ISSN 1996-3343

Asian Journal of Applied Sciences

Knowledgia SCIENTIFIC
A Place to Publish Outstanding Research

Asian Journal of Applied Sciences 8 (4): 240-258, 2015
ISSN 1996-3343 / DOI: 10.3923/ajaps.2015.240.258

Uncertainty and Sensitivity Analysis of Cyclic Creep Models of Concrete

Hem Bahadur Motra
Research Training Group GRK 1462, Assessment of Coupled Experimental and Numerical Partial Models in Structural Engineering, Bauhaus University Weimar, Berkaer Str. 9, Weimar, 99425, Germany

ABSTRACT

This study presents a methodology for uncertainty quantification in cyclic creep analysis. The BP model, the Whaley and Neville model, the modified MC90 for cyclic loading and the modified Hyperbolic function for cyclic loading are used for uncertainty quantification. Three types of uncertainty are included in Uncertainty Quantification (UQ): Natural variability in loading materials properties, data uncertainty due to measurement errors and modeling uncertainty and errors during cyclic creep analysis. With the consideration of all type of uncertainties, a complete measure for the total variation of the model response can be achieved. This study finds that the BP model performs the best for cyclic creep prediction followed by the modified Hyperbolic model and modified MC90 model. Furthermore, a global Sensitivity Analysis (SA) that considers the uncorrelated and correlated parameters is used to quantify the contribution of each source of uncertainty to the overall uncertainty of the prediction as well as to identifying the important parameters. The errors in determining the input quantities and the model itself can produce significant changes in creep prediction values. The influence of the variability of the input of random quantities on the cyclic creep is studied by means of a stochastic uncertainty and sensitivity analysis. The Latin Hypercube Sampling (LHS) is used in the stochastic sensitivity analysis, from which it has been determined that the cyclic creep deformation variability is influenced the most by the Elastic Modulus of concrete, then the compressive strength, mean stress, cyclic stress amplitude and number of cycle, with decreasing influence.

Key words: Cyclic creep, input variables, stochastic, sensitivity analysis, uncertainty quantification

INTRODUCTION

The creep of concrete under a sustained static load is a well-known phenomenon and much research has been carried out in this context (Neville *et al*., 1983; Bazant, 1988). Under actual operating conditions, many structures are subjected to dynamic loads in addition to the static loads. The effect of traffic loads on bridges and pavement, vibrating machinery on floor systems, wave loads on offshore structures and wind load on slender buildings are familiar examples. Such structures experience repeated loads and must be designed to control deformation due to both the static and dynamic creep. Numerous researchers have found an increase in cyclic creep when the concrete is subjected to cyclic loading. It is important to keep in mind that cyclic creep is measured relative to the creep under sustained load equal to the mean cyclic stress and not the creep under a sustained load equal to the upper cyclic stress (Neville *et al*., 1983; Motra *et al*., 2013a, 2014b). Also, time-dependent nonlinearity also grows during cyclic loading especially under higher strains, thus cyclic creep is a nonlinear phenomenon. Many studies have examined the stress-strain behaviour of drying and confined concrete under cyclic compression and tension and numerous concrete models have been proposed but very few studies addressed the long term time-dependent behaviour of concrete under cyclic load. Since the first works attempting to characterize the behaviour of concrete under a rapidly fluctuating (1 Hz) stress of given duration were published which (Probst, 1931, 1933) a significant research effort has been devoted to that field and has found the irreversible deformation to increase with the number of cycles. There is a decrease in non-elastic deformation with an increase in the age of the concrete during the application of cyclic load, this behaviour is similar to that under static loading. Many others mathematical and experimental models have been documents in the literatures like Ross (1937), Lorman (1940), Neville *et al*. (1983), Bazant and Panula (1979), CEB (1990a), Bazant and Kim (1992), Terje and Gordana (1992) and Schwabach (2005) model.

Much research has been carried out to investigate the uncertainties of the time-dependent behaviour of plain concrete under sustained loading (Bazant *et al*., 2012a; Keitel and Dimmig-Osburg, 2010; Yang, 2007a) but less work has been done on concrete under cyclic loading. Some work has been carried out on the cyclic creep and its structural effect by Bazant *et al*. (2012b) and Yu *et al*. (2012). Significant efforts have continuously been inputted into the study about the uncertainties in creep and shrinkage effects. The external or parameters uncertainty and internal (model uncertainty, measurement uncertainty and uncertainty of the creep phenomenon) uncertainty were referenced from Smith and Goodyear (1988), Bjerager and Krenk (1989), CEB (1990b), ACI (1992), Li and Melchers (1992), Tsubaki (1993), Bazant and Baweja (1995), Teply *et al*. (1996), Yang (2005) and Adam and Re da Taha (2011). Uncertainty analysis of creep models under sustained loading with the use of the Latin Hypercube Sampling was proposed (Bazant and Liu, 1985; Keitel *et al*., 2014); however, most of the existing importance analysis techniques assume input variables independence, while a few studies have focused on the importance analysis of correlated input variables and degradation materials behaviour under cyclic loading (Friedhelm *et al*., 2009), which is the common case in concrete structures.

Uncertainty in cyclic modeling has been classified by Madsen and Bazant (1983) into three categories: Parameter uncertainty, measurement uncertainty and model uncertainty. The uncertainties introduced by the model structure and parameterization have received much attention in recent years (Madsen and Bazant, 1983; Al-Manaseer and Ristanovic, 2005; Yang, 2007b; Keitel and Dimmig-Osburg, 2010; Keitel *et al*., 2014; Keitel, 2011; Pan *et al*., 2011; Sankararaman *et al*., 2011; Motra *et al*., 2013a, 2014a). Model uncertainty arises from incomplete understanding of the phenomenon being modeled and/or the inability to accurately reproduce creep phenomenon with mathematical and statistical techniques. In contrast, parameter uncertainty results from deficient knowledge of parameter values, ranges, physical meaning and temporal and spatial variability. But parameter uncertainty is also reflected in the incomplete model representation of the creep phenomenon (model uncertainty) and the inadequacies of parameter estimation techniques in light of uncertain and often limited, measured data. Different UQ and SA techniques will perform better for specific type of models. One method of importance measure of models is to consider the uncorrelated and correlated parameters, proposed by Xu and Gertner (2008), Most (2012) and Motra *et al*. (2013b). The distinction between uncorrelated and correlated contribution of uncertainty for an individual variable is very important and the output response and input variables are approximately linear in this method. One of the most important and basic concepts is that the results of any scientific experiment always has a degree of uncertainty which is known as experimental uncertainty. Although the uncertainty inherent in measured data used to calibrate and validate model predictions is commonly acknowledged, measurement uncertainty is rarely included in the evaluation of model performance. One reason for this omission is the lack of data on the uncertainty inherent in measured cyclic creep data.

