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Study of Reliability with Mixed Standby Components

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ABSTRACT

This study deals with the reliability characteristics of two different series system configurations with mixed standby (include cold and warm standby) components. The failure rates of the primary and warm standby components are assumed to follow the Weibull distribution. The repair time distribution of each server is exponentially distributed. Moreover, we will derive the mean time-to-failure and the steady-state availability for a special case of a serial system of two primary components, two warm standby components and one cold standby component, when the failure and repair rate are constant.

Key words: Reliability, availability, time varying failure, standby components, Markov method

INTRODUCTION

System reliability and system availability have widely been studied because of their prevalence in power plants, manufacturing systems and industrial systems. Maintaining a high or required level of reliability and/or availability is often an essential request. In this chapter, we consider the manufacturing system or the power plant to be serial system with mixed standby (include cold and worm standby) components. A standby component is called a 'cold standby' if its failure rate is zero. The standby component is referred as 'warm standby' when the failure rate is nonzero and is less than the failure rate of a primary component. Primary, warm and cold components can be considered to be repairable.

The present study is differ from past work in that it presents a novel methodology to design a system configuration involving series and mixed standby components. The reliability characteristics of a system with M operating machines, S warm standby spares and R repairmen with exponential failure and exponential repair time distributions was investigated (Wang and Sivazlian, 1989). Srinivasan and Gopalan (1973) studied one on-line unit (operating machine) with general lifetime distribution, w Warm standbys with exponential failure and exponential repair time distributions based on only one assumption, namely, the system fails when no spares are available to replace the failed operating machine. Meng (1993) compared the MTTF of four series-parallel and parallel-series redundant system composed of 2n independent components, general ordering relations between four systems in terms of their MTTF are obtained.

In this study, we are going to study three different system configurations of series and mixed standby components. These configurations 1,2 are compared based on their reliability. In addition, for configuration 3 which is a special case, we are going to develop the explicit expressions for the mean time-to-failure MTTF and the steady-state availability $A_T(\infty)$ and to calculate the cost/benefit ratio (C/B) based on assumed numerical values given to the system parameters, as well as to the costs components.

ESTIMATION OF THE 2-PARAMETER WEIBULL DISTRIBUTION

The hazard function of a component following a 2-parameter Weibull distribution can be described by:

$$h(t) = \frac{\beta\, t^{\beta-1}}{\alpha^{\beta}}$$

The likelihood function for m items begin test at the same time by Farnum and Booth (1997) is:

$$\begin{aligned}
L &= \prod_{i=1}^{\tau} f(t_i) * \prod_{i=\tau+1}^{m} R(t_i) \\
&= \prod_{i=1}^{\tau} [\frac{\beta}{\alpha^{\beta}} t_i^{\beta-1}\, e^{-\left(\frac{t_i}{\alpha}\right)^{\beta}}] * \prod_{i=\tau+1}^{m} e^{-\left(\frac{t_i}{\alpha}\right)^{\beta}} \\
\ln L &= \tau \ln\beta - \beta\tau\ln\alpha + \sum_{i=1}^{\tau}(\beta-1)\ln t_i - \sum_{i=1}^{m} \left(\frac{t_i}{\alpha}\right)^{\beta}
\end{aligned}$$

The partial derivatives of the natural log of the likelihood function are:

$$\begin{aligned}
\frac{\partial \ln L}{\partial \beta} &= \frac{\tau}{\beta} - \tau\ln\alpha + \sum_{i=1}^{\tau}\ln t_i - \sum_{i=1}^{m}\left(\frac{t_i}{\alpha}\right)^{\beta}[\ln t_i - \ln\alpha] = 0 \\
\frac{\partial \ln L}{\partial \alpha} &= \frac{-\beta\tau}{\alpha} + \frac{\beta}{\alpha^{\beta-1}}\sum_{i=1}^{m} t_i^{\beta} = 0
\end{aligned} \tag{1}$$

$$\hat{\alpha} = \left(\frac{\sum_{i=1}^{m} t_i^{\beta}}{\tau}\right)^{\frac{1}{\beta}} \tag{2}$$

$$\ln\alpha = \frac{1}{\beta}\left[\ln\sum_{i=1}^{m} t_i^{\beta} - \ln\tau\right] \tag{3}$$

