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Multi-state System Availability Model of Electricity Generation for a Cogeneration District Cooling Plant

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ABSTRACT

The availability of electricity and chilled water at a cogenerated gas district cooling plant is linked to the availability of the gas turbines of the plant. In this study, multi-state system theory is introduced to analyze the availability of electricity which is produced by the two gas turbines each with 4.2 MW at a cogenerated gas district cooling plant. The states of the turbines and corresponding probabilities are defined. The availability of electricity is estimated based on the multi-state system approach using Markov chain method. The method is applied to the analysis of electricity generated by of two parallel gas turbines. The availability of electricity is simulated based on 1400 days of historical data. The availability analysis is showed that more than 99% of electricity was available to meet the required demand of the system.

Key words: Availability, multi-state system, Markov chain

INTRODUCTION

Availability of electricity at a cogeneration plant relies on the performance of gas turbines. When the turbines are down, electricity must be purchased from another producer. This is generally more expensive. Thus considering importance of the electricity for plant operation and customer need, its availability should be carefully evaluated in order to anticipate the performance of the plant. Availability, as a performance measure, reflects the ability of a power system to meet demand requirements. Barabady and Kumar (2007) stated that the most important performance measures for repairable system are system reliability and availability. Samrout *et al.* (2005) described the availability as good evaluations of a system's performance.

Many availability analysis methods have been developed throughout the years, which can be grouped into binary and multi-state methods. In binary availability modeling, the system is assumed to be either in a working state or in a failed one. However, in many real-life situations the performance of the system equipment can degrade, which results in performance degradation of the system, so there can be several states of degradation. This binary-state assumption may not be adequate. Many papers have been devoted to estimate the availability of multistate systems (Yu *et al.*, 1994; Huang *et al.*, 2000; Levitin, 2004) and optimizing the structure of the multistate systems (Levitin *et al.*, 1998; Ouzineb *et al.*, 2008; Agarwal and Gupta, 2007). In multi-state availability modeling, the system may have more than two levels of performance varying from perfect functioning to complete failure. A multi-State System (MSS) may perform at different

intermediate states between working perfectly and total failure (Lisnianski and Levitin, 2003). The presence of setting and partial operation is a common situation in which a system should be considered to be a MSS.

Practical methods of MSS availability evaluation are based on three different approaches (Hoang, 2003): the structure function approach, where Boolean models are extended for the multi-valued case; the stochastic process (mainly Markov) approach and Monte Carlo simulation. Since the Markov modeling approach can generate all possible states of a system, the number of states can be extremely large even for a relatively small number of Markov elements. In spite of these limitations the above-mentioned models are often used in practice, for example in the field of power systems availability analysis (Billinton and Allan, 1990). In the present work the Markov chain method has been selected for the following reasons: it is appropriate for quantitative analysis of availability and reliability of systems; it can be used with large, complex systems; it is not only useful, but often irreplaceable, for assessing repairable systems. Therefore, this study adopts random process (Markov model) that takes into consideration of Multi-state model to analyze the availability of gas turbines operated at a cogenerated District Cooling (DC) plant.

MODEL DESCRIPTION

System configuration: The gas turbines described in this paper are currently being used at a cogenerated gas district cooling plant of an academic institution. These gas turbines generate electricity with each turbine design capacity of 4.20 MW. The configuration of the gas turbines is as shown in Fig. 1.

The two gas turbines are connected in parallel and homogenous in features (Fig. 1). The output of the system electricity is the sum of the power output of each turbine. The plant system is down when both gas turbines completely failed. The electrical power produced by the plant is highly depending on these two gas turbines. Therefore, the system power availability is depending on the performance of these gas turbines.

