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Mathematical Study of the Effect of Changing Time Step Duration and Oscillating Radon Concentration on the Charcoal Canister's Activity

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ABSTRACT

The activity of a charcoal canister is used to estimate the radon concentration in air. The diffusion equation with a decay term of radon is used to represent diffusion of radon mathematically in the charcoal canister. The finite difference technique is used to solve the equation in the charcoal canister with pre-specified properties and dimensions of canister. A software developed by the authors is used for such simulation. The radon concentration input to the software changes with time either as a step function or a sinusoidal wave superimposed on a constant radon concentration. The activity of the charcoal canister is estimated using both time dependent inputs of concentrations. For step wise input, the change in activity of charcoal does not depend on the start and end time of the step but it depends on the time duration of the step. The maximum change in the activity depends upon both the height of the steps and the duration time. The calibration factor CF after saturation was found to be fairly constant independent off the time duration or the height of the step. For a sinusoidal input, different amplitudes and different half periods are used to study the effect on the activity of the charcoal canister. The increase in the resulting activity over the activity corresponding to a constant average radon concentration in air is compared to the amplitude of the sinusoidal radon concentration. It was found that the delay time in the activity response and the change of activity ΔA divided by $C_1(\Delta A/C_1)$ increases with increasing the half period but they do not depend on the amplitude of the sinusoidal change in radon concentration in air.

Key words: Radon concentration measurements, activated charcoal canisters, modeling and simulation

INTRODUCTION

For an average person, radon is the most important source of exposure to ionizing radiation (UNSCEAR, 2000). The importance of estimating radon concentration in air is also clear in places of high inventories of U-238 like uranium mines, nuclear power reactors and facilities of the nuclear reactors fuel cycle. In such places, the increase in radon concentration may affect workers in these places introducing an internal dose due to inhalation. One of the most efficient and easy to use techniques to measure radon concentration in air is the diffusive sampling of radon in charcoal canisters. The activity measurement of radon using this technique is compromised by the decay and desorption of radon during the sampling process. The following factors affect the

accuracy of the estimation of radon activity in charcoal and the corresponding radon concentration in air (Underhill, 2003):

- The mathematical differential equation model which describes the diffusion of radon in the sampler
- The mathematical procedure which is used to solve such model
- The physical parameters which are used to define the diffusion coefficient and dimensions of the sampler
- The standard protocol which is used in the measurement process

In addition to these factors, the variation of the radon concentration in air with time during the simulation process affects the accuracy of the estimation.

Through out this study, the variation of radon concentration during the process of measurement is studied to evaluate its effect on the calibration factor in m^3 (CF) of the activated charcoal canister which is defined as the ratio of radon activity adsorbed by active charcoal and average radon concentration in the air during the period of exposure (Nikezic and Urosevic, 1998).

MATHEMATICAL MODEL

The model introduced by Urosevic *et al.* (1999) had been used to represent the diffusion of radon gas in charcoal. The governing mathematical equation is:

$$\frac{\partial}{\partial x_i} \left(D_i \frac{\partial C}{\partial x_i} \right) - \lambda C = \frac{\partial C}{\partial t}$$

Where:

C = Radon concentration in the pores of active charcoal

D_i = Diffusion coefficient in coordinate (i)

λ = Decay constant of radon

t = Time

x_i = Coordinate system $i = 1, 2, 3$

Assuming isotropic diffusion coefficient and a single dimension for variation of radon concentration, the equation reduces to:

$$D \frac{\partial^2 C}{\partial Z^2} - \lambda C = \frac{\partial C}{\partial t}$$

The boundary conditions of the equation are:

$$\begin{aligned} \text{at } z = 0 \quad C(0, t) &= k\rho C(t) \\ \text{at } z = l \quad \partial C / \partial Z &= 0 \text{ (non permeable boundary condition)} \end{aligned}$$

Where:

ρ = Charcoal density (kg m^{-3})

k = Adsorption coefficient in charcoal ($\text{m}^3 \text{kg}^{-1}$)

