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The Numerical Solution of Burger's Equation Arising into the Irradiation of Tumour Tissue in Biological Diffusing System by Homotopy Analysis Method

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ABSTRACT

The fundamental Burger's equation's solution has been found by Homotopy Analysis Method (HAM). Here it is assumed the term 'm' the rate of consumption of oxygen per unit volume of the medium, is directly propositional to the rate of change of concentration with respect to 'x'. It is also propositional to the concentration, for moving boundary case. Finally it has been concluded that number of biochemical and medical problem like irradiation of tumour tissue which will be helpful for killing effectiveness of the radiation on cancerous cells.

Key words: Burger's equation, irradiation tumour diffusing, Homotopy analysis method, series solution

INTRODUCTION

The non-dimensional form arise in an oxygen diffusing problem has great importance in biochemical and medical situations. One such situation arises in research into the irradiation of tumour tissue. Governing non-linear partial differential equation has been converted in the form of burger's equation. We have assumed the term 'm' as the rate of consumption of oxygen per unit volume of the medium. It is directly propositional to the rate of change of concentration with respect to 'x'. It is also propositional to the concentration.

The solution of the burger's equation had been given by homotopy analysis method. The HAM is developed in 1992 (Liao, 1992, 1999, 2003a, b, 2004, 2005). Mittal and Singhal (1993) have used a technique of finitely reproducing nonlinearities to obtain a set of stiff ordinary differential equations which are solved by Ruge-Kutta Chebyshev method. Kimiaeifar *et al.* (2009) have used analytical approach and He's Parameter-Expanding Method in 2009. Kelleci and Yildirim (2011) have found solution of coupled Burger's equation using concept of HPM and Pade equation with numerical method for PDE. He (2000). has worked on coupling method of HAM in 2000. Abbasbandy (2007) has used HAM for heat radiation equations and on generalized Hirota-Satsuma coupled Kdv equation. He (2004) has shown the comparison study of HPM and HAM in 2004. Ayub *et al.* (2003) have been worked on porous plate with third grade fluid with HAM.

The HAM contains the auxiliary parameter h, which provides here a single way to adjust and control the convergence region of solution series for large values of x and t. Other numerical methods have given low degree of accuracy for large values of x and t. Therefore, the HAM handles linear and non-linear problem without any assumption and restriction.

MATHEMATICAL BACKGROUND OF THE PROBLEM

The diffusion of oxygen in tissue which simultaneously consumes oxygen occurs in a number of biochemical and medical situations. One such situation arises in research into the irradiation of tumour tissue. The killing effectiveness of the radiation on cancerous cells depends, among other factors, on the oxygen content of the cells. The need to interpret experimental measurements has lead to a mathematical formulation.

The diffusion-with absorption process is represented by the equation

$$\frac{\partial C}{\partial T} = D\frac{\partial^2 C}{\partial X^2} - m \tag{1}$$

With appropriate condition:

$$\begin{aligned} &C(0,T) = C_0 \\ &\frac{\partial C}{\partial X} = C_1, \text{for } X = 0 \text{ and } T > 0 \end{aligned} \tag{2}$$

In this problem, oxygen is allowed to diffusive into a medium and some of the oxygen is absorbed and there by removed from the diffusion process.

The oxygen concentration as the surface of the medium is maintained constant. Therefore the diffusion with absorption process is represented by the equation is (1).

Where, C (X, T) denotes the concentration of the oxygen from the surface time T, D is the diffusion constant. m is the rate of consumption of oxygen per unit volume of the medium is also assumed constant for steady state. It is assume directly propositional to the concentration and the rate of change of concentration with respect to X for moving boundary case. The problem is divided into two parts.

Steady state: During the initial phase, when oxygen is entering through the surface, the boundary condition there is $C = C_0$, X = 0, T = 0.

Where C_0 is constant.