The problem of quantifying the contribution of systematic error and measurement uncertainty considered for the calculation of the uncertainty. In fact, since its first edition (ISO, 1993) of the Guide to the Expression of the Uncertainty in Measurement (GUM) and in the latest one (ISO, 2008), the GUM attempts to completely set aside the concepts of the true value and measurement errors, whose connection with measurement uncertainty is considered (Clause E.5.1). GUM uncertainties are standard deviation of probability distribution and as a degree of belief, quantified by means of a subjective probability distribution (Clause 3.3.5). The GUM Supplement 1 (JCGM, 2008) is based on the general concept of propagating Probability Density Function (PDF) where, in order to obtain the PDF for the measured quantities, the Monte Carlo Method (MCM) (Robert and Casella, 2004) was suggested. Consequently, the law of propagation of uncertainties is based on a construction of a linear approximation of the model function (Wubbeler *et al*., 2008). The GUM uncertainty framework-GUF (JCGM, 2008) and MCM are approximate methods where the first method is more precise than the second one. Apart from that MCM is more valid than the GUF for large class of problems (JCGM, 2008).

In this study, a statistical framework is discussed for the cyclic creep function. As a first step, the four cyclic creep models in plain concrete are discussed briefly: BP model (Bazant and Panula, 1979; Bazant and Kim, 1992), modified MC90/EC2 (CEB, 1990c; Terje and Gordana, 1992), Whaley and Neville model (Whaley and Neville, 1973) and modified Hyperbolic function (Ross, 1937; Schwabach, 2005). Subsequently, the influences of input parameters are discussed in step 2. The Monte Carlo simulation with Latin Hypercube Sampling (LHS) technique is used for determining the parameter uncertainty, SA, measurement, phenomenon and model uncertainties which is explained in step 3. In step 4, the overview of UQ and SA (Xu and Gertner, 2008; Most, 2012) and measurement UQ according to GUM (Monte Carlo) methods are explained. Further, stochastic UQ and SA, are used to determine the uncertainty level of different models and analyse the quality of model and to what degree the randomness of an input quantity influences the variability of the output. The present study has considered the amount of degradation with respect of both strength and stiffness of the concrete.

MATERIALS AND METHODS

Cyclic creep models: Several experimental and mathematical models have been developed for estimating cyclic creep strain. The most widely used mathematical models are the BP model and the Whaley and Neville model, as well as the modified MC90/EC2 and the modified Hyperbolic function experimental cyclic creep models (Gaede, 1962; Mehmel and Kern, 1962; Neville *et al*., 1983; Suter and Mickleborough, 1975; Hirst and Neville, 1977).

Based on the test data the Whaley and Neville model has shown that the cyclic creep strain can be expressed as the sum of the two strain component: A mean strain component and a cyclic strain component. We consider uniaxial stress describe as:

$$\sigma = \sigma_0 + \frac{1}{2}\Delta \sin(2\pi\omega t) \tag{1}$$

where, σ is mean stress, 1/2Δ is cyclic stress amplitude and ω circular frequency.

The mean strain component is the creep strain produced by the static mean stress $(\sigma_m) = (\sigma_{max}-\sigma_{min})/2$. The additional cyclic creep component is found to dependent on both mean stress (σ_m) and the stress range $(\Delta) = \sigma_{max}-\sigma_{min}$. They proposed the following predictive equation for the total cyclic creep strain:

$$\Phi(t-t_0) = \frac{1}{\sigma}\left[\varepsilon_{el}(t_0) + \varepsilon(t-t_0)\right] = \frac{1}{E_c(t_0)} + \frac{\varepsilon(t-t_0)}{\sigma} \tag{2}$$

$$\varepsilon(t-t_0) = 129\sigma_m(1+3.87\Delta)t^{\frac{1}{3}} \times 10^{-6} \tag{3}$$

where, ε (t-t_0) is the cyclic creep strain, σ_m is the mean stress expressed as a fraction of the compressive strength and Δ is the stress-range expressed as a fraction of the compressive strength Φ (t-t_0) is the creep function.

Shown above the static and dynamic components of dynamic creep is a function of time. It can also be expressed as a function of number of cycles:

$$\varepsilon(t-t_0) = 129\ \sigma_0 t^{\frac{1}{3}} + 17.8\ \sigma_0 \Delta N^{\frac{1}{3}} \tag{4}$$

The above equation is fits when $\sigma_m<0.45$ and $\Delta<0.3$. The dimensions of the cyclic creep specimens are 76×76×203 mm cast vertically which is fog-cured for 14 days at 20±1°C. The specimens were enclosed in polyethylene bags containing some water which was not in direct contact with the specimens. The cyclic load varied sinusoidally at 9.75 (Hz) cycles per second. The BP model takes into consideration both shrinkage strain and mechanical strain. According to the BP model, cyclic creep function (t-t_0) = $\varepsilon/\sigma_{mean}$, where, ε is the strain mean level of cycle, is as follows:

$$\phi(t-t_0) = \begin{bmatrix} \frac{1}{E} + C_{oc}(t-t_{oc}) + C_d(t-t_0-t_d) \\ g\sigma - C_p(t-t_o-t_d) \end{bmatrix} f\sigma \tag{5}$$

Where:

$$C_{oc}(t-t_0) = \frac{\varphi_1}{E_0}(t_0^{-m}+\alpha)\ (1+k_\omega\,\varphi_\sigma\,\sigma_{pp}^2\,\omega^n)\ (t-t_0)^n \tag{6}$$

And this equation modified:

$$\phi(t,t_0,\sigma) = q_1 + F(\sigma)\left[C_{0c}(t,t^{'} + C_d(t_{dc},t^{'},t_0) + C_p(t_{dc,}t^{'},t_0)\right] \tag{7}$$

In which t_{dc} can be calculated as:

$$t_{dc} = t^{'} + (t-t^{'})\left[1+10\,\omega^{\frac{1}{4}}\Delta^2F^3\sigma_{max}\right] \tag{8}$$

where, ω is the frequency (Hz), k_ω is the empirical constant and the function $F(\sigma_{max})$ is the nonlinearity over proportionality factors.