The standard dimensions and properties of charcoal which had been used in the study are (Nikezic and Urosevic, 1998):

$$\begin{aligned} D &= 1.27 \times 10^{-9} \text{ (m}^2 \text{ sec}^{-1}\text{)}, \\ k &= 3.9 \text{ (m}^3 \text{ kg}^{-1}\text{)}, \\ \rho &= 500 \text{ (kg m}^{-3}\text{)}, \\ l &= 0.017 \text{ m,} \end{aligned}$$

where, l is the canister thickness:

$$S = 0.0081 \text{ m}^2$$

where, S is the canister exposed surface area:

$$C_o = 1 \text{ (Bq m}^{-3}\text{)},$$

where, C_o is the normalized value of radon concentration in air:

$$\lambda = 2.1 \times 10^{-6} \text{ s}^{-1}$$

During such study, a software developed by the authors using finite difference technique is used to solve the governing equations along with the appropriate boundary conditions and the proposed values of constants and dimensions for the charcoal canister. Initially, the input radon concentration step wise function is used with different time durations and different heights. Figure 1 shows the shape and definitions which have been used for the time step input radon concentration in air.

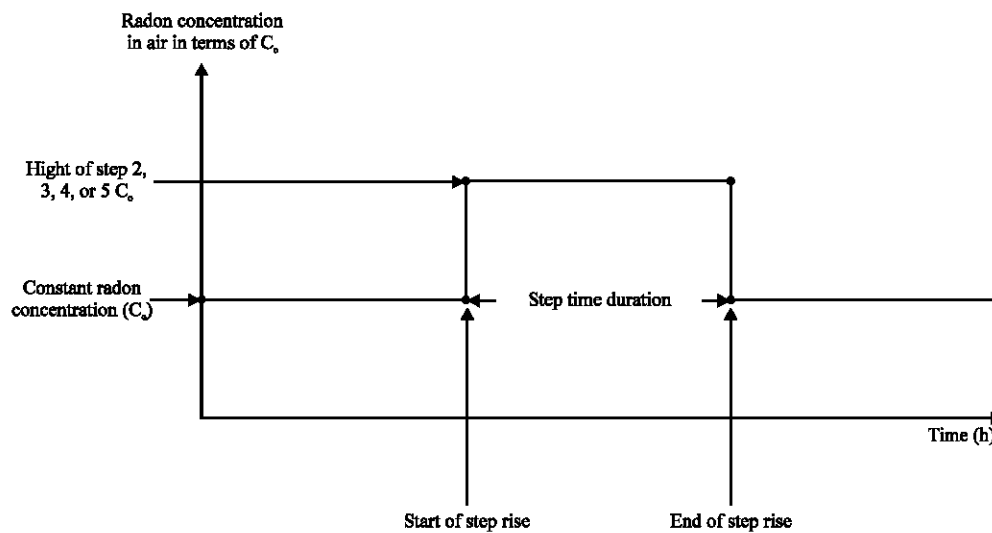


Fig. 1: Schematic diagram of radon concentration in air as a step function

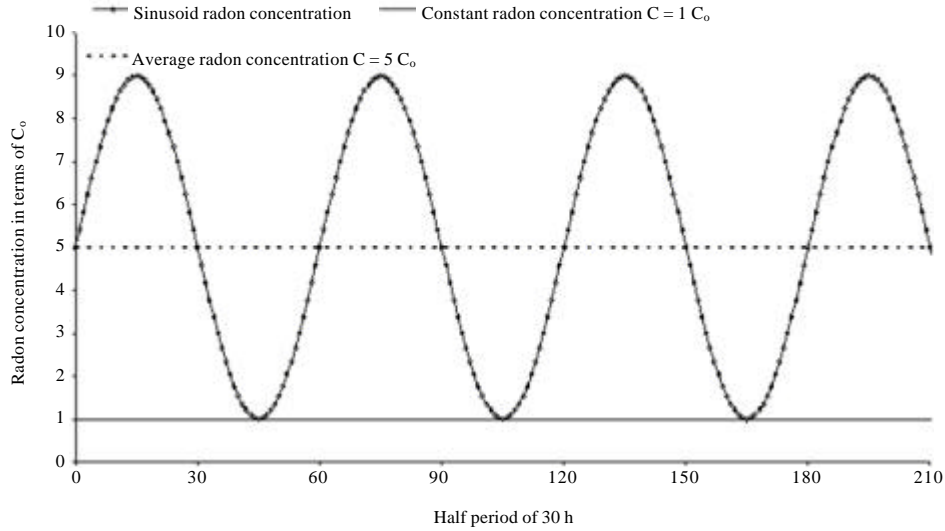


Fig. 2: Sinusoidal radon concentration with amplitude $4 C_o$ and half period of 30 h super imposed on a constant radon concentration $1 C_o$.