Substituting the results from Eq. 2, 3 into Eq. 1, then we have:

$$\begin{aligned}
\frac{\partial \ln L}{\partial \beta} &= \frac{\tau}{\beta} - \frac{\tau}{\beta}\left[\ln\sum_{i=1}^{m} t_i^{\beta} - \ln\tau\right] + \sum_{i=1}^{\tau}\ln t_i - \sum_{i=1}^{m}\left(\frac{t_i^{\beta}\tau}{\sum_{i=1}^{m} t_i^{\beta}}\right)\left[\ln t_i - \frac{1}{\beta}\left(\ln\sum_{i=1}^{m} t_i^{\beta} - \ln\tau\right)\right] \\
0 &= \frac{\tau}{\beta} + \sum_{i=1}^{\tau}\ln t_i - \tau\sum_{i=1}^{m}\left(\frac{t_i^{\beta}\ln t_i}{\sum_{i=1}^{m} t_i^{\beta}}\right) \\
\frac{\tau}{\beta} &= \tau\sum_{i=1}^{m}\left(\frac{t_i^{\beta}\ln t_i}{\sum_{i=1}^{m} t_i^{\beta}}\right) - \sum_{i=1}^{\tau}\ln t_i \\
\beta &= \left[\sum_{i=1}^{m}\left(\frac{t_i^{\beta}\ln t_i}{\sum_{i=1}^{m} t_i^{\beta}}\right) - \frac{1}{\tau}\sum_{i=1}^{\tau}\ln t_i\right]^{-1} = [h(\beta)]^{-1}
\end{aligned} \tag{4}$$

Table 1: Compute estimate of the parameters α and β for number of failures times t_i

Order failures times t_i For i = 1,2,...,10												
1	2	3	4	5	6	7	8	9	10	v	$\hat{\beta}=\frac{2}{V}$	$\hat{\alpha}=\left(\sum_{i=1}^{10}\frac{t_i^{\beta}}{10}\right)^{\frac{1}{\beta}}$
37	58	72	88	115	136	152	165	185	213	0.682	2.933	138.07
31	43	56	65	73	82	96	101	111	135	0.948	2.120	97.22
27	35	66	83	96	101	131	145	199	222	0.884	2.260	128.41
24	32	41	66	79	89	98	120	180	235	1.117	1.790	111.66
18	26	39	53	77	93	108	135	220	253	1.216	1.640	118.84

For censoring, t_i is a recorded failure time for $i\le\tau$ and $t_i = t_S$ for $\tau+1\le i\le m$, where t_S is the maximum test time for censoring, τ is the number of items that fail before t_S. When all t_i (i = 1,2,3..., m) are available, the data are complete; complete data are a special case of right concerning for $\tau = m$.

Our empirical investigations suggest that choosing:

$$v=\lim_{\beta\to\infty} h(\beta)$$

Then from Eq. 4, we have:

$$v=\ln t_s-\frac{1}{\tau}\sum_{i=1}^{\tau}\ln t_i \tag{5}$$

and:

$$\begin{aligned}\lim_{\beta\to 0} h(\beta) &= \frac{\sum_{i=1}^{\tau}\ln t_i+(m-\tau)\ln t_s}{m}-\frac{1}{\tau}\sum_{i=1}^{m}\ln t_i\\ &=\left[\ln t_s-\frac{1}{\tau}\sum_{i=1}^{m}t_i\right]-\frac{\tau}{m}\left[\ln t_s-\frac{1}{\tau}\sum_{i=1}^{\tau}t_i\right]\\ &=v-\frac{\tau}{m}v=\left(1-\frac{\tau}{m}\right)v\end{aligned} \tag{6}$$

Using Eq. 5-6 to obtain $\hat{\beta}$:

$$\hat{\beta}=\left[\frac{v+\left(\frac{1-\tau}{m}\right)v}{2}\right]^{-1}=\frac{2}{v\left(\frac{2-\tau}{m}\right)} \tag{7}$$

This approximation simplifies to $\hat{\beta}=\frac{2}{v}$.

Equation 7 provides a quick approximation to $\hat{\beta}$ and can be used as an initial estimate of $\hat{\beta}$ for iterative MLE routines. Table 1 presents compute estimate of the parameters α and β for number of failures times t_i.

DESCRIPTION OF THE SYSTEM

For the sake of discussion, we consider the requirements of a 10 MW power plant. We also assume that generators are available in units of both 10 and 5 MW. Standby generators are always necessary in case of failure. We assume that the switch is perfect (Wang and Kuo, 2000). We also assume that the switchover time from warm standby component to primary component, from cold standby component to warm standby component, from failure to repair, or from repair to cold standby component (or primary component if the system is short) is instantaneous. Primary components and warm standby components can be considered to be repairable by Wang *et al.* (2006) and Xie *et al.* (2004). Each of the primary components fails independently of the state of the others, has time-dependent failure rate $\lambda_1(t)$ and follow Weibull distribution with parameters θ_1, η_1. Whenever, one of the primary components fails, a warm standby moves into operation if any is moves into operation if any is available and a cold standby is put on warm standby state if any is available, we now assume that when a warm standby moves into a primary component state, its failure characteristic will be that of the primary component and when a cold standby moves into a warm standby state, its failure characteristic will be that of a warm standby. We assume that each of the available warm standby components fails independently of the state of all the others and has time-dependent failure rate $\lambda_2(t)$ and follow Weibull distribution with parameters θ_2, η_2. Whenever a primary component or a warm standby component fails, it is immediately repaired in the order of breakdowns with a time-to-repair which is exponentially distributed with parameter μ. Once a component is repaired, it is "as good as new", notice that a failed system is never repaired.