The long-time material model presented in the 1990 CEB Model Code (MC90) was chosen as the model and the static creep tests within the previously mentioned and the modified by (Terje and Gordana, 1992) cyclic creep function is defined as:

$$\phi(t-t_0)=\frac{1}{E_c(t_o)}+\frac{\varphi_c(t-t_0)}{E_c(28\ d)}+\frac{\varphi_{cc}(t-t_0)}{E_c(28\ d)} \tag{9}$$

In these expression φ_c $(t\text{-}t_o)$ is the static creep ratio and φ_{cc} is the cyclic creep ratio, t' the concrete age at loading and t the actual time. The cyclic creep ratio is defined as:

$$\varphi_{CC}(t\text{-}t_0) = \beta(t_0)\ \beta(f_{cm})\ \beta(S_m)\ \beta(\Delta)\ \varphi_{CC}\ \beta(N, \omega) \tag{10}$$

In this expression f_{cm} is the average compressive cylinder strength at 28 days, S_m the ratio between the mean stress and the concrete strength at the start of testing, Δ the relative stress amplitude, N is the number of load cycles and ω is the frequency $N = (t\text{-}t_0)\omega$:

$$\beta(N, \omega) = N^n 1 = ((t\text{-}t_0)\ 86400\ \omega)^n\text{-}1,\ \text{with, } n = 0.022 \tag{11}$$

The general expression for cyclic creep term is then written as:

$$\varphi_{CC}(t\text{-}t_0) = 1.39\ \beta(t_0)\ \beta(f_{cm}\ 81+10.5\ (S_m\text{-}0.4)^2\ \Delta(N^n\text{-}1) \tag{12}$$

This expression is derived for high strength concrete and it is applicable also plain concrete with different constants parameters.

The hyperbolic function form German code 1045-1 or DAfStb booklet 525 (DIN5) modified by (Schwabach, 2005) gives the final equation as:

$$\varphi(t-t_0)=\left(\frac{t-t_0}{a+(t-t_0)}\right)^b\varphi_\infty(t_0)=\left(\frac{t-t_0}{a+(t-t_0)}\right)^b\times c\times\frac{1}{d+t_0^e} \tag{13}$$

$$c=\varphi_{RH}\times\beta(f_{cm})=\left(1+\frac{1-\frac{RH}{100}}{0.1\times h_0^{\frac{1}{3}}}\times\left(\frac{35}{f_{cm}}\right)^{0.7}\right)\times\left(\frac{35}{f_{cm}}\right)^{0.2}\times\left(\frac{16.8}{(f_{cm})^{\frac{1}{2}}}\right) \tag{14}$$

The constant a, b, d and e are determined from cyclic creep experimental data.

For concrete with a compressive strength 52.00 MPa, the value of a, b, d and e are found to be 318.22, 0.30, 0.10 and 0.20, respectively.

Sources of uncertainty: This section describes the method used to include the different sources of uncertainty in cyclic creep prediction. These sources of uncertainty can be classified into three different types; physical or natural uncertainty, data uncertainty and model uncertainty, as shown in Fig. 1.

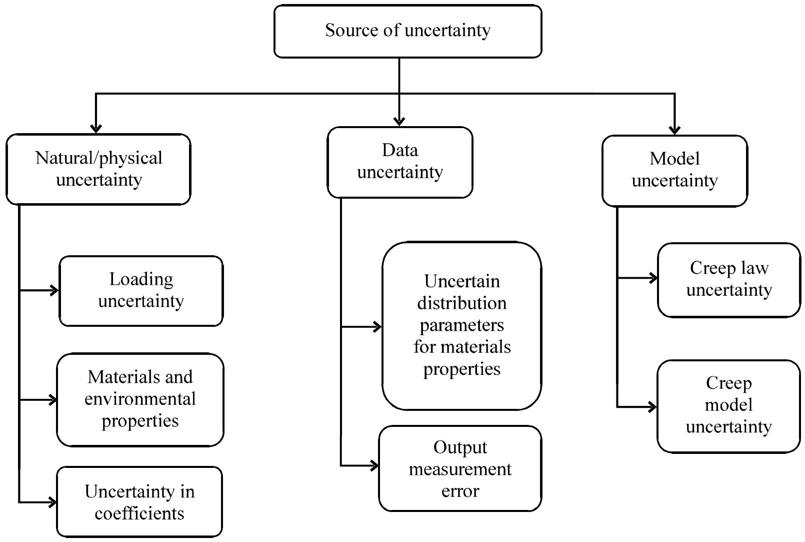

Fig. 1: Sources of uncertainty in creep prediction

Figure 1 illustrates different sources of error and uncertainty considered and the proposed methodology used.

Physical or natural uncertainty: Physical or natural uncertainty refers to the uncertainties or fluctuations in the environment, test procedures, instruments, observer, etc.

Hence, repeated observations of the same physical quantity do not yield identical results. This study considers the physical uncertainty in loading and materials properties. The uncertainty in the systematic errors to the measurement, human error, variability in others materials properties such as Poisson ratio, supplementary cementing materials, the curing time period, temperatures, etc. are not considered.

Data uncertainty: Experimental data is available in literature to characterize the distribution of materials properties such as Young's modulus of elasticity, compressive strength of concrete, etc. This data may be sparse and cause uncertainty regarding the probability distribution type and parameters; these errors are not considered in this paper and the quantification of these errors is trivial and will be considered in future study. The measurement uncertainties are calculated using the GUM (ISO, 1993, 2008; JCGM, 2008; Motra *et al*., 2013b) and Monte Carlo method. Bayesian model screening is implemented using the Monte Carlo method which is described in literature (JCGM, 2008; Motra *et al*., 2014a). The study found that the experimental error between 0.062 and 0.121 is reasonable for different tests.

Model uncertainty: More than 10 different creep prediction laws have been proposed in the literatures; each with its own limitations and uncertainties. The uncertainty in cyclic creep prediction can be subdivided into two different types: creep model error and uncertainty in model coefficients. These errors are assumed to represent the difference between the model prediction and the experimental observations. The variation from the experiments have been determined by Madsen and Bazant (1983), Li and Melchers (1992), Motra *et al*. (2013c) for the comparison with measured data. No one has done statistical analysis of cyclic creep data and there is no existing data bank for cyclic loading. All previous comparisons were based on the RILEM data bank for sustained loading. So far, the effects of cyclic loading on the calculation of variation in experiments have been neglected but they may be non-negligible for large structures, such as bridges with many lanes or with dense traffic and heavy trucks. The measurement error ($Cv_{\varphi,\alpha} \approx 0.08$) and internal uncertainty ($Cv_{\varphi,\beta} \approx 0.05$) were assumed. The coefficient of variation of the creep phenomenon α and measurements β are used to determine the coefficient of variation of the model uncertainties. The model uncertainty factor is normally distributed with an expected value of $E(\varphi_{cr,cyc}) = 1$. Frequency of loading also appears to have an influence on cyclic creep. Creep generally decreases with an increase in frequency so that under very rapid cycles the behaviour of concrete becomes more elastic.

Data assessment which is composed of test description, determination of error sources, estimating of uncertainty and documentation of the results is a key part of the entire experimental testing. Furthermore, uniform cyclic loading causes less creep than loading in an irregular pattern within the same range of stresses. Table 1 lists the comparisons of the total coefficient of variation of four models based on statistic input variables. Using the coefficients of variations, a generally valid comparison of the cyclic creep prediction of different models is enabled. The variation of the model response to the measured creep functions, $CV_{\text{Ø},cr,cyc}$, is given in the first row of Table 1. In these comparisons, the BP model is found to be the best model.