A steady state is ultimately achieved in which:

$$\frac{\partial C}{\partial T} = 0$$

Every where when both the concentration and its space derivative are zero at a point $X = X_0$. No oxygen diffusion beyond this point and

$$C = \frac{\partial C}{\partial X} = 0, X \geq X_0$$

The required solution in the steady state is easily found to be

$$C = \frac{m}{2D}\left(X - X_0\right)^2 \tag{3}$$

Where:

$$X_0 = \sqrt{\frac{2DC_0}{m}}$$

Moving boundary problem: In the second phase of problem,

$$m = C\frac{\partial C}{\partial X} \tag{4}$$

So,

$$\frac{\partial C}{\partial T} + C\frac{\partial C}{\partial X} - \varepsilon\frac{\partial^2 C}{\partial X^2} = 0$$

Where:

$$\varepsilon = D \tag{5}$$

With boundary conditions:

$$C(0,T) = C(1,T) = 0 \qquad T > 0 \tag{6}$$

And the initial condition:

$$C(X,0) = \pi X \tag{7}$$

The Eq. 5 is the non-linear burger's equation for the biological diffusing problem together with boundary condition.

BASIC IDEA OF HAM

The concept of HAM (Liao, 1992) have been applied to the burgers' equation with boundary and initial conditions. The following differential equation has been considered:

$$N\,[C\,(X, T)] = 0 \tag{8}$$

Where N is a non linear operator for this problem X and T denote independent variables, C(X,T) is an unknown function, for simplicity, we ignore here all boundary and initial conditions, which can be treated in the similar way. By means of the HAM, one first constructs zero-order deformation equation:

$$(1-p)£[\varphi(X,T,p)\text{-}C_0(X,T)] = phN\left[\phi(X,T,p)\right] \tag{9}$$

where, £ is an auxiliary linear operator is C_0 (X, T) an initial guess. $h \neq 0$ is an auxiliary parameter. $p\in[0, 1]$ is the embedding parameter. when p = 0 and p = 1, it holds.

$$\varphi(X,T,0) = C_0(X,T) \quad \varphi(X,T,1) = C(X,T) \tag{10}$$

The solution ρ (X, T, p) varies from the initial guess C_0 (X, T) to the solution C (X, T). Liao (1992, 1999, 2003a, b, 2004, 2005) expanded (X, T, p) in Taylor's series about the embedding parameter,

$$\varphi(X,T,p) = C_0(X,T) + \sum_{m=1}^{+\infty} C_m(X,T)p^m \tag{11}$$

Where:

$$C_m(X,T) = \frac{1}{m!}\frac{\partial^m \varphi(X,T;p)}{\partial p^m} \text{ at} p = 0 \tag{12}$$

The convergence of the series (11) depends upon the auxiliary parameter h. If it is convergent at p = 1, one has:

$$C(X,T) = C_0(X,T) + \sum_{m=1}^{+\infty} C_m(X,T) \tag{13}$$

Define the vectors:

$$\overrightarrow{C_n} = [C_0(X,T), C_1(X,T), \_\_\_\_\_\_\_ C_n(X,T)]$$

Differentiating the zeroth-order deformation Eq. 9 m-times with respect to p and then diving them by m! And finally setting p = 0, we get the following m th order deformation equation:

$$£\left[C_m(X,T) - \chi_m C_{m-1}(X,T)\right] = hR_m\left(\overline{C}_{m-1}\right) \tag{14}$$

Where:

$$R_m\left(\overline{C}_{m-1}\right) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N\left[\varphi(X,T;p)\right]}{\partial p^{m-1}} \text{at } p = 0 \tag{15}$$

And

$$\chi_m = \begin{cases} 0, & m \le 1 \\ 1, & m > 1 \end{cases} \tag{16}$$

It should be emphasized C (X, T) for m≥1 is governed by the linear Eq. 14 with the linear boundary conditions that come from original problem, which can be easily solved by symbolic computation software such as maple and mathematica.