As a time dependent input of radon concentration a sinusoidal variation in radon concentration is considered. The choice of such variation is based upon the variation of radon concentration in closed areas that are being opened in regular time periods. During closing the area, radon concentration increases, while during opening the area and refreshing air inside, radon concentration decreases.

To represent such behavior mathematically, a sinusoidal wave super-imposed on a constant radon concentration is being used. This input is shown in Fig. 2 and the concentration of radon in air is represented by the mathematical form:

$$C_{\text{air}}(t) = C_o + C_1 \left(1 + \sin \frac{\pi t}{T} \right)$$

Where:

C_o = Basic constant radon concentration

C_1 = Radon concentration amplitude of the sinusoidal wave

T = Half-period of the variation in time of the sinusoidal wave

To study the effect of sinusoidal wave, the amplitude of the sinusoidal wave C_1 has been changed to 1, 2, 3, 4 and $5 C_o$.

The half period of the sinusoidal wave (T) has been given the values 6, 12, 18, 24, 30 and 36 h. For long half periods seven half periods had been used in the simulation process to show the effect of the variation in radon concentration on charcoal activity. For the sake of comparison, the results are compared to the results of constant radon concentration with a value equal to the average of the concentration in air ($C_o + C_1$).

RESULTS AND DISCUSSION

Step wise in radon concentration in air: In the course of studying the effect of a step rise in radon concentration, the step height is changed to take values of 2, 3, 4 and 5 C_o .

Figure 3 shows the output of the activity of the charcoal for different step heights with step time duration of 3 h starting at 3 h time. The peak activity increases with the increase in step height.

To study the effect of the step function duration and height on the charcoal activity, the increase in the activity of charcoal is calculated for different step heights and step durations. The increase in activity is represented by $\delta A = A_{\text{step}} - A_{\text{constant}}$ at constant step time duration of 3, 6, 9 and 12 h. Where, A_{step} is the activity of canister in case of a step increase in radon concentration and A_{constant} is the corresponding value for the case of constant normalized radon concentration (C_o). The start time of the step takes the values of 0, 12, 24, 36, 48 and 60.

Figure 4 shows the change in δA for different step heights and different start times of the step for step duration of 12 h. It is obvious that the increase in activity of charcoal over that

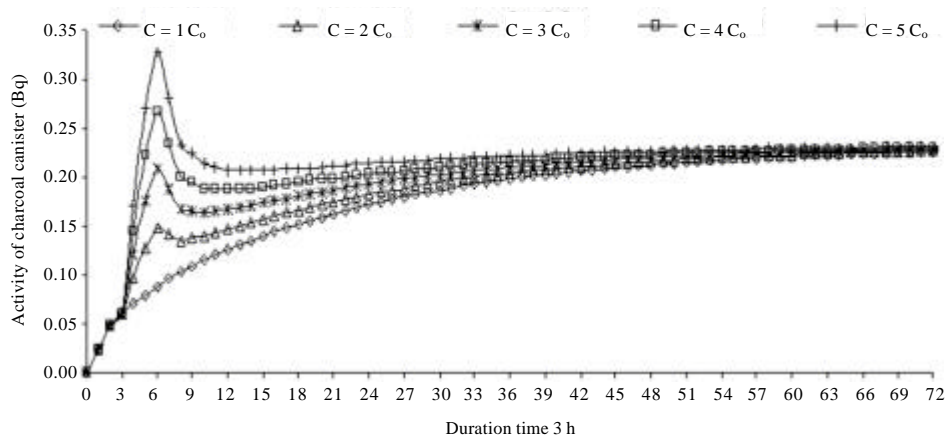


Fig. 3: Changing charcoal canister activity with a step of time duration 3 hours at different step heights