States definition and probabilities: The electricity production highly depends on the gas turbine states. Subtractive clustering (Romera *et al.*, 2007) analysis was used to cluster the performance for the gas turbines to find the system states. The subtractive clustering method assumes each performance data point is a potential cluster center and calculates a measure of the likelihood that

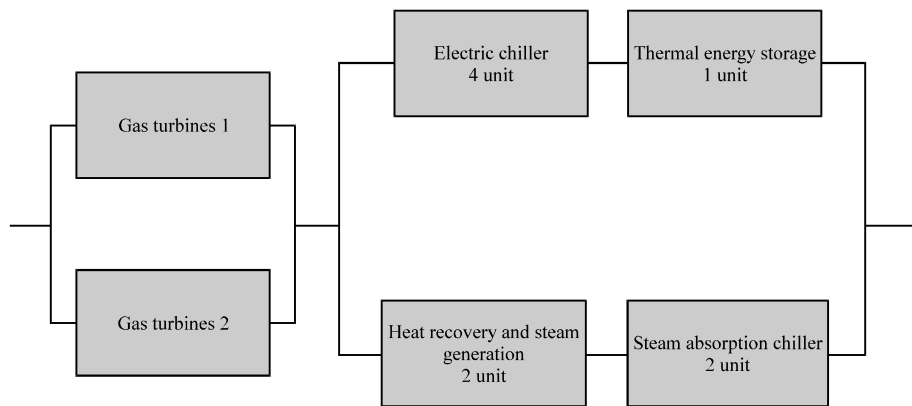


Fig. 1: System block diagram for the gas turbines at the cogenerated DC plant

Table 1: Performance data cluster for gas turbines

Cluster	Time	Electricity production (k W)			Average (kW)
1	2	0	0	0	0
2	10	3188	3359	3509	3297
	11	3143	3272	3313	
3	23	2905	2248	2560	2571

each performance data point would define the cluster center, based on the density of surrounding data points. Thus, the measure of potential for a data point is a function of its distances to all other data points as per Eq. 1 and 2;

$$p_i = \sum_{j=1}^n \exp(-\alpha \|x_i - x_j\|^2) \tag{1}$$

$$\alpha = \frac{4}{r_a^2} \tag{2}$$

Where:

p_i = Potential value of data point i

x_i = i th data points

n = Total number data points

r_a = Radii or radius defining a neighborhood

Three performances for gas turbines were developed using subtractive cluster analysis as shown in Table 1. Therefore, the corresponding performance, g_s , associated with each state (s) are 0, 2.6 and 3.3 MW. Let $P_s(t)$, $s = \{1, 2, \dots, K_j\}$ is the state probabilities of the element's performance process $G(t)$ at time t as shown in Eq. 3:

$$P_s(t) = \Pr \{G_j(t) = g_s\}, \quad s = \{1, 2, \dots, K_j\}; t \geq 0 \tag{3}$$

System of differential equations for finding the state probabilities $P_s(t)$, $s = \{1, 2, \dots, K_j\}$ for the homogeneous Markov process (Lisnianski and Levitin 2003) defined as follows:

$$\frac{dP_s(t)}{dt} = P_s(t)V(t) \tag{4}$$

Where:

$P_s(t)$ = System-state probability vector at time t , whose entries are the system state probabilities at t

$V(t)$ = Transition-rate matrix, whose entries are the component failure, repair and intensity rate

State-space model for multi-state elements: A state-space method using Markov Model (MM) was used for multi-state system availability analysis. This method is flexible and gives realistic and

dynamic model for availability. The model enclosed intensity of reduced capacity, repair times and failure rates. The state-space method is not limited to two states only, such as up and down. Furthermore, components contained different states such as operational, partial and down. Therefore, the Markov model of two parallel gas turbines was developed as shown in Fig. 2. The following assumptions and conditions are adopted for the model.

- The system is repairable
- The system is subject to repair and maintenance
- The outcome of each individual repair and maintenance measure is random
- The system can be in a working state but not operating at full capacity

Where:

- μ = Repair rate
- λ = Failure rate
- T_c = Cycle time
- PO = System operating at reduced capacity (61% of the design capacity)
- t_p = The mean duration of the peak
- ψ = Transition intensity rate from PO to up state
- ϵ = Transition intensity rate from up state to PO
- Up state = The turbine working at nominal capacity 3.3 MW as shown Table 1 which means 78.5% of the design capacity

In this model, all transitions are caused by the element's failures and repairs corresponding to the transition intensities are expressed by the element's failure and repair rates. Every element state there is associated performance of the element as in Table 1. Failure and repairs cause element transition from one state to only adjacent state. As can be seen in Fig. 2, with assumption state 1 is the best state of the system, there is transition from state 1 to state 2 and 3, if failure (λ_1) and ϵ occurs in the state 1, if the repair (μ_1) will be completed, the system will be back to the previous highest state 1. Similarly, if state 3 fail or subjected to demand variation, it goes to state