The following configurations are considered:

- The first configuration is a serial system of one primary 10 MW component, one warm standby 10 MW component and one cold standby 10 MW component
- The second configuration is a serial system of two primary 5 MW components, one warm standby 5 MW component and one cold standby 5 MW component
- The third configuration (a special case): of two primary 5 MW components, two warm stand by 5 MW components and one cold standby 5 MW component, with constant failure rate λ_1, λ_2 and constant repair rate μ

THE RELIABILITY OF THE SYSTEM

The state probability $P_j(t)$, for j = 0,1,2,3 can be viewed as a result of solving a set of four first order linear differential equations given by the following identity:

$$\frac{d_{P_j}(t)}{dt} = \dot{P}_j(t) = -_{P_j}(t)\sum_{\substack{i=0 \\ i\neq j}}^{3} a_{ji} + \sum_{\substack{i=0 \\ i\neq j}}^{3} P_i(t) a_{ij} \tag{8}$$

where, α_{ji} is the transition rate from state j to state i.

Calculations for configuration 1: For configuration 1, let $P_3(t)$ be the probability that exactly 3 components are working at time t, ($t \geq 0$). if we let P(t) denote the probability row vector at time t, then the initial conditions for this problem are:

$$P(0) = [P_3(0), P_2(0), P_1(0), P_0(0)] = [1, 0, 0, 0] \tag{9}$$

The system-state equations for a Markov model which is the set of the first-order linear differential equations given by:

$$\dot{P} = QP$$

The transition rate matrix Q for reliability according to configuration 1 is given by:

$$Q = \begin{pmatrix} -\lambda_1(t)-\lambda_2(t) & \mu & 0 & 0 \\ \lambda_1(t)+\lambda_2(t) & -\lambda_1(t)-\lambda_2(t)-\mu & 2\mu & 0 \\ 0 & \lambda_1(t)+\lambda_2(t) & -\lambda_1(t)-2\mu & 0 \\ 0 & 0 & \lambda_1(t) & 0 \end{pmatrix}$$

We will take the matrix Q and delete the rows and columns for the absorbing state. The new matrix is called $\Lambda(t)$:

$$\Lambda(t) = \begin{pmatrix} -\lambda_1(t)-\lambda_2(t) & \mu & 0 \\ \lambda_1(t)+\lambda_2(t) & -\lambda_1(t)-\lambda_2(t)-\mu & 2\mu \\ 0 & \lambda_1(t)+\lambda_2(t) & -\lambda_1(t)-2\mu \end{pmatrix}$$

We can write the system in the form:

$$\dot{P} = \Lambda(t).P(t) \tag{10}$$

where:

$$P(t) = \begin{bmatrix} P_3(t) \\ P_2(t) \\ P_1(t) \end{bmatrix}$$

To solve Eq. 10 with the initial condition:

$$P(t=0) = \begin{bmatrix} P_3(0) \\ P_2(0) \\ P_1(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

Multiplying both sides of Eq. 10, $e^{-\int_0^t \Lambda(s)ds}$ by then we have:

$$\frac{d}{dt}\left[e^{-\int_0^t \Lambda(s)ds}. P(t) \right] = 0 \tag{12}$$

and hence:

$$P(t) = e^{\int_0^t \Lambda(s)ds}. P(t=0) \tag{13}$$

Our problem now is how to determine the value of $e^{-\int_0^t \Lambda(s)ds}$:
So assume that:

$$D = \int_0^t \Lambda(s)ds \tag{14}$$

where:

$$D=\int_0^t \Lambda(s)\,ds=\begin{pmatrix}-H_1(t)-H_2(t) & \mu t & 0\\ H_1(t)+H_2(t) & -H_1(t)-H_2(t)-\mu t & 2\mu t\\ 0 & H_1(t)+H_2(t) & -H_1(t)-2\mu t\end{pmatrix}$$

and:

$$H_1(t)=\int_0^t \lambda_1(t)dt,\; H_2(t)=\int_0^t \lambda_2(t)dt$$

The method of solving we follow gives us the value of e^D by the following relation:

$$e^D=\alpha_0 I+\alpha_1 D+\alpha_2 D^2 \tag{15}$$

where, I is the identity matrix of rank 3 and α_0, α_1, α_2 are the parameters obtained from the solution of the following system:

$$e^{S_1}=\alpha_0+\alpha_1 S_1+\alpha_2 S_1^2 \tag{16}$$

$$e^{S_2}=\alpha_0+\alpha_1 S_2+\alpha_2 S_2^2 \tag{17}$$

$$e^{S_3}=\alpha_0+\alpha_1 S_3+\alpha_2 S_3^2 \tag{18}$$

where, S_1, S_2, S_3 are the characteristic roots of the matrix D. These roots are obtained from the characteristic equation g(s) of the matrix D given by:

$$g(s)=\begin{pmatrix}-H_1(t)-H_2(t)-s & \mu t & 0\\ H_1(t)+H_2(t) & -H_1(t)-H_2(t)-\mu t-s & 2\mu t\\ 0 & H_1(t)+H_2(t) & -H_1(t)-2\mu\, t-s\end{pmatrix} \tag{19}$$

By solving Eq 16-18, we have:

$$\alpha_0=\frac{S_2S_3(S_2-S_3)e^{S_1}+S_1S_3(S_3-S_1)e^{S_2}+S_1S_2(S_1-S_2)e^{S_3}}{(S_1-S_2)(S_1-S_3)(S_2-S_3)} \tag{20}$$

$$\alpha_1=\frac{(S_3^2-S_2^2)e^{S_1}+(S_1^2-S_3^2)e^{S_2}+(S_2^2-S_1^2)e^{S_3}}{(S_1-S_2)(S_1-S_3)(S_2-S_3)} \tag{21}$$

$$\alpha_2=\frac{(S_2-S_3)e^{S_1}+(S_3-S_1)e^{S_2}+(S_1-S_2)e^{S_3}}{(S_1-S_2)(S_1-S_3)(S_2-S_3)} \tag{22}$$

Now we can obtain the value of e^D from Eq. 15 and obtain the values of required states probabilities from Eq. 13 which are:

$$P_3(t)=\alpha_0-\alpha_1\left(H_1(t)+H_2(t)\right)+\alpha_2\left(\left(H_1(t)+H_2(t)\right)^2+\mu\, t\left(H_1(t)+H_2(t)\right)\right)$$
$$P_2(t)=\alpha_1\left(H_1(t)+H_2(t)\right)-\alpha_2\left(\left(H_1(t)+H_2(t)\right)^2+\left(H_1(t)+H_2(t)\right)\left(H_1(t)+H_2(t)+\mu\, t\right)\right)$$
$$P_1(t)=\alpha_2\left(H_1(t)+H_2(t)\right)$$

where:

$$H_i(t)=\left(\frac{t}{\theta_i}\right)^{\eta_i},\ i=1,2$$

The system reliability function of configuration 1 is:

$$R_1(t)=\sum_{i=1}^{3}P_i(t)=\alpha_0 \tag{23}$$

where:

$$S_1=\frac{\frac{1}{6}(36ab-108c-8a^3+12\sqrt{12b^3-3b^2a^2-54bac+81c^2+12ca^3})^{\frac{1}{3}}-6\left(\frac{1}{3}b-\frac{1}{9}a^2\right)}{(36ba-108c-8a^3+12\sqrt{12b^3-3b^2a^2-54bac+81c^2+12ca^3})^{\frac{1}{3}}-\frac{1}{3}a}$$

$$S_2,S_3=\frac{-\frac{1}{12}(36ab-108c-8a^3+12\sqrt{12b^3-3b^2a^2-54bac+81c^2+12ca^3})^{\frac{1}{3}}+3\left(\frac{1}{3}b-\frac{1}{9}a^2\right)}{(36ba-108c-8a^3+12\sqrt{12b^3-3b^2a^2-54bac+81c^2+12ca^3})^{\frac{1}{3}}-\frac{1}{3}a}$$

$$\pm\frac{1}{2}I\sqrt{3}=\left(\frac{\frac{1}{6}(36ab-108c-8a^3+12\sqrt{12b^3-3b^2a^2-54bac+81c^2+12ca^3})^{\frac{1}{3}}+6\left(\frac{1}{3}b-\frac{1}{9}a^2\right)}{(36ba-108c-8a^3+12\sqrt{12b^3-3b^2a^2-54bac+81c^2+12ca^3})^{\frac{1}{3}}}\right)$$

and:

$$a=3H(t)_1+2H_2(t)+3\mu t$$
$$b=3\left[H_1(t)\right]^2+\left[H_2(t)\right]^2+4H_1(t).H_2(t)+3\mu tH_1(t)+2\mu tH_2(t)+2\mu^2t^2$$
$$c=\left[H_1(t)\right]^3+2\left[H_1(t)\right]^2.H_2(t)+H_1(t).\left[H_2(t)\right]^2$$

with:

$$H_1(t)=\left(\frac{t}{97.22}\right)^{2.12},H_2(t)=\left(\frac{t}{111.66}\right)^{1.79},\mu=0.05$$

The relation between reliability and time as shown in Fig. 1.