Uncertainty quantification in model parameters

Bayes methods: This section explains the Bayesian technique used in the uncertainty analysis of the measurements, in which the MCM was used with the experimental data. In order to obtain reliable results through MCM, the number M of trails, or evaluations performed by the model, of 10^6 is often considered appropriate in order to provide a coverage interval of 95%. However, the random nature of the process and the nature of the probability distribution of the output quantity Y have an influence on the values needed for M which varies for each case. Each value of standard uncertainty $y_r = (r = 1,..., M)$ is obtained by performing a random sampling from each of the probability density functions from the input quantities X_i and evaluating the model with the values found. The M values of Y thus obtained must be arranged in an increasing order. The output quantity and the associated standard uncertainty can be calculated as follows:

The average:

$$\bar{y} = \frac{1}{M}\sum_{r=1}^{M} y_r \tag{15}$$

And the standard deviation is taken as the standard uncertainty u(y) associated with y:

$$u^2(\overline{y}) = \frac{1}{m-1}\sum_{r=1}^{M}(y_r - \bar{y})^2 \tag{16}$$

Table 1: Model uncertainties

Model	BP model	MC90 model	Hyperbolic	Neville
$CV_{\Psi,cr,cyc}$	0.283	0.306	0.300	0.380
$Cv_{\varphi,\alpha}$	0.080	0.080	0.080	0.080
$Cv_{\varphi,\beta}$	0.062	0.086	0.093	0.121

The variation, according to Madsen and Bazant (1983), is composed of the uncertainty the creep model itself $CV_{mod,cr,cyc}$ the measurement uncertainty $_{Cv\varphi,\beta}$ and an internal uncertainty of the creep phenomenon $_{Cv\varphi,\alpha}$. Using the decomposition of the variance, or the decomposition of the coefficients of variation as follows:

$$CV^2_{Z,cr,cyc} = CV^2_{mod,cr,cyc} + CV^2_{\varphi,\beta} + CV^2_{\varphi,\alpha} \tag{17}$$

In order to carry out the MCM, the program is run in MATLAB for n_digit = 1, performing 10^6 evaluations of the different models until there is a stabilization in the results. The program gives the estimated cyclic creep with the associated standard uncertainty, measurement uncertainty which ($_{Cv\varphi,\beta}$) or $u(E_x)$. For simplification in this work, the standard uncertainty $u(E_x)$, is written as measurement uncertainty ($_{Cv\varphi,\beta}$) which is shows in the last row of Table 1 with the shortest 95% coverage interval. For the result validation, GUF and MCM give an estimation of cyclic creep that is noticeably different. Graphical approximations, in the form of a histogram of the probability density function of the output quantity, were created from the parameters from the GUF and MCM, onto which the curve of a Gaussian distribution has been superimposed. The probability density functions obtained from these two methods are in good agreement with each other. The basis of this method is the theory that cyclic creep models should not be evaluated against the values of measured data which are uncertain but against the inherent measurement uncertainty, particularly, the deviation calculation of the probability distribution (MCM) of measured data.

Global sensitivity analysis: The objective of the SA is to identify critical inputs variables of a model and quantify how input uncertainty impacts model outcomes. The sensitivities are solved at nominal values and cannot account for the variation effect of the input variables and thus these sensitivities are local. In contrast, the uncertainty importance measure is defined as the uncertainty in the output that can be apportioned to different sources of uncertainty in the model input and thus measures is also called global sensitivity. Xu and Gertner (2008) and Most (2012) methods are used in this study and have an approximately linear output response to the input variables. For a model $y = (x_1, x_2, x_3, ..., x_i, ..., x_k)$ and the main effect of each variable, the model can be simplified as the following:

$$y = \beta_0 + \Sigma_{i=1}^{K} \beta_i x_i + e \tag{18}$$

where, $\beta_0, ..., \beta_k$ are regression coefficients and e is the error. The partial variance (V_i) and total variance (V) can be estimate for uncorrelated variables as follows:

$$\hat{V}\,\mathrm{var}(y) = \beta_i^2\,\mathrm{var}(x_i) = \frac{1}{N-1}\,\beta_i^2 \sum_{j=1}^{N} (x_{ji} - \bar{x}_i)^2 \tag{19}$$

$$\hat{V}\,\mathrm{var}(y) = \frac{1}{N-1} \sum_{j=1}^{N} (y_i - \bar{y})^2 \tag{20}$$

The sensitivity of variable x_i can be calculated as:

$$S_i = \frac{\hat{V}_i}{\hat{V}} \tag{21}$$

where, x_{ji} is the j^{th} sample element for variable x_i, $\overline{x}_i$ is the sample mean of the variable from the LHS and β_i is the least-square estimate of the regression.

The partial variance V_i is decomposed into partial variance V_i^U due to uncorrelated variation of input variables X_i and partial variance V_i^C due to correlated variation of the input variables:

$$V_i = V_i^U + V_i^C \tag{22}$$

The partial variance V_i can be estimated as follows:

$$\hat{V} = \frac{1}{N-1}\sum_{j=1}^{N}(\hat{y}_j - \overline{y})^2 \tag{23}$$

The partial variance (V_i^U) can be estimated as follows:

$$\hat{V} = \frac{1}{N-1}\sum_{j=1}^{N}(\hat{y}_j^{(-i)} - \overline{y})^2 \tag{24}$$

The sensitivity indices can be calculated as follows:

$$S_i = \frac{V_i}{\hat{V}} \tag{25}$$

$$S_i^U = \frac{V_i^U}{\hat{V}} \tag{26}$$

$$S_i^C = \frac{V_i^C}{\hat{V}} \tag{27}$$

where, V_i, V_i^U, V_i^C are the partial variances, uncorrelated variance and correlated variance respectively.

UQ and SA of cyclic creep function

Input parameter and parameter correlation: The uncertainty factors of the cyclic creep models that are assumed to be random are: The compressive strength of concrete (f_c), the Young's modulus of elasticity (E_c), the relative humidity (RH), the water-cement ratio (w/a), the sand-aggregate ratio (a/c), the geometry factor (k_s), the cement content (c), the frequency of loading (ω), the mean stress (σ_m), the stress amplitude (Δ) and the number of cycle (N). The statistical properties of the material properties of the concrete are given in Table 2. The dynamic modulus of concrete, E_d and the dynamic compressive shear strength of concrete, f_d are crucial parts for the analysis of the cyclic creep function because these quantities are depend on the strain rate, number of cycle.

