s_{0,0}-s_{0,1})+(s_{1,0}-s_{1,1})}{4} \\ \frac{(s_{0,0}+s_{0,1})-(s_{1,0}+s_{1,1})}{4} & \frac{(s_{0,0}+s_{0,1})-(s_{1,0}+s_{1,1})}{4} \end{bmatrix}\Rightarrow.....$$

We can repeat this algorithm and estimate function f.

EXAMPLE

Example 1: let for approximating f we chose sample values:

$$f(0,0)=9,\ f\left(0,\frac{1}{2}\right)=7,\ f\left(\frac{1}{2},0\right)=5,\ f\left(\frac{1}{2},\frac{1}{2}\right)=3$$

The square-step approximation $\tilde{f}$ is:

$$\tilde{f}=s_{0,0}\phi_{0,0}^{(1)}+s_{0,1}\phi_{0,1}^{(1)}+s_{1,0}\phi_{1,0}^{(1)}+s_{1,1}\phi_{1,1}^{(1)}=9\phi_{0,0}^{(1)}+7\phi_{0,1}^{(1)}+5\phi_{1,0}^{(1)}+3\phi_{1,1}^{(1)} \tag{20}$$

Consider again the above data. By the algorithm, we can write:

$$\begin{bmatrix} 9 & 7 \\ 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{9+7}{2} & \frac{9-7}{2} \\ \frac{5+3}{2} & \frac{5-3}{2} \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 4 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{8+4}{2} & \frac{1+1}{2} \\ \frac{8-4}{2} & \frac{1-1}{2} \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 2 & 0 \end{bmatrix}$$

The one-dimensional Fast wavelet transform extends to two- dimensional fast wavelet transform with tensor products, through alternating applications of the one-dimensional transform to each row and then to each new column.

So by the tensor product wavelet,

$$\tilde{f} = 6\varphi_{(0,1)} \otimes \varphi_{(0,1)} + 1\varphi_{(0,1)} \otimes \psi_{(0,1)} + 2\psi_{(0,1)} \otimes \varphi_{(0,1)} + 0\psi_{(0,1)} \otimes \psi_{(0,1)} \tag{21}$$

Example 2: supposed that a function f sampled at 4×4 matrix values

$$\tilde{f} = \begin{bmatrix} 3 & 5 & 4 & 8 \\ 1 & 3 & 4 & 4 \\ 2 & 6 & 4 & 2 \\ 6 & 2 & 6 & 0 \end{bmatrix}$$

On a square grid, and we consider approximating f, by square-step function as following:

$$\begin{bmatrix} 3 & 5 & 4 & 8 \\ 1 & 3 & 4 & 4 \\ 2 & 6 & 4 & 2 \\ 6 & 2 & 6 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{3+5}{2} & \frac{3-5}{2} & \frac{4+8}{2} & \frac{4-8}{2} \\ \frac{1+3}{2} & \frac{1-3}{2} & \frac{4+4}{2} & \frac{4-4}{2} \\ \frac{6+6}{2} & \frac{6-6}{2} & \frac{4+2}{2} & \frac{4-2}{2} \\ \frac{6+2}{2} & \frac{6-2}{2} & \frac{6+0}{2} & \frac{6-0}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & -1 & 6 & -2 \\ 2 & -1 & 4 & 0 \\ 6 & 0 & 3 & 1 \\ 4 & 2 & 3 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{4+2}{2} & \frac{-1+(-1)}{2} & \frac{6+4}{2} & \frac{-2+0}{2} \\ \frac{4-2}{2} & \frac{-1-(-1)}{2} & \frac{6-4}{2} & \frac{-2-0}{2} \\ \frac{6+2}{2} & \frac{0+2}{2} & \frac{3+3}{2} & \frac{1+3}{2} \\ \frac{6-2}{2} & \frac{0-2}{2} & \frac{3-3}{2} & \frac{1-3}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -1 & 5 & -1 \\ 1 & 0 & 1 & -1 \\ 5 & 1 & 3 & 2 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$

The intermediate result just obtained corresponds to one basic two dimensional wavelet transform on each of the four adjacent 2×2 square matrixes.

RESULTS

- All of the sums from the upper left hand corners of the four 2×2 blocks go into the upper left hand corner of the larger grade, corresponding to wavelets of the form $\varphi_{m,n} \otimes \varphi_{k,l}$.
- All of the difference from the upper right-hand corners of the four 2×2 blocks go into the upper right-hand corner of the larger grade, corresponding to wavelets of the form $\varphi_{m,n} \otimes \psi_{k,l}$.
- All of the difference from the lower right-hand corners of the four 2×2 blocks go into the lower right-hand corner of the larger grade, corresponding to wavelets of the form $\varphi_{m,n} \otimes \varphi_{k,l}$.
- All of the difference from the lower left-hand corners of the four 2×2 blocks go into the lower left-hand corner of the larger grade, corresponding to wavelets of the form $\varphi_{m,n} \otimes \psi_{k,l}$.

Proposition: We can complete the two dimensional transform by the following method:

$$\begin{bmatrix} 3 & -1 & 5 & -1 \\ 1 & 0 & 1 & -1 \\ 5 & 1 & 3 & 2 \\ 1 & -1 & 0 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & 5 & -1 & -1 \\ 5 & 3 & 1 & -1 \\ 1 & 1 & 0 & 2 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$

Finally, perform a two dimensional wavelet transform only the four entries iv the upper left-hand corner as follows:

$$\begin{bmatrix} 3 & 5 \\ 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{3+5}{2} & \frac{3-5}{2} \\ \frac{5+3}{2} & \frac{5-3}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 4 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{4+4}{2} & \frac{-1+1}{2} \\ \frac{4-4}{2} & \frac{-1-1}{2} \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & -1 \end{bmatrix}$$

Thus the matrix:

$$\begin{bmatrix} 4 & 0 & -1 & -1 \\ 0 & -1 & 1 & -1 \\ 1 & 1 & 0 & 2 \\ 1 & -1 & 0 & -1 \end{bmatrix}$$ Completed the two dimensional wavelet transform.

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REFERENCES

Afshari, M., 2008. Wavelets Kernel estimation of regression function for uniformly mixing process. World Applied Sci. J., 3: 441-447.

Antoniadis, A., G. Gregoire and I. McKeague, 1994. Wavelet methods for curve estimation. J. Am. Stat. Assoc., 89: 1340-1353.

Clyed, M.A., G. Parmigiana and B. Vidakovic, 1998. Multiple shrinkage and subset selection in wavelets. Biometrika, 85: 391-401.

Cohen, A., I. Daubechies and P. Vial, 1993. Wavelets on the interval and fast wavelet transform. Applied Comput. Harmon. Anal., 1: 54-81.

Daubechies, I., 1992. Ten Lectures on Wavelets. 1st Edn., SIAM., Philadephia, ISBN: 0-89871-274-2, pp: 1-52.

Daubechies, I., 1988. Orthonormal bases of compactky supported wavelets. Comun. Pure Applied Math., 8: 909-996.

Doosti, H., M. Afshari and H.A. Niroumand, 2008. Wavelets for nonparametric stochastic regression with mixing stochastic process. Commun. Stat. Theory Methods, 37: 373-385.

Meyer, Y., 1990. On de Lettes et Operateurs. Hermann Press, Paris.

Vidakovic, B., 1999. Statistical Modeling by Wavelets. Wiley, New York.

Walter, G., 1993. A new tool in applied mathematics. UMPA J., 2: 155-178.