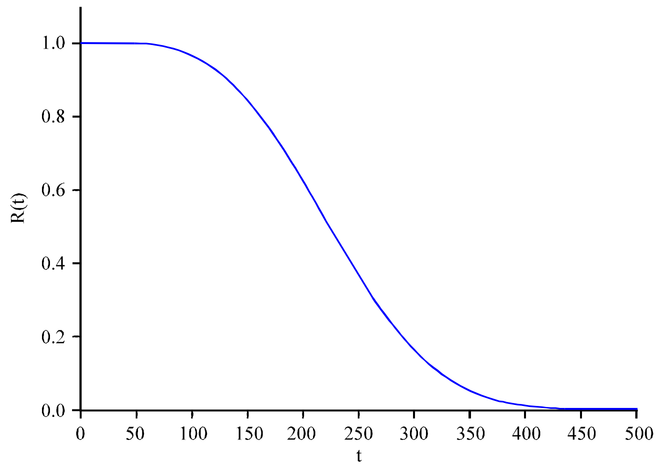

Fig. 1: Relationships between $R_1(t)$ and time in configuration 1

Calculations for configuration 2: Let $P_3(t)$ be the probability that exactly 4 components are working at time t, $(t\geq 0)$. if we let P(t) denote the probability row vector at time t, then the initial conditions for this problem are:

$$P(0) = [P_3(0), P_2(0), P_1(0), P_0(0)] = [1, 0, 0, 0] \tag{24}$$

The transition rate matrix Q for reliability according to configuration 2 is given by:

$$Q = \begin{pmatrix} -2\lambda_1(t) - \lambda_2(t) & \mu & 0 & 0 \\ 2\lambda_1(t) + \lambda_2(t) & -2\lambda_1(t) - \lambda_2(t) - \mu & 2\mu & 0 \\ 0 & 2\lambda_1(t) + \lambda_2(t) & -2\lambda_1(t) - 2\mu & 0 \\ 0 & 0 & 2\lambda_1(t) & 0 \end{pmatrix}$$

We will take the matrix Q and delete the rows and columns for the absorbing state. The new matrix is called $\Lambda(t)$:

$$\Lambda(t) = \begin{pmatrix} -2\lambda_1(t) - \lambda_2(t) & \mu & 0 \\ 2\lambda_1(t) + \lambda_2(t) & -2\lambda_1(t) - \lambda_2(t) - \mu & 2\mu \\ 0 & 2\lambda_1(t) + \lambda_2(t) & -2\lambda_1(t) - 2\mu \end{pmatrix}$$

We can write the system in the form:

$$\dot{P} = \Lambda(t).P(t) \tag{25}$$

where:

$$P(t) = \begin{bmatrix} P_3(t) \\ P_2(t) \\ P_1(t) \end{bmatrix}$$

To solve Eq. 25 with the initial condition:

$$P(t=0)=\begin{bmatrix}P_3(0)\\P_2(0)\\P_1(0)\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix} \tag{26}$$

We will solve Eq. 25 with the aid of the method used in the previous section and hence:

$$D=\int_0^t \Lambda(s)ds=\begin{pmatrix}-2H_1(t)-H_2(t) & \mu t & 0\\ 2H_1(t)+H_2(t) & -2H_1(t)-H_2(t)-\mu t & 2\mu t\\ 0 & 2H_1(t)+H_2(t) & -2H_1(t)-2\mu t\end{pmatrix}$$

and:

$$H_1(t)=\int_0^t \lambda_1(t)dt,\ H_2(t)=\int_0^t \lambda_2(t)dt$$

Here the value of e^D will be given by the same relation which is:

$$e^D=\alpha_0 I+\alpha_1 D+\alpha_2 D^2 \tag{27}$$

where, I is the identity matrix of rank 3 and α_0, α_1, α_2 are the parameters obtained from the solution of the following system:

$$e^{r_1}=\alpha_0+\alpha_1 r_1+\alpha_2 r_1^2 \tag{28}$$

$$e^{r_2}=\alpha_0+\alpha_1 r_2+\alpha_2 r_2^2 \tag{29}$$

$$e^{r_3}=\alpha_0+\alpha_1 r_3+\alpha_2 r_3^2 \tag{30}$$

where, r_1, r_2, r_3 are the characteristic roots of the matrix D. these roots are obtained from the characteristic equation g(s) of the matrix D given by:

$$g(s)=\begin{pmatrix}-2H_1(t)-H_2(t)-r & \mu t & 0\\ 2H_1(t)+H_2(t) & -2H_1(t)-H_2(t)-\mu t-r & 2\mu t\\ 0 & 2H_1(t)+H_2(t) & -2H_1(t)-2\mu t-r\end{pmatrix} \tag{31}$$