Numerous empirical relationships are available in the literatures. The "Deterioration of Materials and Structures" (Friedhelm *et al*., 2009) provides an overview of degradation of concrete under cyclic loading and is used in this paper. The deformation of concrete at any instant is defined as follows:

$$\text{Total strain} = \text{Elastic strain+Creep+Shrinkage} \tag{28}$$

Table 2: Statistic properties of the input variables

Variables	Mean	Std	CoV	Distribution	Models	Sources
$f_{c,28}$	52.00 MPa	3.12	0.06	Log-normal	1,2,3,4	30
f_d	50.70 MPa	3.00	0.06	Log-normal	1,2,3,4	Assumed
$E_{ci,\ 28}$	34144 Mpa	3414.4	0.1	Log-normal	1,2,3,4	30
$E_{cm,28}$	29394 MPa	2994.0	0.1	Log-normal	1,2,3,4	30
$E_{c,d}$	33290 MPa	3329.0	0.1	Log-normal	1,2,3,4	Assumed
Humidity	0.65 [-]	0.026	0.04	Normal	1,2,3	28
Cement content	362 (kg m^{-3})	36.20	0.1	Normal	1,3	29
Water-cement ratio	0.50 [-]	0.05	0.1	Normal	1	29
sand-cement ratio	5.16 [-]	0.516	0.1	Normal	1	29
Fine-aggregate ratio	0.50 [-]	0.05	0.1	Normal	1	29
Geometry factor ks	1.15 [-]	0.057	0.05	Normal	1,2,3	29
Frequency	9 Hz	0.72	0.08	Normal	1,3	Assumed
Mean stress	$0.35f_c$ [-]	0.035	0.1	Normal	1,2,3,4	Assumed
Stress amplitude	$0.3f_c$ [-]	0.03	0.1	Normal	1,2,3,4	Assumed
Number of cycles	10^6 Number	80000	0.08	Normal	1,2	Assumed
a	318.22	31.82	0.1	Normal	3	Assumed
b	0.3	0.03	0.1	Normal	3	Assumed
d	0.1	0.01	0.1	Normal	3	Assumed
e	0.2	0.02	0.1	Normal	3	Assumed

1: BP, 2: Modified MC90/CE 2, 3: Modified hyperbolic, 4: Neville

Table 3: Correlation matrix Neville model

Variables	f_c	E_c	σ_m	Δ
f_c	1	0.8	0	0
E_c		1	0	0
σ_m			1	0
Δ	Symm.			1

Table 4: Correlation matrix modified MC90/EC2 model

Variables	RH	k_s	f_c	E_c	σ_m	Δ	N
RH	1	0	0	0	0	0	0
k_s		1	0	0	0	0	0
f_c			1	0.8	0	0	0
E_c				1	0	0	0
σ_m					1	0	0
Δ	Symm.					1	0
N							1

Table 5: Correlation matrix modified hyperbolic model

Variables	RH	k_s	f_c	E_c	a	b	d	e
RH	1	0	0	0	0	0	0	0
k_s		1	0	0	0	0	0	0
f_c			1	0.8	0	0	0	0
E_c				1	0	0	0	0
a					1	0	0	0
b						1	0	0
d	Symm.						1	0
e								1

If the elastic strain under a constant stress is assumed to diminish with time, then the creep is increased by a corresponding amount to insure that the total strain is constant.

Under cyclic loading, the precise interpretation of elastic strain is very important, because changes in elastic strain due to change in elastic modulus are generally small compared with the sum of others quantities. Only the correlations of ñ = 0.4 and $\rho = 0.8$ were used and those under $\rho = 0.1$ were neglected in the stochastic analysis due to insignificance. These models are not intended to include the effects of temperature on creep.

The input variables correlation of the Neville model, modified MC90/CE2 model, modified Hyperbolic function model and BP model are shown in Table 3-6.

Table 6: Correlation matrix BP model

Variables	RH	c	w/c	a/c	k_s	f_c	ω	Δ
RH	1	0	0	0	0	0	0	0
c		1	-0.4	-0.4	0	0.4	0	0
w/c			1	0	0	-0.4	0	0
a/c				1	0	-0.4	0	0
k_s					1	0	0	0
f_c	Sym					1	0	0
ω							1	0
Δ								1

RESULTS AND DISCUSSION

Uncertainty of cyclic creep strain: The mean value of the predicted cyclic function for the four models over a short period time is presented in Fig. 2. Since the initial elastic strains were not reported, because of the pronounced short time creep duration, the strains had to be assumed and so the compressions are only relevant to the part of the strain that represents the creep increase due to strain cycling. Significant errors have often been caused by combining the creep coefficient with an incompatible value of the conventional elastic modulus, thus analysis must properly be based on the cyclic creep function. In Fig. 2 the data of all four models shows very different values in the first hour of testing and at 100 h the difference shown is small which may be the fluctuation in time to the physical mechanism of creep. The modified MC90/EC2, Neville and modified hyperbolic models are based only on the set of data and may not be applicable for conditions substantially different than that of the experiments.

Figure 3 and 4 show the results of the uncertainty analysis of four different models. Both figures show that the correlated and uncorrelated contributions of input variables have important contributions to the uncertainty in model output. The uncorrelated input variable uncertainty of Neville model is very small with only the contribution of four variables. On the other hand the input variables have a notable effect on the output because there are more variables and complex models and the model uncertainty is small. The correlated and uncorrelated input variables for model Neville shows largest uncertainty $Cv_{par,\ cr,\ cyc}(t\text{-}t_0) = 0.08$ at t = 1 h and uncertainty $Cv_{par,\ cr,\ cyc}$ $(t\text{-}t_0) = 0.06$ at t = 100 h, the uncertainty decreases with the increase in time under load. The uncorrelated input quantities uncertainty of the modified MC90 model and modified Hyperbolic model have a $Cv_{par,\ cr,\ cyc}$ $(t\text{-}t_0) = 0.10$ and are almost independent of time.

The uncertainty of the BP model is strongly time-dependent, varying in the range of $Cv_{par,\ cr,\ cyc}$ $(t\text{-}t_0) = 0.11\text{-}0.08$.

Taking into account the input variables and the real correlation of model Neville, the input variables increase significantly $Cv_{par,cr,cyc}(t\text{-}t_0) = 0.08$ which may strongly influence the strength and Young's modulus of elasticity due to its correlation. Comparing the all uncertainty of the models in Fig. 4, we conclude that the model and measurement play an important role on the uncertainty behaviour of models. In comparison, of all models the BP model has the lowest total uncertainty $Cv_{par,\ cr,\ cyc}(t\text{-}t_0) = 0.30$ and model Neville has highest total uncertainty $Cv_{par,\ cr,\ cyc}(t\text{-}t_0) = 0.40$. The modified MC90 model, modified Hyperbolic model and Neville model are based on the experimental data and also assumed that the strain-time equation does not always fit satisfactorily with the experimental data, so that long-term values cannot be estimate with confidence. Generally, if creep is measured over a longer time interval, the better the prediction of creep. The CV of the initial time of loading is higher and decreases with the time because there are more uncertainties in the measurements at the initial time. The most important variable in short-time creep is model uncertainty factor for all the models.