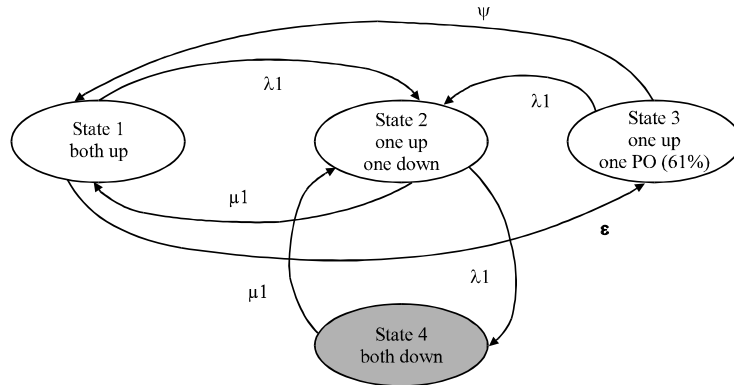


Fig. 2: State-space diagram for the system

with failure and intensity rate λ_1 and ψ . Based on the developed state space diagram, the mathematical equations using Markov chain were developed. The corresponding system of equations are written as Eq. 5-7:

$$[P_i] = \begin{bmatrix} -(\lambda_1 + \varepsilon) & \mu_1 & \psi & 0 \\ \lambda_1 & -(\mu_1 + \lambda_1) & \lambda_1 & \mu_1 \\ \varepsilon & 0 & -(\lambda_1 + \psi) & 0 \\ 0 & \lambda_1 & 0 & -\mu_1 \end{bmatrix} \quad (5)$$

$$\psi = \frac{1}{T_c - t_p} \quad (6)$$

$$\varepsilon = \frac{1}{t_p} \quad (7)$$

Assume that the initial state is the state 1 with the best performance. Therefore, by solving system (5) using Laplace transformation under the initial condition $P_k(0)=1, P_{k-1}(0)=\dots = P_2(0) = P_1(0) = 0$, the state probabilities were determined.

Equations for multi-state system availability: Based on state probabilities which are determined in Markov model for all elements, availability defined as a measure which depicts the probability of maintaining normal working of systems/components under determinate time and task conditions. Therefore, the availability of the system is defined as Eq. 8 and 9,

- MSS availability $A(t, w)$ at instant $t > 0$ for random constant demand w

$$A(t, w) = \sum_{i=1}^k P_i(t), (g_i - w \geq 0) \quad (8)$$

- MSS expected output performance at instant $t > 0$ for arbitrary constant demand w

$$E(t) = \sum_{i=1}^k P_i(t)g_i \quad (9)$$

RESULTS AND DISCUSSION

Estimation of state probabilities: In order to evaluate the availability of the entire system, it is necessary to determine the probability of each system states with corresponding system performance. Using Eq. 5 with initial conditions $p_i(0)$ is zero, for all $i \neq 1$ and $P_1(0) = 1$, the evaluated states probabilities as function of time are shown in Fig. 3.

In Fig. 3 each value of performance corresponds to the probability that the element provides a performance rate. As can be observed from Fig. 3, state 1, which is the best state of the system was greater than 90.23% during this operation days. Analogously, State 4 (worst state of the system) almost did not occur in the system. The probability that the plant runs under state 4 conditions is almost negligible or zero.

