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Review Article
Derivation of Heisenberg Uncertainty Relations in the Non-local Approach of String Theory

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Abstract
The derivation of the Heisenberg uncertainty relations in the nonlocal approach in string theory was proposed. This approach, in contrast to the local one, makes it possible to trace the influence of the dimension of space-time on the form of the uncertainty relations.

Key words: Quantum-mechanical phenomena, uncertainty relations, Heisenberg uncertainty relations, Heisenberg, quantum mechanics

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INTRODUCTION

From the end of the 19th century, the accumulation of results that did not fit into the old classical ideas began. Their understanding led to the birth of quantum mechanics, which makes it possible to give reasonable explanations for the phenomena of the microworld. It birth can be considered 1900, when an explanation of the radiation of an absolutely black body was given by M. Planck. Its final formation took place[1-4] in 1925-1927. So, about 100 years have passed since it birth.

The main result of the development of quantum mechanics was the understanding of the non-locality of our world. Local description was very good to understand the classical behavior of the objects surrounding us. Of course, with this local description frame of classical physics had to start studying the phenomena of the quantum world. It turned out that the apparatus of the local description is also suitable for describing non-local phenomena. Thus, for example, the theoretical value of the anomalous magnetic moment of an electron in relativistic quantum electrodynamics coincides with the experimental value with an accuracy of up to 11 significant digits (relative accuracy $2\ 10^{-10}$)[5-8].

However, with the development of the quantum theory, some illogicality of such a description has become more and more apparent. You use a local approach but in doing so, you come to the fact that it is impossible exact determination of the positions and momenta of particles at the same time (the Heisenberg uncertainty relations). You can describe, if necessary, the corpuscular nature of light, or, if necessary, its wave nature but you can not in this approach say why the corpuscles suddenly begin to exhibit wave properties[9], etc.

In the work an attempt was made to clarify the nature of the Heisenberg uncertainty relations in the non-local approach.

LOCAL DESCRIPTION

Significant advances in the physics associated with the use of Lagrangian functions with local density. The Lagrange function (Lagrangian) or its density was determined on spatial structures, the simplest of which was the point of space. Such Lagrange functions were called local. In the local approach, the overwhelming number of results known to us was obtained.

Thanks to the local description, a significant advance in understanding the phenomena of the microworld was achieved. Nevertheless, the world was non-local. Firstly, this was reflected by the very construction of the apparatus of quantum mechanics. The developed apparatus of quantum mechanics deals with wave functions. But the approach based on the use of wave functions was related to the probability description. For processes that were taking place the probabilities could be only predicted. This in itself already suggested that since exact localization, for example, the position and momentum of a particle was impossible, this circumstance may be due precisely to its non-locality or its non-local interaction with the environment.

The non-locality of the world, in particular, was revealed, in addition to the already mentioned probability description, in: the corpuscular-wave duality and the Heisenberg uncertainty relations, the non-commutativity of quantum-mechanical operators, the necessity of taking into account all possible positions of the system (an alternative approach based on path integrals[4]), EPR-paradox[10,11], Bell's inequalities[12,13] etc.

So, the world surrounding us was non-local. It is better to describe a nonlocal nature in the corresponding nonlocal approach. Although, until now, the local approach in nonlocal phenomena understanding was rather acceptable but it was not entirely natural to investigate the nonlocal nature in the local description.

NONLOCAL DESCRIPTION

Therefore, the non-local schemes should be initially developed. The main focus of research with nonlocal Lagrangians while focused on the examination of the strings. But should be remembered that the one-dimensional objects just a special case of non-local objects. In such approach it is necessary to analyze also objects of higher dimension-membranes ($n \geq 2$)[14]. String theory can really be productive if the contribution of membrane theories in the description of physical processes will be small. But the development of membrane theories practically not moving and this is due to their significant nonlinearity[15].