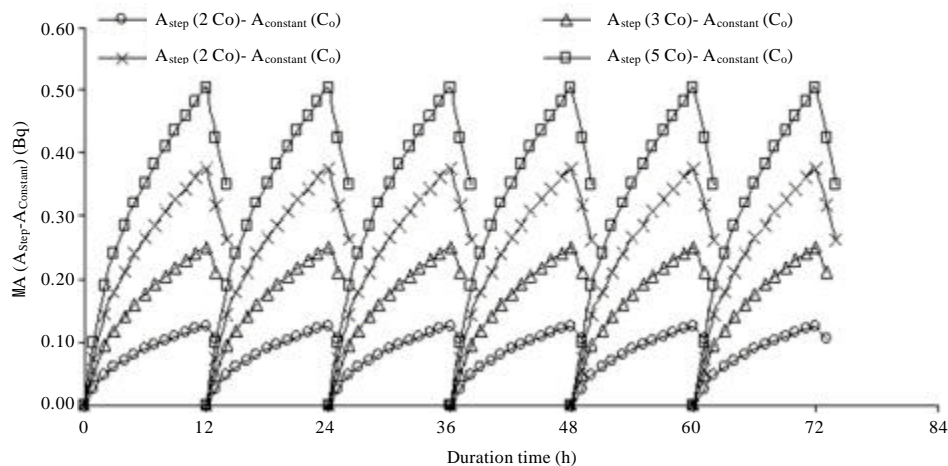


Fig. 4: Step rise effect on canister's increase in activity ($\delta A = A_{\text{step}} - A_{\text{constant}}$) at duration time equals 12 h

corresponding to the constant radon concentration δA depends on the step height. The curve shows that the increase in charcoal activity δA does not depend on the start and end time for the same height and duration. The time duration of the step is changed to 3, 6 and 9 h. The output curves (not included) for each of these values of time durations show the same conclusion.

Figure 5 shows the maximum values of δA for different values of step time duration, namely, 3, 6, 9 and 12 h. The maximum values of δA depend on the height of the step and on the duration of the step. For the same duration they increase with step height. For the same height they increase with the step duration time.

Figure 6 shows the value of the calibration factor CF of the charcoal canister for different values of step time duration. The step has a height of 3 C_o . The time durations of the step are taken as 3, 6, 9 and 12 h. The curve shows that CF increases with time that the charcoal spends in air. The rate of increase decreases with time because of the saturation of radon in charcoal canister.

