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Estimation Some Parameters of K-station Series Model

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ABSTRACT

This study deals with the problem of statistical point estimation of K-station series model with unlimited inter queue capacity. K-station series model all over the world, both public and private, are facing continuously increasing traffic loads. The maximum likelihood function is used to estimate the arrival rates, service rates and the utilization factor in steady state when the service discipline is general.

Key words: Maximum likelihood function, arrival rate, service rate, utilization factor

INTRODUCTION

This study considers a system with K-station in series, supposes that arrivals at station 1 are generated from an infinite population according to a Poisson distribution with mean serviced units will move successively from one station to the next until they are discharged from station K. Service time at each station I has exponential distribution. The problem of point estimation is formulated as follows: A random sample is drawn from a population whose distribution has a known mathematical form but involves a certain number of unknown parameters. On the basis of this random sample, it is required to estimate the value of unknown parameters of the population.

Research has done on parameter estimation of the extended generalized Gaussian family distributions using maximum likelihood scheme (Lee, 2010). Time domain DS-UWB channel maximum likelihood algorithm was in working by Yin *et al.* (2011) but the work of Shi *et al.* (2011) was in solving project scheduling problems using estimation of distribution algorithm with local simplex search. Desta *et al.* (2011) carried out review on selected channel estimation algorithms for orthogonal frequency division multiplexing system. Wang *et al.* (2011) deal with a novel joint estimation algorithm for multi-parameter of underwater acoustic channels. Abd El-Salam (2011) deals with an efficient estimation procedure for determining ridge regression parameter. Ojo (2010) has work done in on the estimation and performance of one-dimensional autoregressive integrated moving average bilinear time series models. Oyeyemi *et al.* (2010) consider on the estimation of power and sample size in test of independence. Ojo (2008) carried out on the performance and estimation of spectral and bispectral analysis of time series data.

DESCRIPTION OF THE PROBLEM AND ITS SOLVING

Under these conditions, the out put of station I follows Poisson process with arrival rate λ, where station I follows the model $(M/M_i/1):(GD/\infty/\infty)$.

This means for the ith station, the steady state probabilities P_{n_i} are given by:

$$P_{n_i} = (1-\rho_i)\,\rho_i^{n_i},\ n_i = 0, 1, 2, \cdots$$

For I = 1, 2,..., k, where n_i is the number in the system consisting of station I only. Steady state results will exist only if utilization factor $\rho_i = \lambda/\mu_i<1$.

To find the likelihood function, it is necessary to find three basic components, Namely:

- The initial number of units n_i with probability $\Pr(n_i) = P_{n_i}$ for I = 0, 1, 2,..., k
- The inter-arrival time of lengths t_1 for R arrival units with probability $\prod_{l=1}^{R}\lambda e^{-\lambda t_l}$
- The service time of durations q_j for M units with probability:

$$\left(\prod_{j=1}^{M}\mu_i e^{-\mu_i q_{ij}}\right) \text{for } i = 1, 2, \cdots, k$$

It follows that the likelihood function is given by:

$$L\ (\theta) = \prod_{l=1}^{R}\lambda e^{-\lambda t_l}\prod_{j=1}^{M}\mu_i e^{-\mu_i q_{ij}}\ P_{n_i};\ \ \theta \equiv (\lambda,\ \mu_i)$$

Hence:

$$L\ (\theta) = \prod_{l=1}^{R}\lambda e^{-\lambda t_l}\prod_{j=1}^{M}\mu_i e^{-\mu_i q_{ij}}\ P_0\left(\frac{\lambda^{n_i}}{\mu_i^{n_i}}\right)$$

Hence:

$$L\ (\theta) = P_0 e^{-\lambda T} e^{-\mu_i Q_i}\lambda^{R+n_i}\mu_i^{M-n_i}$$

Where:

$$T = \sum_{i=1}^{k} T_i = \sum_{i=1}^{k}\ \sum_{l=1}^{R} t_{il} \text{ and } Q = \sum_{i=1}^{k}\ Q_i = \sum_{i=1}^{k}\ \sum_{j=1}^{M} q_{ij}$$