String theory: The object following on complexity after a point is the one-dimensional structure-the line. Using theories with local Lagrange functions greatly simplifies the analysis but it does not follow from anywhere that it is the only possible physical theories. Lines on which Lagrange's function is defined, are called as strings, respectively, corresponding theories are the string theories[15-18]. The main emphasis of research with nonlocal Lagrangians was still focused on the consideration of strings. Basically, optimism when considering string theory is related to the fact that this approach can lead to quantization of gravity. Note that while all the advantages of string theory remain without experimental confirmation.

Nevertheless, this approach is undoubtedly also necessary to develop, in order to understand what opportunities in the description of the real world it can provide.

Dimension of space: The non-locality of the phenomena of the microworld must have very nontrivial consequences. When describing nonlocal objects in string theory, fundamental aspects of this theory is revealed. The point is that string theories can not exist in a space of arbitrary dimension. The dimension of the space depends on the sector of interactions considered in this theory. For example, in the bosonic sector of theory this dimension must be 26, in the fermionic supersymmetric sector of theory[15-17] it must be 11 (the dimensionality of our real space can take greater values[19,20]). Thus, the non-locality of the phenomena of the microworld indicates the fact that a space has a certain dimension. In the local description, multidimensional spaces also appeared[19,20] but in this case the transition to multidimensional spaces can be interpreted as the possibility of a more convenient description of the processes under consideration.

Cosmological constant: So, the problem of non-locality in quantum mechanics led to a consideration of a non-local description of micro phenomena. This consideration, in particular, led to the construction of non-local Lagrangians or in other words to the construction of a string theory as the simplest variant of non-locality. In this theory, space can not have an arbitrary dimension-the dimension is determined by commutation relations for the generators of Lorentz charges[15-17]. But that’s not all. Consecutive consideration of this issue gives arguments in favor of the possibility of changing the dimensionality of space. The string theory, because of its nonlocal specifics, contains additional symmetries that are not contained in local theories. The T-duality of the string theory imposes a ban on the possibility of an electric field strength to accept infinitely large values. In thermodynamic equilibrium, each degree of freedom of the system, on the average, has the same energy (the law of equidistribution). If, for very large energy densities, this law is also valid, this will lead to the fact that a finite number of degrees of freedom for a finite number of fields will contain only a finite energy density. At an infinite energy density of the Big Bang (BB), a finite number of degrees of freedom at the initial moments of the explosion can not accumulate the released energy. As applied to the initial instant of time in the theory of BB, this leads to an infinite (or very large) dimensionality of the space:

$$N(t) \geq 16\pi^3 \alpha'^2 w(t) \quad (1)$$

where α' is so called slope parameter in string theory, which is proportional to the string tension and w(t) is an energy density of BB at some time moment t near BB initiation and consequently, to the possibility of changing it[19,20]. The change in the dimension of space gives, in turn, the key to describing the nature of the cosmological constant[19,20]. Thus, the phenomena of micro and macro-cosm are related in a non-trivial way.

Of all the above mentioned set of characteristic manifestations of the non-locality of our world. Below two of them, closely related questions-the question of the nature of the corpuscular-wave dualism and the question of the nature of the Heisenberg uncertainty relations will be considered.

CORPUSCULAR-WAVE DUALISM

The corpuscular-wave dualism reflects the fact that the system participates in all its possible realizations. This is one of the brightest manifestations of nonlocality. For example, particles in interference experiments feel at once both (or all, if there are more than two) slits. An attempt to describe these experiments according to the local nature of particles does not lead to an explanation of the observed effects. Conducting ingenious experiments that seemingly should "force" photons or particles to behave locally-experiments on the interference of single particles, experiments on anticoincidence, experiments with deferred choice, etc., each time lead to the opposite situation. Particles can not be deceived. They feel all their possible positions and participate in them and do not succumb to the provocation of these ingenious experiments[9].

Note that in the local description the wave properties of particles can be only described but not explained[9]. In addition, the typical corpuscular behavior of particles also remains questionable. For example, Einstein’s explanation of the phenomenon of the photoelectric effect from the point of view of the corpuscular theory of photons is not the only possible one. Note[9] that Planck also had suspicions about this and he was much more cautious in his statements, despite the fact that Einstein convinces him of his concept of light. All the features of the photoelectric effect experiments can also be explained on the basis of the wave nature of light[9,21-23].