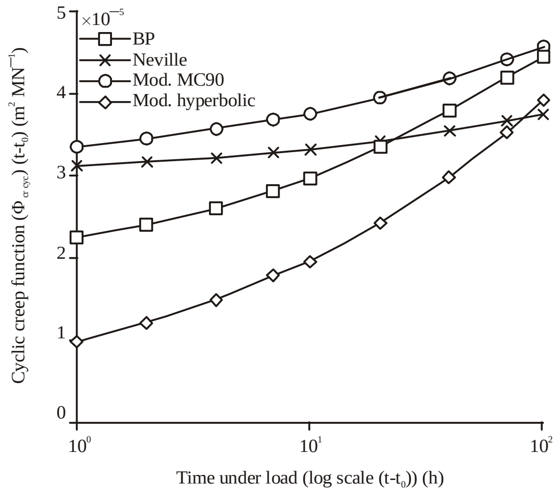

Fig. 2: Mean value of creep function

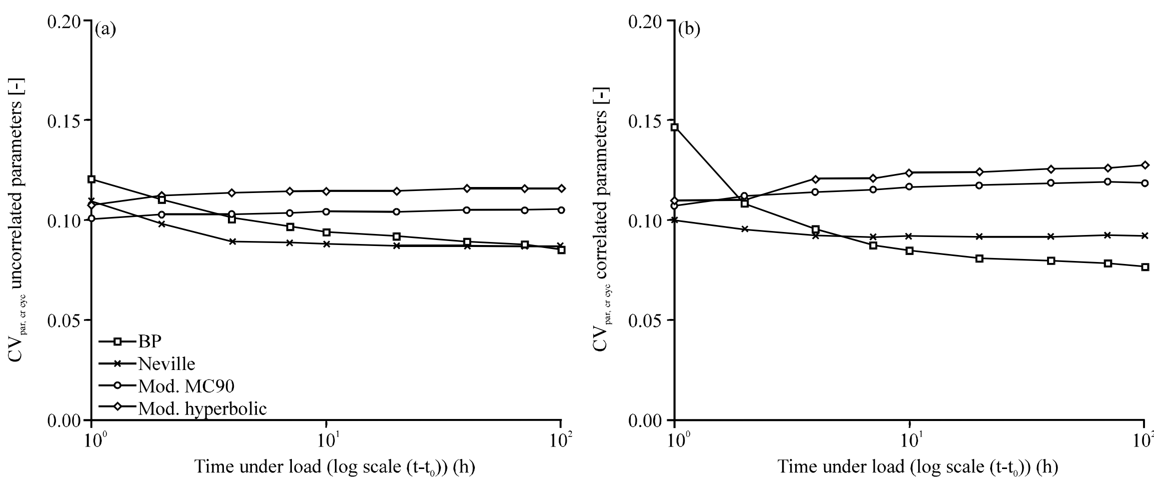

Fig. 3(a-b): Input variables uncertainty of cyclic creep prediction, (a) Uncorrelated and (b) Correlated parameters

Total Model Quality (MQ) can be used to balance the better response of the model to its uncertainty in order to select the model that is best model quality for a certain response. Figure 5 shows the time-dependent model quality. The MQ is dependent on total uncertainty considering the correlated input quantities. The MQ is slight time dependent and thus the time integration according to the (Keitel, 2011) is used and is shown in the results given in Fig. 5. In all these comparisons, the BP model is found to be the best. The CEB-MC90/EC2 model (CEB, 1990a) which modifies his original MC90/EC 2 model (Terje and Gordana, 1992) by co-opting key aspects of cyclic loading (the mean stress and stress amplitude function and dependence on the number of cycles would simply mean a loading frequency), comes out as the second best. Considerably worse but the third best overall is seen to be the modified Hyperbolic model. Since the current Neville model, labelled Neville, is the simplest model, introduced in 1973 on the basis of Neville's research (Neville *et al*., 1983), it is not surprising that it comes out as the worst because it is based on only four variables and there is no consideration of concrete composition and environmental variables.

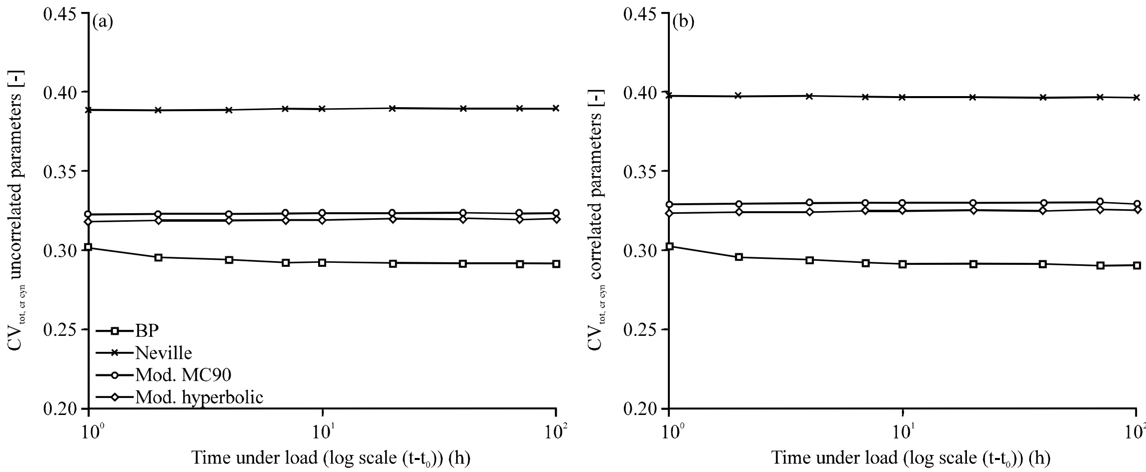

Fig. 4(a-b): Input variables and model uncertainty of cyclic creep prediction, (a) Uncorrelated and (b) Correlated parameters