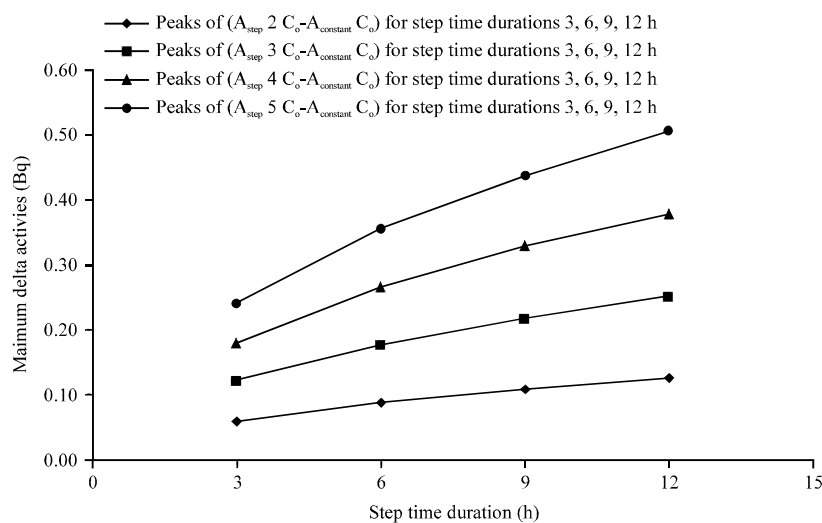


Fig. 5: Peak of δA with time for different step time durations

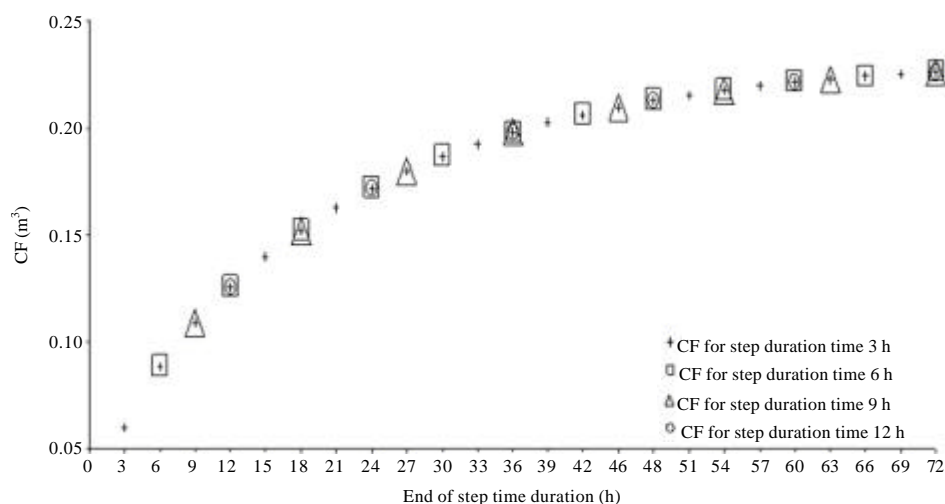


Fig. 6: Calibration factor at step height 3 C_o and 3, 6, 9 and 12 step time durations

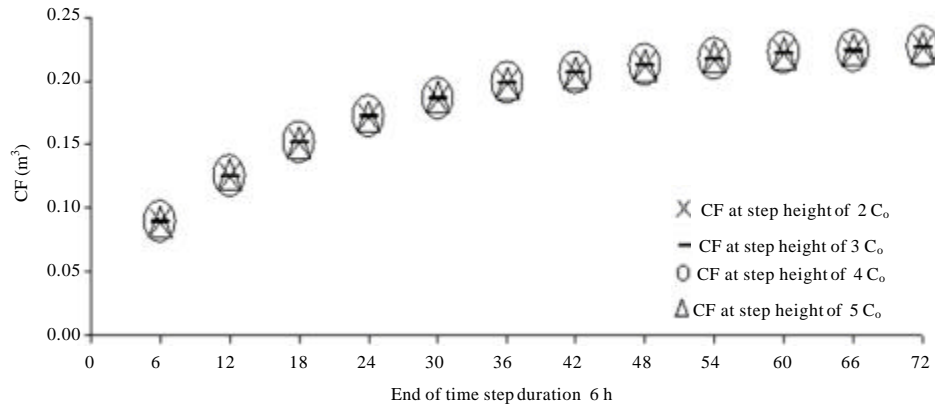


Fig. 7: Calibration factor at step heights of 2, 3, 4 and 5 C_0 with a step duration time of 6 h

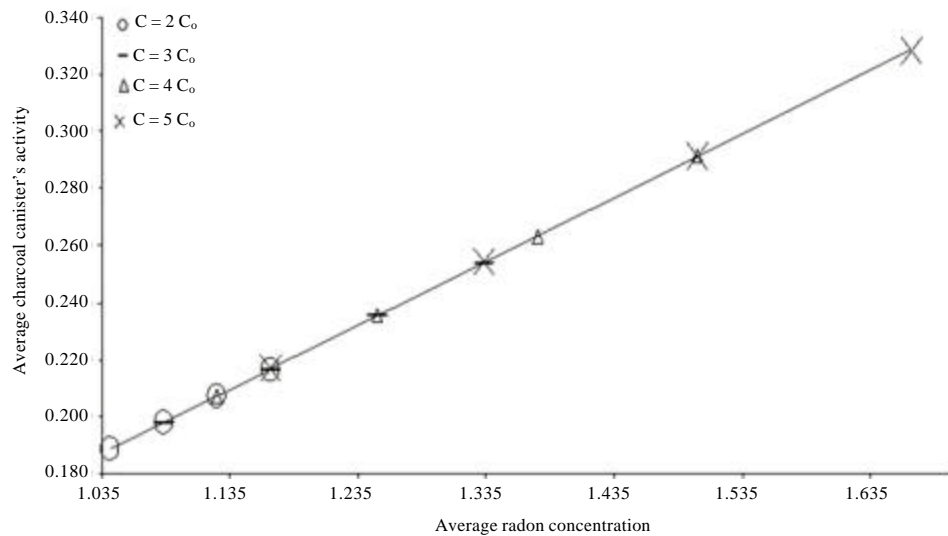


Fig. 8: Average charcoal activity versus average radon concentration

Figure 7 shows the values of the calibration factor CF for a time step duration of 6 h and different step heights 2, 3, 4 and 5 C_0 . The curve gives the same values as in Fig. 6 which indicates that CF does not depend on step height or time duration.