In most cases, the corpuscular approach of describing particles is quite acceptable, in particular, experimental data on the radiation of both laser and lamp sources are reproduced theoretically quite well[24-26]. However, this does not remove the question of the nature of particle-wave dualism.

HEISENBERG UNCERTAINTY RELATIONS

As a consequence of a probabilistic description, Heisenberg uncertainty relations are received[1-3,27-30]. There are some disagreements in the interpretation of the relationship on the coordinates-momenta and energy-time. However, despite the difference in their interpretation by some authors, they formally have the same form. The impossibility of an accurate prediction of the characteristics of a particle it was natural to associate with the fact that it did not represent a local object.

Local approach: The justification of the Heisenberg uncertainty relations is contained and can be seen from relation[3] in the case of non-commuting operators A and B:

$$m_A m_B \geq \frac{1}{2}\left|<[A,B]>\right| \quad (2)$$

where, m_A, m_B are the dispersions of the physical values A and B, on the right side there is an average value of the commutator of the operators corresponding to the given values.

However, the Schrödinger equation in the local approach was not derived but postulated. The matrix mechanics of Heisenberg is equivalent to the Schrödinger approach. Consequently, matrix mechanics was also postulated together with the presence of non-commuting operators in it. Thus, the nature of the uncertainty relation hides behind the nature of the Schrödinger equation.

Non-local approach: Earlier, the close connection between the wave behavior of particles and the Heisenberg uncertainty relations was mentioned. The uncertainty relations are a serious argument in favor of the non-locality of microworld processes. Note, that in the overwhelming number of attempts to disprove the uncertainty relations, for example, in Bohr's disputes with Einstein (see also the supplement to the Russian edition[1]), thought experiments with light exhibiting wave properties took place.

This suggested that the wave nature of particles (massless (photons) or massive) is closely related to the nature of the uncertainty relation. If the uncertainty relations are a consequence of non-locality, then the wave structure of particles (both mass and massless) should be also described in a non-local way. Conversely, if the wave nature of the particles can be explained in a non-local description, then, since it must be closely related to the nature of the Heisenberg uncertainty relations, the latter can also be clarified in a non-local description. Indeed, it can be shown that the both wave nature of particles and the nature of uncertainty relations can be explained in a non-local approach (see below).

WAVE BEHAVIOR OF PARTICLES

In the local approach L. de Broglie from the analogy of the description of particles in classical mechanics and the wave motion of light, concludes that massive as well as massless particles, should exhibit wave properties and obtain a relation connecting the momentum of a particle with its wavelength[3].

This analogy is the heuristic factor of such a substantiation of the wave properties of particles. There is no derivation as such in the local approach. The derivation of the wave equation for particles in a non-local approach is contained[17] but it is contained among general considerations of various problems in string theory. Due to the importance of this issue in relation to presented consideration, this derivation is briefly repeated below. A significant addition to this derivation is given at the end of this paragraph.

When constructing the density of the Lagrangian of string theory, it is assumed that its most general form is proportional to the area swept out by the string as it moves. A string as a line can be specified as X^μ, $\mu = 1, 2, \ldots, N$, maps in the enclosing space $(x_1, x_2, \cdots x_{N-1}, t)$. In an arbitrary flat two-dimensional parametric space it is possible to work in Cartesian coordinates ξ_1, ξ_2 and the element of the area is the product $d\xi_1 d\xi_2$. The functions X^μ themselves depend on ξ_1, ξ_2. When ξ_1, ξ_2 is mapped into the enclosing space, a rectangle with sides ξ_1 and ξ_2 is transformed in the general case into a parallelogram based on the vectors $v_i^\mu = \partial X^\mu/\partial\xi_i$. According to the general theory of differential calculus, the area element under such a mapping is a quantity:

$$df = \sqrt{(\vec{v}_1)^2(\vec{v}_2)^2 - (\vec{v}_1\vec{v}_2)^2}\, d\xi_1 d\xi_2 \quad (3)$$