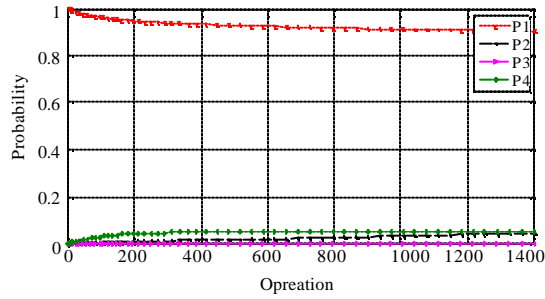


Fig. 3: Probability different performance level

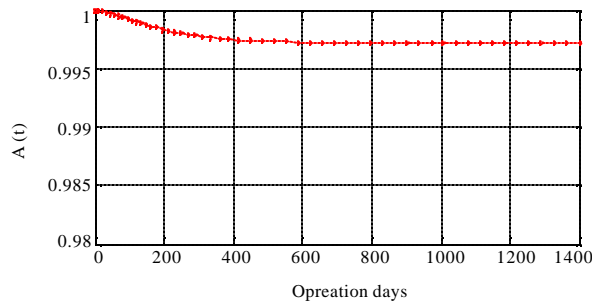


Fig. 4: Availability of generated electricity

Availability of generated electricity: Depending on the demand required, each state constitute the set of acceptable states. The states which have the output performance lower than the demand required will be combined in one state called absorbing state (unacceptable states). Therefore, the instantaneous Eq. 8 is defined by the sum of probability of only acceptable state. The availability of generated electricity is shown in Fig. 4.

Figure 4 shows that the availability of the system with respect to time between 0 to 1400 operation days. As an over all trend it is clear that the availability of electricity decreased through time due to either the performance degradation or high demand need. If the required demand or generation of electricity between 0 and 3.3 MW, all states delivered the required amount of electricity except state 4. In this case state 4 would be an unacceptable state and the Eq. 8 of the system was the sum of probabilities state one, state 2 and state 3. So the system availability is greater than 99%.

Expected output performance of the system: The expected output performance as a function of time is shown in Fig. 5 using Eq. 9. The expected output decreases at the beginning for 400 days of operation due to setting and degradation and then constant for the remaining operation days. The expected output performance was compared with the actual output. The results were statistically evaluated using T-test for comparison of actual and expected output for evidence of significant difference. Table 2 displays summary of statistics and the variances of the two samples. According to the data presented in Table 2, the P value for actual and expected output ($p = 0.0865$) is greater than the conventional p-value (0.05). Therefore, the actual output is significantly more precise with expected output. Deviation of mean and the standard error of the outputs were minimal which means the model can predict the output of the plant without significant difference.

Table 2: T-test: Two-sample assuming unequal variances

Statistical values	Actual	Model
Mean	6497.849	6500.919
Variance	15457.06	1081.778
Hypothesized mean difference	0	
t-Stat		-0.17051
P(T≤t) one-tail		0.432606
t Critical one-tail		1.672029
P(T≤t) two-tail		0.865212
t Critical two-tail		2.002465

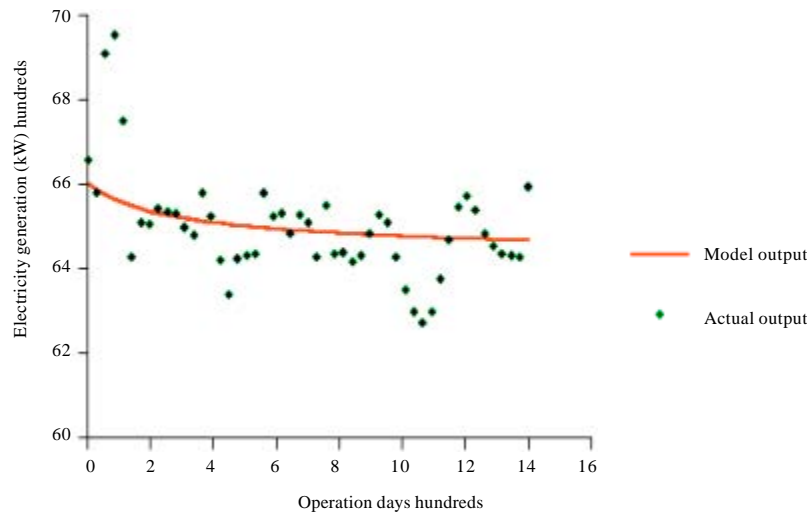


Fig. 5: Instantaneous mean performance (kW)

CONCLUSION

This study shows that random process method takes into account multistate models for all system components to predict the availability of the electricity in a gas district cooling plant. Results indicate that the availability of the system is more than 99.5% and the expected performance is greater than 6.6 MW for 1400 operation days. As an overall trend, availability of the system decreased with time due to performance degradation and setting.

Comparison of the outputs from the model and actual plant was done using statistical analysis. Result shows that expected output is more precise with the actual mean output. Therefore, the model can predict the availability and expected output performance of the plant.

Even though this model is very essential to analyze availability of the system, it has its own limitation particularly when the numbers of states or components increase. To address these difficulties this model can be integrated with universal generating function to reduce the number of states and iteration.

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