Figure 8 gives the average charcoal canister activity versus the average radon concentration in air for different values of step heights. The figure shows a straight line with a constant slope which indicates that CF does not depend on the height or the step duration. The value of CF after saturation can be used to determine the average value of radon concentration in air during exposure time once the canister's activity is measured.

Sinusoidal change of radon concentration in air: To check the effect of radon concentration change with time on the activity of charcoal, a sinusoidal wave superimposed on a constant radon concentration is applied. The constant radon concentration is taken as C_0 . The amplitude of the

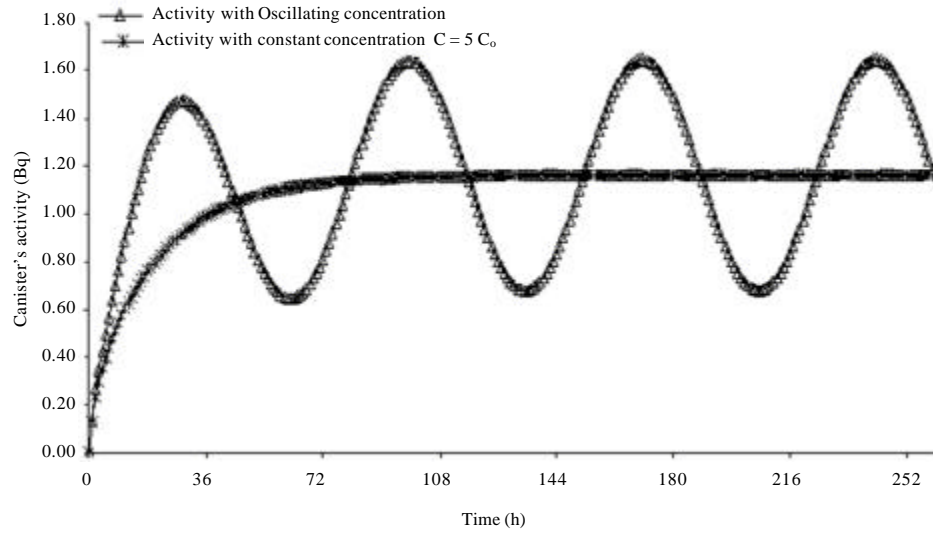


Fig. 9: Response of activity of charcoal as a result of sinusoidal change in radon concentration with amplitude $4 C_o$ and half period 36 h compared to response due to average constant radon concentration $C = 5 C_o$.

sinusoidal wave C_1 are taken as 1, 2, 3, 4 or $5 C_o$. The half periods of the sinusoidal wave T are taken as 6, 12, 18, 24, 30 and 36 h. The amplitude values are normalized to the constant radon concentration C_o ($C_o = 1$).

Figure 9 gives the response of the charcoal activity as a result of a sinusoidal change with amplitude $4 C_o$ and half period 36 h compared to the response of the charcoal activity as a result of constant radon concentration in air equal to $5 C_o$ which is the average value of the radon concentration in air. The results show oscillations around the activity of charcoal corresponding to the constant average concentration which equals $(C_o + C_1)$ in this run. The curve indicates that the use of the average value of the charcoal activity is fair enough to determine the average value of concentration in air. Defining the maximum change in activity as:

$$\Delta A = A_{\text{sin, max.}} - A_{\text{constant, average}}$$

Where:

$A_{\text{sin, max.}}$ = The maximum activity due to a sinusoidal change in radon concentration after saturation

$A_{\text{constant, average}}$ = Activity of charcoal canister due to a constant radon concentration equals to the average value of radon concentration in air $(C_o + C_1)$. The maximum change in radon concentration over the average concentration is C_1

Figure 10 shows the change in activity ΔA dividing by the amplitude of the sinusoidal wave C_1 . The curve indicates that for different amplitudes the value of $(\Delta A/C_1)$ does not change, while increasing the half period T leads to increase in the value of $(\Delta A/C_1)$. So, the value of $(\Delta A/C_1)$ depends only on the half period T . Practically, the half period depends on the closing time for which the radon concentration increases in the place where it is monitored.