After differentiating we get:

$$-\frac{\lambda\partial \ln L\ (\theta)}{\partial\lambda}\bigg|_{\theta=\hat{\theta}} = -\hat{\lambda}\left[\frac{\partial \ln P_0}{\partial\lambda} - T + \frac{(R+n_i)}{\hat{\lambda}}\right] = 0 \tag{1}$$

And:

$$-\frac{\mu_i\partial \ln L\ (\theta)}{\partial\mu_i}\bigg|_{\theta=\hat{\theta}} = -\hat{\mu}_i\left[\frac{\partial \ln P_0}{\partial\mu_i} - Q_i + \frac{(M-n_i)}{\hat{\mu}_i}\right] = 0 \tag{2}$$

where, $\hat{\lambda}$ and $\hat{\mu}_i$ are the maximum likelihood estimators of the parameters λ and μ_i, respectively. Then:

$$E\ (n_i) = \frac{-\lambda \partial \ln P_0}{\partial \lambda} = \frac{\mu_i \partial \ln P_0}{\partial \mu_i}$$

Where:

$$E\ (n_i) = \frac{\lambda}{\mu_i - \lambda}$$

Using the Eq. 1 and 2, then:

$$\hat{\lambda} = \frac{1}{T}\left[R + n_i - \hat{E}\ (n_i)\right] \tag{3}$$

And:

$$\hat{\mu}_i = \frac{1}{Q_i}\left[M - n_i + \hat{E}\ (n_i)\right] \tag{4}$$

where, $\hat{E}\ (n_i)$ is the maximum likelihood estimator of $E\ (n_i)$.
From the Eq. 3 and 4 we get:

$$\frac{Q_i \hat{\mu}_i - (M - n_i)}{-T\hat{\lambda} + (R + n_i)} = 1 \tag{5}$$

And:

$$\hat{\rho}_i = \frac{Q_i (R + n_i - \hat{E}\ (n_i))}{T\ [M - n_i + \hat{E}\ (n_i)]} \tag{6}$$

Substituting $\hat{E}\ (n_i)$ for its value in the Eq. 6, we get the equation:

$$\hat{\rho}_i^2\ (T - \alpha_i) + \hat{\rho}_i\ (\alpha_i + \beta_i + Q_i) - \beta_i = 0$$

Where:

$$\alpha = \sum_{i=1}^{k}\ \alpha_i = \sum_{i=1}^{k}\ T_i\ (M - n_i) \text{ and } \beta = \sum_{i=1}^{k}\ \beta_i = Q_i\ (R + n_i)$$

Hence:

$$\hat{\rho}_i = \frac{-(\alpha_i + \beta_i + Q_i) + \sqrt{(\alpha_i + \beta_i + Q_i)^2 + 4\beta_i (T_i - \alpha_i)}}{2\ (T_i - \alpha_i)} \tag{7}$$

Suppose that $M = R = n_i$ and using the Eq. 5 and 7, it follows that:

$$\hat{\rho} = \frac{-(\beta + Q) + \sqrt{(\beta + Q)^2 + 4\beta T}}{2T}$$

$$\hat{\lambda} = \frac{\hat{\rho}\ (R + M)}{T\hat{\rho}\ + Q}$$

And:

$$\hat{\mu}\ = \frac{(M + R)}{Q\ + T\hat{\rho}}$$

In addition, we can obtain:

$$\text{The mean of service time in the system} \approx \frac{1}{M + R}\left[Q + T\hat{\rho}\right]$$

$$\text{The mean of the numbers of the units in the system} \approx \frac{\lambda\ (Q\ + T\hat{\rho}\ )}{M + R - \lambda\ (Q\ + T\hat{\rho}\ )}$$

Table 1: Application with numerical results*

Arrival times of customers to server 1	Service time of customers from server 1	Arrival times of customers to server 2	Service time of customers from server 2	Arrival times of customers to server 3	Service time of customers from server 3
2	6	8	9	17	22
3	5	8	10	18	21
4	4	8	11	19	20
5	6	11	10	21	20
6	4	10	9	19	22
7	5	12	11	23	19

*Suppose that the following data access times of units were given

In Table 1, we can obtain the following results:

- Arrival rate of all customers to server 1 = 4.5
- Arrival rate of all customers to server 2 = 9.5
- Arrival rate of all customers to server 3 = 19.5
- Service rate for server 1 = 5
- Service rate for server 2 = 10
- Service rate for server 3 = 20.6666666666667
- Number of servers = 3
- Sum of arrival times of customers to server 1 = 27
- Sum of arrival times of customers to server 2 = 57
- Sum of arrival times of customers to server 3 = 117
- Sum of arrival times of customers to all servers = 201
- Sum of service times of server 1 = 30
- Sum of service times of server 2 = 60
- Sum of service times of server 3 = 124

- Sum of service times of all servers = 214
- Number of arrival customers to the system = 6
- Number of arrival customers to any server = 6
- Number of departure customers from the system = 6
- Utilization factor (traffic intensity) = 0.868570344017331

CONCLUSION

The mentioned above results show that the system is stable because the coefficient of efficiency has ρ (traffic intensity) is less than or equal to 1 and if it is greater than 1, then that leads to instability system and losses, so the system must be stopped or modify the service times in this server or change the arrival times of the units. Therefore, we can control the system to obtain less expensive, rapid processes and stability of the system.

In Fig. 1-3, the following formula was obtained:

$$y = ax^4+bx^3+cx^2+dx+e$$

where, x represents the arrival times, y represents the times of service received by the server, thus we can forecast the length of service time required from the server if the arrival time of the unit was given.

Figure 4 shows the distribution of the service times. Therefore, we can obtain the average service time of any unit or client within the system from the equation:

$$y = ae^{bx}$$

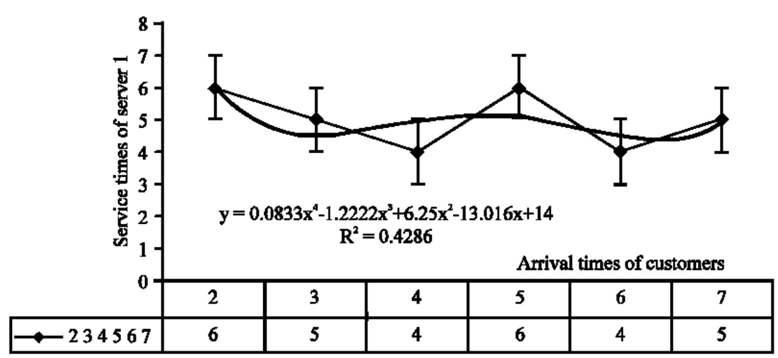


Fig. 1: The relation between arrival times of all customers and service times of server 1