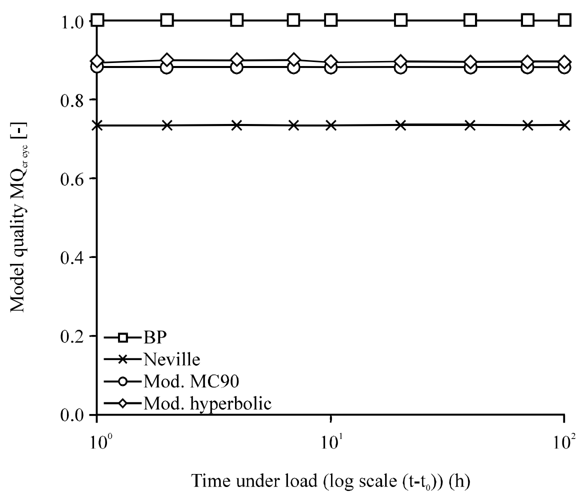

Fig. 5: Model Quality (MQ) of cyclic creep prediction

Sensitivity analysis of the cyclic creep strain: A SA is required to find out the dominant effect of the variability of input random variables on the cyclic creep strain. Figure 6-9 show the results of the sensitivity analysis of uncorrelated and correlated variables. For the calculation of the sensitivity, the model uncertainty is not considered and it is assumed that the sensitivity indices are upto:

$$\sum_{p'=1}^{pK} S_p = 1$$

The normalization is necessary due to consideration of correlation which may vary the results of sensitivity indices $S_p \geq 1$. From this arise the difficulty to compare the uncorrelated and the correlated indices. High values of sensitivity S_p means highly influential on the uncertainty, for example S_p= 1 means only this quantities affect the output. The input quantities sensitivity of the

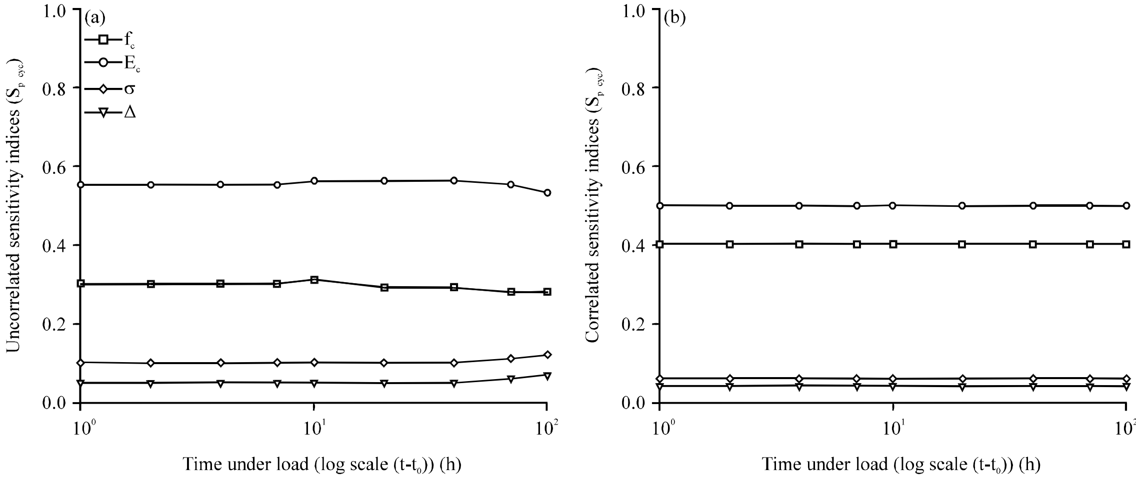

Fig. 6(a-b): (a) Uncorrelated and (b) Correlated sensitivity indices of Neville model

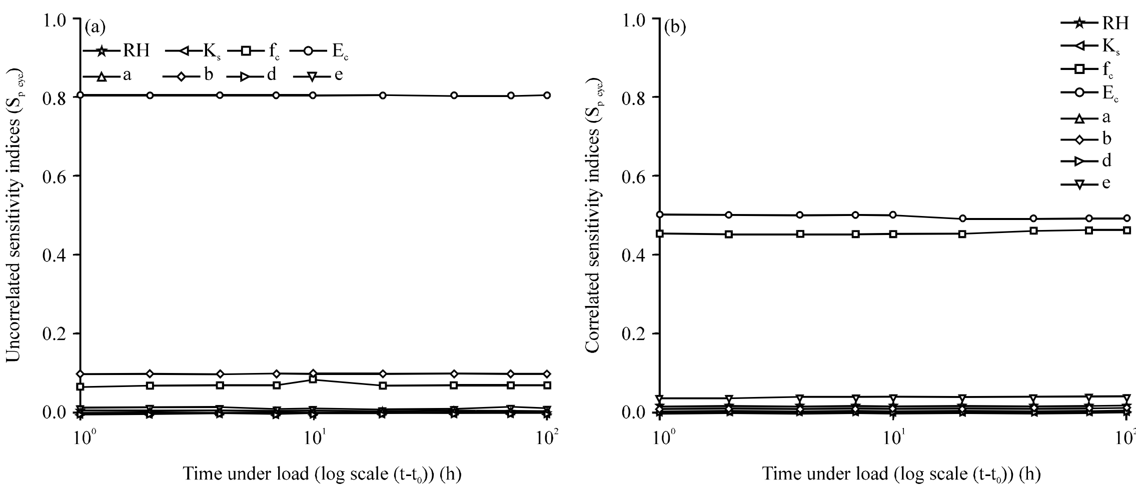

Fig. 7(a-b): (a) Uncorrelated and (b) Correlated sensitivity indices of model modified hyperbolic

Neville model is presented in Fig. 6. All input quantities are approximately time-independent. The reason behind this is that the expression depends on the value of the mean stress, stress amplitude, compressive strength and modulus of elasticity of concrete as well as other input quantities considered in this model which are assumed to be constant with respect to time. The strength and modulus of elasticity is not exactly constant over the time but it is much complicated to consider otherwise. It is seen that the most sensitive quantities turn out to be elastic modulus and followed by compressive strength. The mean stress and stress amplitude do not influence as much in comparison to the two quantities above. The variable correlations strongly influence the sensitivity indices, among which, E_c and f_c are most influential quantities.

The modified Hyperbolic model also show constant sensitivity indices for all the input quantities over the time. In this model the time is account only this quantity $((t-t_0)/(a+(t-t_0))^b$ and the influence of both a and b is much smaller than compared to that of the other quantities. The