By solving Eq. 28-30, we have:

$$\alpha_0=\frac{r_2r_3(r_2-r_3)e^{r_1}+r_1r_3(r_3-r_1)e^{r_2}+r_1r_2(r_1-r_2)e^{r_3}}{(r_1-r_2)(r_1-r_3)(r_2-r_3)} \tag{32}$$

$$\alpha_1 = \frac{(r_3^2 - r_2^2)e^{r_1} + (r_1^2 - r_3^2)e^{r_2} + (r_2^2 - r_1^2)e^{r_3}}{(r_1 - r_2)(r_1 - r_3)(r_2 - r_3)} \tag{33}$$

$$\alpha_2 = \frac{(r_2 - r_3)e^{r_1} + (r_3 - r_1)e^{r_2} + (r_1 - r_2)e^{r_3}}{(r_1 - r_2)(r_1 - r_3)(r_2 - r_3)} \tag{34}$$

Now we can obtain the value of e^D from Eq. 27 and obtain the values of required states probabilities from Eq. 25 which are:

$$P_3(t) = \alpha_0 - \alpha_1(2H_1(t) + H_2(t)) + \alpha_2\left((2H_1(t) + H_2(t))^2 + \mu\, t(2H_1(t) + H_2(t))\right)$$
$$P_2(t) = \alpha_1(2H_1(t) + H_2(t)) - \alpha_2\left((2H_1(t) + H_2(t))^2 + (2H_1(t) + H_2(t))(2H_1(t) + H_2(t) + \mu\, t)\right)$$
$$P_1(t) = \alpha_2(2H_1(t) + H_2(t))$$

where:

$$H_i(t) = \left(\frac{t}{\theta_i}\right)^{\eta_i},\ i = 1, 2$$

The system reliability function of configuration 2 is:

$$R_2(t) = \sum_{i=1}^{3} P_i(t) = \alpha_0 \tag{35}$$

where:

$$r_1 = \frac{\frac{1}{6}(36ab - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{\frac{1}{3}} - 6\left(\frac{1}{3}b - \frac{1}{9}a^2\right)}{(36ba - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{\frac{1}{3}} - \frac{1}{3}a}$$

$$r_2, r_3 = \frac{-\frac{1}{12}(36ab - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{\frac{1}{3}} + 3\left(\frac{1}{3}b - \frac{1}{9}a^2\right)}{(36ba - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{\frac{1}{3}} - \frac{1}{3}a}$$

$$\pm\frac{1}{2}I\sqrt{3} = \left(\frac{\frac{1}{6}(36ab - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{\frac{1}{3}} + 6\left(\frac{1}{3}b - \frac{1}{9}a^2\right)}{(36ba - 108c - 8a^3 + 12\sqrt{12b^3 - 3b^2a^2 - 54bac + 81c^2 + 12ca^3})^{\frac{1}{3}}}\right)$$

and:

$$a = 6H(t)_1 + 2H_2(t) + 3\mu t$$
$$b = 12[H_1(t)]^2 + [H_2(t)]^2 + 8H_1(t).H_2(t) + 6\mu t H_1(t) + 2\mu t H_2(t) + 2\mu^2 t^2$$
$$c = 8[H_1(t)]^3 + 8\ [H_1(t)]^2.H_2(t) + 2H_1(t).[H_2(t)]^2$$

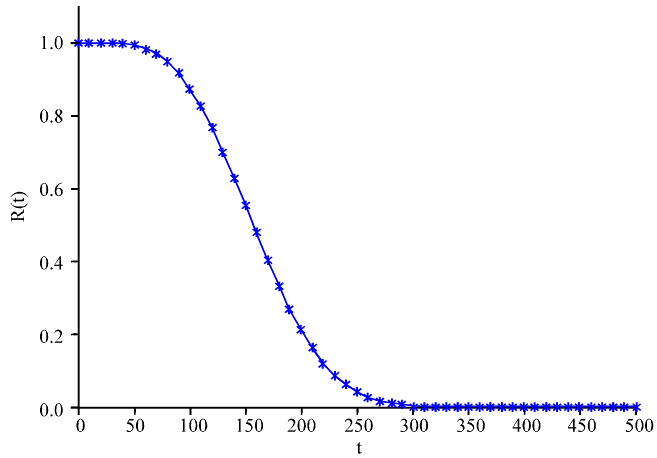

Fig. 2: Relationships between $R_2(t)$ and time in configuration 2

with:

$$H_1(t)=\left(\frac{t}{97.22}\right)^{2.12}, H_2(t)=\left(\frac{t}{111.66}\right)^{1.79}, \mu=0.05$$

The relation between reliability and time as shown in Fig. 2.