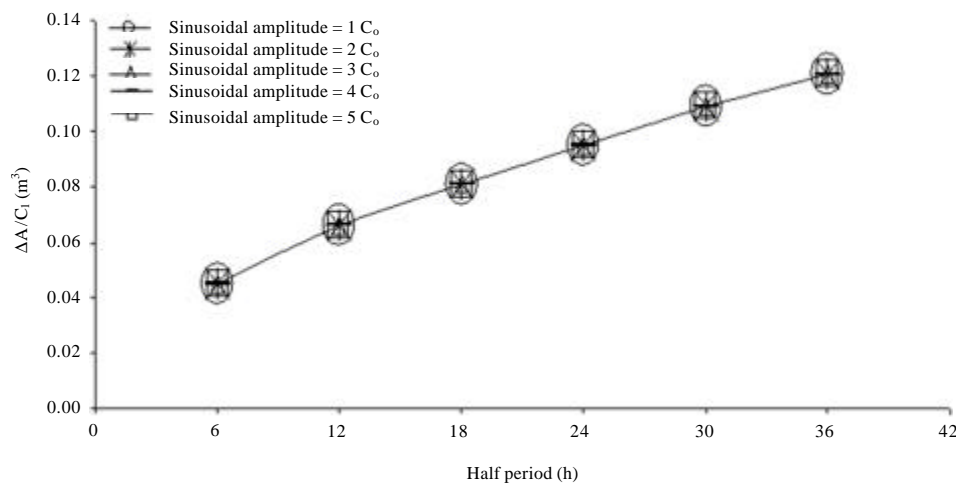


Fig. 10: ($\Delta A/C_1$) versus half period T for a sinusoidal change in radon concentration with different amplitudes and different half periods

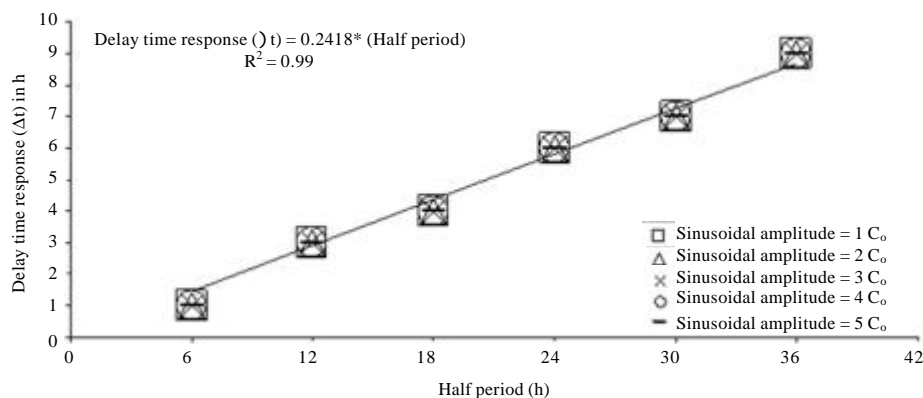


Fig. 11: Delay time response (Δt) versus half period for a sinusoidal change in radon concentration with different amplitudes and different half periods

Defining the delay time response (Δt) as:

Δt = (Time of the peak in radon concentration sinusoidal wave in air) - (Time of corresponding peak in activity of charcoal canister)

Figure 11 shows the delay time (Δt) of the response compared to the sinusoidal change in radon concentration in air. Again, the delay in time increases with increasing the half period T. It does not depend on the amplitude of the sinusoidal wave for the same half period.

CONCLUSION

For step wise change of radon concentration in air, it is obvious that the increase in activity of charcoal over that corresponding to the constant radon concentration δA depends on the step height. δA does not depend on the start and end time for the same height and duration. The

maximum values of δA depend on the height of the step and on the duration of the step. For the same duration they increase with step height. For the same height they increase with the step duration time. The calibration factor CF of the charcoal canister increases with time that the charcoal spends in air. But, the rate of such increase decreases with time because of the saturation of radon in charcoal canister. CF does not depend on step height or time duration. The value of CF after saturation can be used to determine the average value of radon concentration in air during exposure time once the canister's activity is measured. In the case of sinusoidal wave change of radon concentration in air, the use of the average value of the charcoal activity is fair enough to determine the average value of radon concentration in air. For different amplitudes the value of $(\Delta A/C_1)$ does not change, while changing the half period T changes the value of $(\Delta A/C_1)$. The value of $(\Delta A/C_1)$ depends only on the half period T. The delay in time response Δt increases with increasing the half period T. It does not depend on the amplitude of the sinusoidal wave of radon concentration in air for the same half period.

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