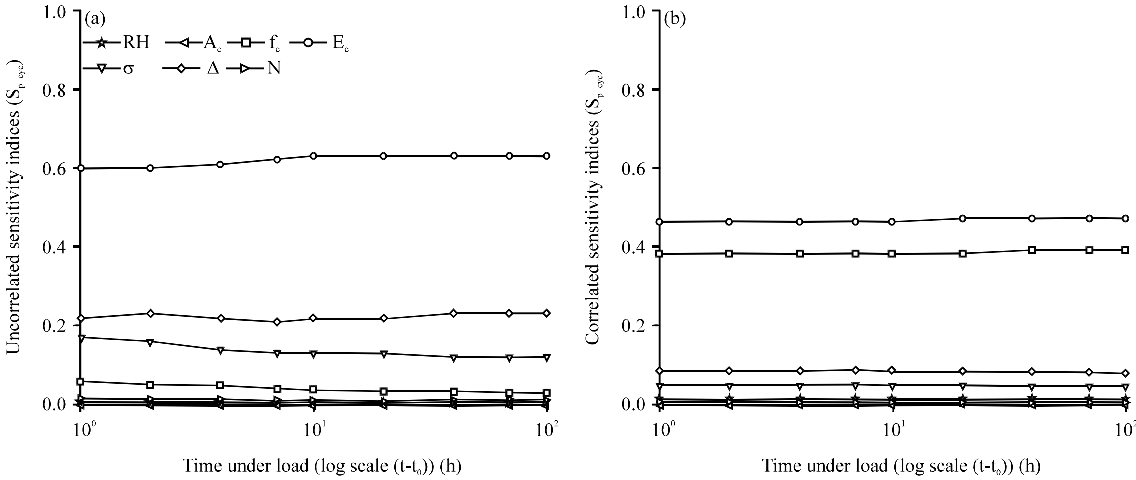

Fig. 8(a-b): (a) Uncorrelated and (b) Correlated sensitivity indices of modified MC90/CE2 model

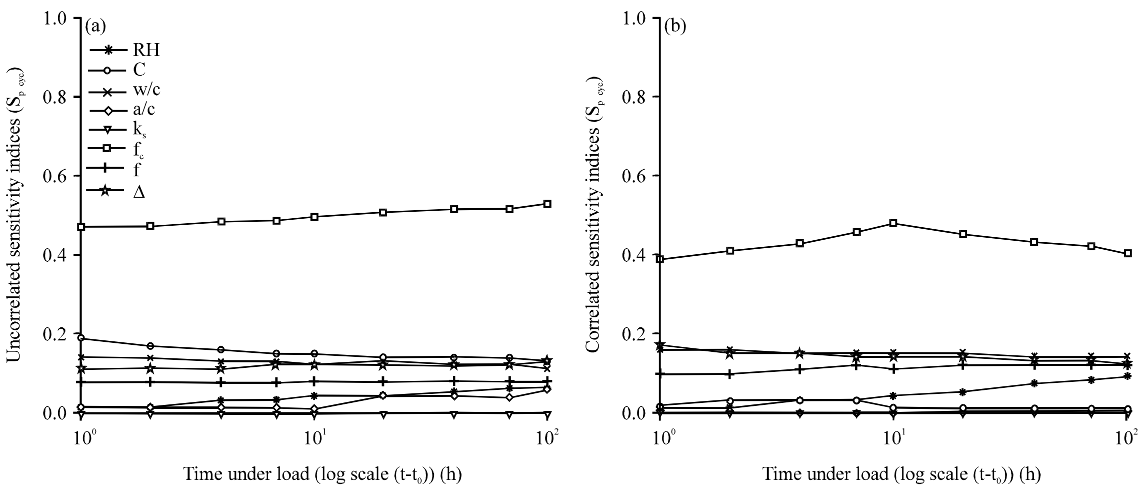

Fig. 9(a-b): (a) Uncorrelated and (b) Correlated sensitivity indices of the (BP) model

elastic modulus is the most influential variable and followed strength of concrete. The correlations show the variable's influences on the sensitivity indices. Figure 7 shows the sensitivity indices of all input variables of modified hyperbolic model.

The sensitivities of the modified MC90 model remain approximately constant over the time. The humidity influences the time function by factor β_H but the influence is relatively small. The sensitivity indices of E_c, f_c and σ_m fluctuate over the time. The main reason for this is that these variables are affected by the time the specimen is under loading but the effects are small. There is clearly a large difference between the sensitivity indices between the uncorrelated and correlated variables for the most influential quantities. In the case of the uncorrelated input quantities, E_c is the most dominating input quantities. On the other hand, the E_c and f_c are the most sensitive quantities due to the strong correlation with each other. The numbers of cycle, mean stress and stress amplitude have small influence.

The sensitivity indices of the BP model are shown to be more time dependent. The main reason behind this is that there are more combinations of time function with the input quantities. It is seen that the most influential quantities turn out to be concrete strength. Second is the content of the cement, when quantities are assuming the uncorrelated. Further, the stress amplitude and frequency is the third and fourth most influential quantities. The influences of the water cement ratio, aggregate-sand ratio and humidity not insignificant. The concrete strength is most dominating of the quantities when considering the quantities correlation. The second dominant quantities are the cement content and stress amplitude. The sensitivity indices of cement content and stress amplitude decrease with increasing time. The cyclic parameter is also seen to have considerable influence.

CONCLUSION

In the present study, a probabilistic framework is suggested for the prediction of the cyclic creep of plain concrete considering four different cyclic creep models. Different sources of uncertainty; physical variability, data uncertainty and model error/uncertainty, were included in the cyclic creep analysis. The input quantities which drive the cyclic creep such as, elastic modulus, concrete strength, mean stress, cyclic stress amplitude, number of cycle, humidity, cement content, water-cement ratio, sand-cement ratio, geometric factor have been considered as random variables. The uncertainty and sensitivity analysis are computed using the LHS sampling technique. It is seen from the uncertainty analysis the complex cyclic creep the BP model has the good MQ and less uncertainty whereas the simple Neville model has higher uncertainty and lower MQ. In contrast, the complex model needs computational effort and more input variables. Stochastic sensitivity analysis is performed to determine the predominant factor amongst the input variables which influences the cyclic creep prediction. It is observed that cyclic creep is more sensitive to the elastic modulus and strength of concrete, followed by mean stress, stress amplitude, frequency, cement content, humidity and water cement ratio. Further, the present study of cyclic creep models brings some interesting point. Most of the creep analysis is only sustained load; the cyclic loading effect is neglected. Cyclic effect, neglected so far, might not be negligible for long span bridge with many lanes or with a dense traffic of heavy trucks. This may cause the excessive time-dependent deflection of concrete structures. The concrete structure can lose their stiffness by (1) The degradation of concrete and (2) The creep of concrete etc. The relationship between the frequency of the structure and its age is important for the study of the long-term behaviour of materials and possibly for the detection of its damage. There are significant changes of the modulus of elasticity of concrete due to cyclic creep.

Also, the proposed approach for UQ and SA is applicable to several engineering disciplines and the domain of cyclic creep analysis was used only as an illustration to develop the methodology. In general, the proposed methodology provides a fundamental framework in which multiple models can be connected through a Bayes network and the confidence in the overall model prediction can be assessed quantitatively.

ACKNOWLEDGMENT

This research is supported by the German Research Foundation (DFG) via research training group "Assessment of Coupled Experimental and Numerical Partial Models in Structural Engineering (GRK 1462)" which is gratefully acknowledged by the authors.

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