Calculations for configuration 3

Mean time to failure of the system: For configuration 3, let $P_4(t)$ be the probability that exactly 5 components are working at time t, $(t\geq 0)$. if we let P(t) denote the probability row vector at time t, then the initial conditions for this problem are:

$$P(0)=[P_4(0), P_3(0), P_2(0), P_1(0), P_0(0)]=[1,0,0,0,0] \quad (36)$$

where, the transition rate matrix Q for reliability according to configuration 3 is given by:

$$Q=\begin{pmatrix} -2\lambda_1-2\lambda_2 & \mu & 0 & 0 & 0 \\ 2\lambda_1+2\lambda_2 & -2\lambda_1-2\lambda_2-\mu & 2\mu & 0 & 0 \\ 0 & 2\lambda_1+2\lambda_2 & -2\lambda_1-\lambda_2-2\mu & 3\mu & 0 \\ 0 & 0 & 2\lambda_1+\lambda_2 & -2\lambda_1-3\mu & 0 \\ 0 & 0 & 0 & 2\lambda_1 & 0 \end{pmatrix}$$

To evaluate the transient solution is too complex. Therefore, we will restrict ourselves in calculating the MTTF. Therefore, we will take the transpose matrix of Q and delete the rows and columns for the absorbing state. The new matrix is called A:

$$A=\begin{pmatrix} -2\lambda_1-2\lambda_2 & 2\lambda_1+2\lambda_2 & 0 & 0 \\ \mu & -2\lambda_1-2\lambda_2-\mu & 2\lambda_1+2\lambda_2 & 0 \\ 0 & 2\mu & -2\lambda_1-\lambda_2-2\mu & 2\lambda_1+\lambda_2 \\ 0 & 0 & 3\mu & -2\lambda_1-3\mu \end{pmatrix} \quad (37)$$

The expected time to reach an absorbing state is calculated from:

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)(-A^{-1})\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \tag{38}$$

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTTF_3 = \frac{\lambda_1\left(3\mu^3 + 6\mu^2\lambda_2 + 8\mu^2\lambda_1 + (6\mu + 4\lambda_2)\lambda_2^2 + (18\mu + 24\lambda_2)\lambda_2\lambda_1 + (12\mu + 36\lambda_2 + 16\lambda_1)\lambda_1^2\right)}{8(\lambda_1 + \lambda_2)^3} \tag{39}$$

Availability analysis of the system: For the availability case of configuration 3, we will use the initial condition initial conditions for this problem from Eq. 36:

The deferential equations form can be expressed as:

$$\begin{pmatrix}\dot{P}_4(t)\\\dot{P}_3(t)\\\dot{P}_2(t)\\\dot{P}_1(t)\\\dot{P}_0(t)\end{pmatrix} = \begin{pmatrix}-2\lambda_1 - 2\lambda_2 & \mu & 0 & 0 & 0\\ 2\lambda_1 + 2\lambda_2 & -2\lambda_1 - 2\lambda_2 - \mu & 2\mu & 0 & 0\\ 0 & 2\lambda_1 + 2\lambda_2 & -2\lambda_1 - \lambda_2 - 2\mu & 3\mu & 0\\ 0 & 0 & 2\lambda_1 + \lambda_2 & -2\lambda_1 - 3\mu & 4\mu\\ 0 & 0 & 0 & 2\lambda_1 & -4\mu\end{pmatrix}.\begin{pmatrix}P_4(t)\\P_3(t)\\P_2(t)\\P_1(t)\\P_0(t)\end{pmatrix}$$

The steady state availability can be obtained using the following procedure. In the steady state, the derivatives of the state probabilities become zero. That allows us to calculate the steady state probabilities with:

$$A_T(\infty) = 1 - P_0(\infty) \tag{40}$$

and $QP(\infty) = 0$. Or, in the matrix form:

$$\begin{pmatrix}-2\lambda_1 - 2\lambda_2 & \mu & 0 & 0 & 0\\ 2\lambda_1 + 2\lambda_2 & -2\lambda_1 - 2\lambda_2 - \mu & 2\mu & 0 & 0\\ 0 & 2\lambda_1 + 2\lambda_2 & -2\lambda_1 - \lambda_2 - 2\mu & 3\mu & 0\\ 0 & 0 & 2\lambda_1 + \lambda_2 & -2\lambda_1 - 3\mu & 4\mu\\ 0 & 0 & 0 & 2\lambda_1 & -4\mu\end{pmatrix}.\begin{pmatrix}P_4(\infty)\\P_3(\infty)\\P_2(\infty)\\P_1(\infty)\\P_0(\infty)\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} \tag{41}$$

Using the following normalization condition:

$$\sum_{i=0}^{4} P_i(\infty) = 1 \tag{42}$$

We substitute Eq. 42 in any one of the redundant rows in Eq. 41 to yield:

Table 2: The size-proportional cost for the components

Component	Cost (in $)
Primary 5 MW	5E+6
Warm standby 5 MW	3E+6
Cold standby 5 MW	2E+6

$$\begin{pmatrix} -2\lambda_1-2\lambda_2 & \mu & 0 & 0 & 0 \\ 2\lambda_1+2\lambda_2 & -2\lambda_1-2\lambda_2-\mu & 2\mu & 0 & 0 \\ 0 & 2\lambda_1+2\lambda_2 & -2\lambda_1-\lambda_2-2\mu & 3\mu & 0 \\ 0 & 0 & -2\lambda_1-\lambda_2 & -2\lambda_1-3\mu & 4\mu \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} P_4(\infty) \\ P_3(\infty) \\ P_2(\infty) \\ P_1(\infty) \\ P_0(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{43}$$

Solution of Eq. 43 provides the steady-state probabilities in the availability case. The explicit expression for $A_T(\infty)$ is given by:

$$A_T(\infty) = \frac{2\mu(3\mu^3 + (6\mu^2\lambda_2 + (6\mu + 2\lambda_2)\lambda_2^2 + (6\mu^2 + (12\mu + 8\lambda_2)\lambda_2 + (6\mu + 10\lambda_2 + 4\lambda_1)\lambda_1)\lambda_1}{(6\mu^4 + 12\mu^3\lambda_2 + (12\mu^2 + \mu\lambda_2)\lambda_2^2 + (12\mu^3\lambda_1 + (24\mu^2\lambda_2\lambda_1 + (10\mu^2 + 2\lambda_2)\lambda_2^2\lambda_1 + 12\mu^2\lambda_1^2 + (17\mu + 8\lambda_2)\lambda_2\lambda_1^2 + (8\mu + 10\lambda_2 + 4\lambda_1)\lambda_1^3} \tag{44}$$

Cost/benefit ratio: The notion of cost-benefit analysis is simple in principle. We assume the size-proportional cost for the primary components, warm standby components and cold standby components, respectively, shown in Table 2 with this we calculate the costs for configuration 3. It utilizes the cost/benefit ratio (C/B) as a means to rank alternatives, let:

C = The cost for the configuration 3
B = The MTTF of configuration 3
B = The $A_T(\infty)$ of configuration 3

The cost for configuration 3 (where there is two primary components, two warm standby components and one cold standby component= 18E+6$).

A numerical illustration is provided by considering the following parameters:

$\lambda_1 = 0.6$
$\lambda_2 = 0.05$
$\lambda_3 = 1.0$

Given these values, we can calculate for configuration 3:

- cost/MTTF = 1.39E+6
- cost/$A_T(\infty)$=18.5E+6

CONCLUSION

We have provided in this paper, the reliability of two configurations, when the components have time-dependent failure rate and a constant repair rate. By comparing the R(t) in both

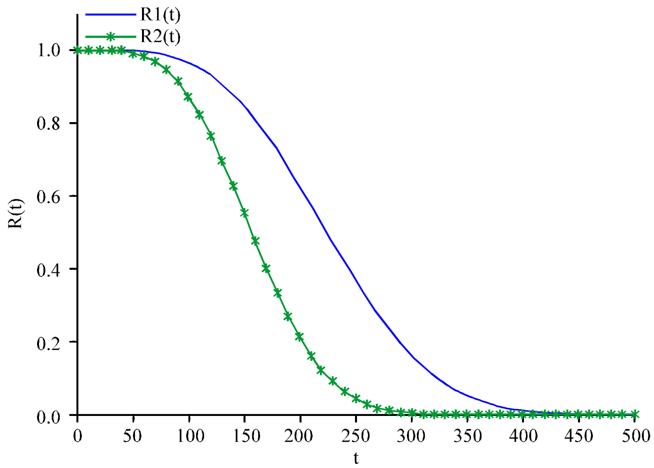

Fig. 3: Relationships between R(t) and time in configuration 1 and 2

configurations, we can see that in the first configuration the reliability is higher than reliability in second configuration as shown in Fig. 3.

Moreover, from numerical results for the cost/benefit measure have been obtained for configuration 3 (special case), we have provided a systematic methodology to develop the mean time to system failure and the steady-state availability of series system with mixed standby components. By comparing the cost/MTTF is smaller than cost/$A_T(\infty)$. Consider the configurations in [6], we have the configuration 3 gives small cost/MTTF than the cost/MTTF by configuration (two primary 5 MW components, one warm standby 5 MW component and one cold standby 5 MW component) and the configuration 3 gives smallest cost/$A_T(\infty)$ than cost/$A_T(\infty)$ by configurations (one primary 10 MW component, one warm standby 10 MW component and one cold standby 10 MW component), (one primary 10 MW component, one warm standby 10 MW component and two cold standby 10 MW components) and (one primary 10 MW component, two warm standby 10 MW components and one cold standby 10 MW component).

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