ANALYSIS OF THE METHOD BY THE HAM

The HAM has been applied to Eq. 5 to illustrate the strength of the method and to establish exact solutions for this problem. Choosing the linear operator:

$$£[\varphi(X,T;p)] = \frac{\partial \varphi(X,T;p)}{\partial T} \tag{17}$$

With the properly:

$$£[C'] = 0$$

where, C' is a constant, so a non linear operator can be defined as:

$$N[\varphi(X,T;p)] = \frac{\partial \varphi(X,T;p)}{\partial T} + \varphi(X,T;p)\frac{\partial \varphi(X,T;p)}{\partial X} - v\frac{\partial^2 \varphi(X,T;p)}{\partial X^2} \tag{18}$$

Using above definition, the zeroth-order deformation equation can be constructed as

$$(1-p)£[\varphi(X,T;p) - C_0(X,T)] = phN[\varphi(X,T;p)]$$

For p = 0 and p = 1, It can be written as:

$$\begin{cases} \varphi(X,T;0) = C_0(X,T) = C(X,0) \\ \varphi(X,T;1) = C(X,T) \end{cases} \tag{19}$$

Thus, the m th- order deformation equations can be obtained.

$$£[C_m(X,T) - \chi_m C_{m-1}(X,T)] = hR_m(\bar{C}_{m-1})$$

Where:

$$R_m(\bar{C}_{m-1}) = \frac{\partial \varphi_{m-1}(X,T;p)}{\partial T} + \sum_{n=0}^{m-1} \varphi_n(X,T;p)\frac{\partial \varphi_{m-1-n}}{\partial X} - v\frac{\partial^2 \varphi_{m-1}(X,T;p)}{\partial X^2} \tag{20}$$

Now the solution of the mth-order deformation Eq. 20 for m≥1 become

$$C_m(X,T) = \chi_m C_{m-1}(X,T) + h\pounds^{-1}\left[R_m\left(\overline{C}_{m-1}\right)\right] \tag{21}$$

So a few terms of series solution are as follows:

$$C_0(X,T) = n\pi X$$

$$C_1(X,T) = n^2 hT\pi X\left[1 - n^2\frac{\pi X^2}{2} + U\pi\right]$$

$$C_2(X,T) = \frac{1}{4}\pi hT\left[4\left(1+h+3h\pi^2+v\right)\left(1-n^2\frac{\pi X^2}{2}\right) + \pi\left(4v + h\left(T+4v+2\pi^2+v^2\right) + 3hT\left(1-n^2\frac{\pi X^2}{2}\right)(n\pi X)\right)\right]$$

And hence the form of C (X, T) is given by:

$$C(X,T) = \frac{2\pi v\sum_{n=1}^{\infty} a_n e^{-n^2\pi^2 vT} n^2\pi X}{a_0 + \sum_{n=1}^{\infty} a_n e^{-n^2\pi^2 vT}\left(1 - \frac{n^2\pi X^2}{2}\right)} \tag{23}$$

Where:

$$a_0 = \int_0^1 \exp\left[-(2\pi v)^{-1}\left(1 - \left(1 - \frac{n^2\pi X^2}{2}\right)\right)\right]dX$$

And

$$a_n = 2\int_0^1 \exp\left[-(2\pi v)^{-1}\left(1 - \left(1 - \frac{n^2\pi X^2}{2}\right)\right)\right]\left(1 - \frac{n^2\pi X^2}{2}\right)dX$$

CONCLUSION

The solution (3) and (23) represent the concentration of oxygen free to diffuse at a distance from the outer surface at time T in case of steady-state and moving boundary, respectively.

The solution will be helpful to control the diffusion of oxygen in tissue which simultaneously consumes oxygen occurs in a number of biochemical and medical problem like irradiation of tumour tissue, which will be helpful to killing effectiveness of the radiation on cancerous cells depends among other factors on the oxygen content of the cells.

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