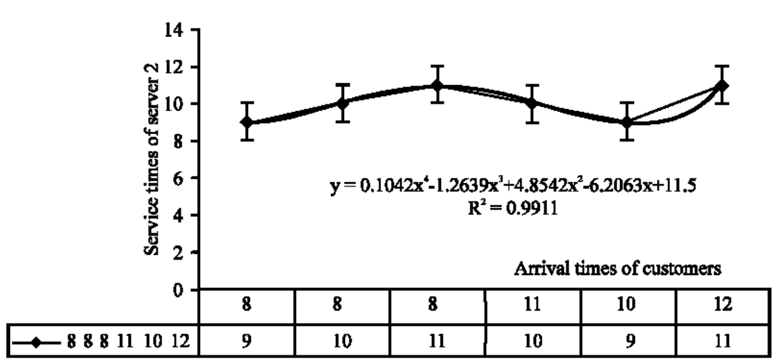
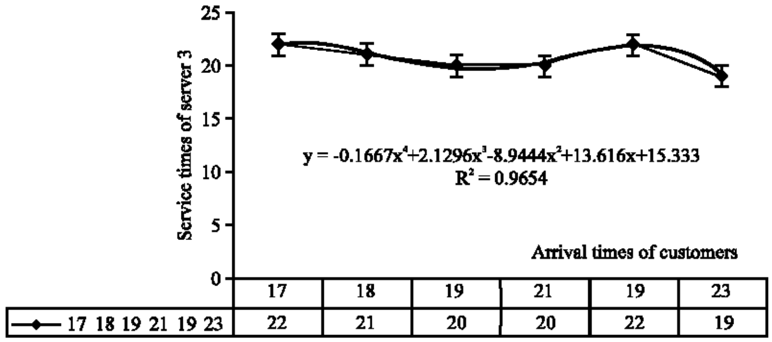
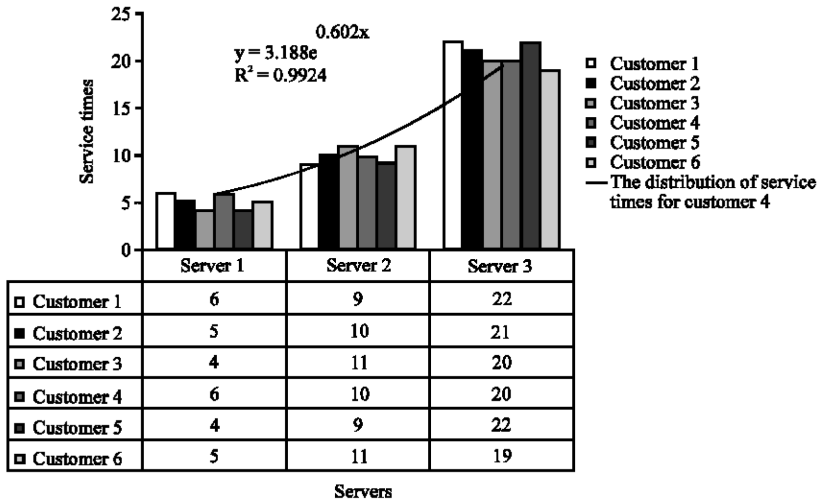


Fig. 2: The relation between arrival times of all customers and service times of server 2



	17	18	19	21	19	23
17 18 19 21 19 23	22	21	20	20	22	19

Fig. 3: The relation between arrival times of all customers and service times of server 3



	Server 1	Server 2	Server 3
Customer 1	6	9	22
Customer 2	5	10	21
Customer 3	4	11	20
Customer 4	6	10	20
Customer 5	4	9	22
Customer 6	5	11	19

Fig. 4: The distribution of service times of servers

In Fig. 4, the values of a and b depend on the unit within the system.

The fourth unit has the values a = 3.188 and b = 0.602, if the average of the arrival time was given, then we can obtain the average service time in the system. As a result, we can predict the time of service required from the system and know the number of servers that allows us to manage the financial cost to add or delete a server to the system, like as the cost of traffic loads and waiting the cargo ships in the port to get the service from several servers.

REFERENCES

Abd El-Salam, M.E.F.A., 2011. An efficient estimation procedure for determining ridge regression parameter. Asian J. Math. Stat., 4: 90-97.

Desta, Y.T., J. Tao and W. Zhang, 2011. Review on selected channel estimation algorithms for orthogonal frequency division multiplexing system. Inform. Technol. J., 10: 914-926.

Lee, J.Y., 2010. Parameter estimation of the extended generalized gaussian family distributions using maximum likelihood scheme. Inform. Technol. J., 9: 61-66.

Ojo, J.F., 2008. On the performance and estimation of spectral and bispectral analysis of time series data. Asian J. Math. Statist., 1: 150-158.

Ojo, J.F., 2010. On the estimation and performance of one-dimensional autoregressive integrated moving average bilinear time series models. J. Applied Sci., 3: 225-236.

Oyeyemi, G.M., A.A. Adewara, F.B. Adebola and S.I. Salau, 2010. On the estimation of power and sample size in test of independence. Asian J. Math. Stat., 3: 139-146.

Shi, Y.J., G.J. Shen and W. Chen, 2011. Solving project scheduling problems using estimation of distribution algorithm with local simplex search. Inform. Technol. J., 10: 1374-1380.

Wang, B., G. Yang and G. Liu, 2011. A novel joint estimation algorithm for multi-parameter of underwater acoustic channels. Inform. Technol. J., 10: 440-445.

Yin, Z., Z. Shi, G. Qi and Y. Zhou, 2011. Time domain DS-UWB channel estimation using maximum likelihood algorithm. Inform. Technol. J., 10: 127-133.