In string theory, these parameters ξ_1, ξ_2 are denoted as τ, σ. Taking into account the agreement on the form of the scalar product under relativistic transformations, the general expression for such a Lagrangian density, called the Nambu-Goto Lagrangian density, is therefore taken in the form:

$$L(\dot{X}^\mu, X^{\mu'}) = -\frac{T_0}{c}\sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2} \quad (4)$$

Where:

$$\dot{X}^{\mu} = \frac{\partial X^{\mu}}{\partial \tau} \tag{5}$$

$$X^{\mu}{}' = \frac{\partial X^{\mu}}{\partial \sigma} \tag{6}$$

T_0 is the string tension, c is the speed of light. Accordingly, the action for describing the string is represented in the form:

$$S = \int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma L(\dot{X}^{\mu}, X^{\mu'}) \tag{7}$$

In the variational principle, the equations of motion of a string are obtained from the condition of equality to zero of the variation of the action:

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \left[\frac{\partial L}{\partial \dot{X}^{\mu}} \frac{\partial(\delta X^{\mu})}{\partial \tau} + \frac{\partial L}{\partial X^{\mu'}} \frac{\partial(\delta X^{\mu})}{\partial \sigma} \right] \tag{8}$$

where, when this expression was received, was taken:

$$\delta \dot{X}^{\mu} = \delta\left(\frac{(\partial X^{\mu})}{\partial \tau} \right) = \frac{\partial(\delta X^{\mu})}{\partial \tau} \tag{9}$$

$$\delta X^{\mu'} = \delta\left(\frac{(\partial X^{\mu})}{\partial \sigma} \right) = \frac{\partial(\delta X^{\mu})}{\partial \sigma} \tag{10}$$

Partial derivatives of the Lagrangian density are the densities of the string momentum and have the form:

$$P_{\mu}^{\tau} \equiv \frac{\partial L}{\partial \dot{X}^{\mu}} = -\frac{T_0}{c} \frac{(\dot{X} \cdot X')X'_{\mu} - (X')^2 \dot{X}_{\mu}}{\sqrt{(\dot{X} \cdot X')^2 - (\dot{X})^2 (X')^2}} \tag{11}$$

$$P_{\mu}^{\sigma} \equiv \frac{\partial L}{\partial X^{\mu'}} = -\frac{T_0}{c} \frac{(\dot{X} \cdot X')\dot{X}_{\mu} - (\dot{X})^2 X_{\mu}'}{\sqrt{(\dot{X} \cdot X')^2 - (\dot{X})^2 (X')^2}} \tag{12}$$

Integration by parts for the variation of the action yields the expression:

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \left[\frac{\partial}{\partial \tau}(\delta X^{\mu} P_{\mu}^{\tau}) + \frac{\partial}{\partial \sigma}(\delta X^{\mu} P_{\mu}^{\sigma}) - \delta X^{\mu} \left(\frac{\partial P_{\mu}^{\tau}}{\partial \tau} + \frac{\partial P_{\mu}^{\sigma}}{\partial \sigma} \right) \right] \tag{13}$$

The parameter τ, as will be seen from the following discussion, is related to the time component of the described space; therefore, as is done in the usual variational approach, at the initial and final moments of this parameter the characteristics of the string do not vary:

$$\delta X^{\mu}(\tau_f, \sigma) = \delta X^{\mu}(\tau_i, \sigma) = 0 \tag{14}$$

consequently:

$$\delta S = \int_{\tau_i}^{\tau_f} d\tau \left[\delta X^{\mu} P_{\mu}^{\sigma} \right]_0^{\sigma_1} - \int_{\tau_i}^{\tau_f} d\tau \int_0^{\sigma_1} d\sigma \delta X^{\mu} \left(\frac{\partial P_{\mu}^{\tau}}{\partial \tau} + \frac{\partial P_{\mu}^{\sigma}}{\partial \sigma} \right) \tag{15}$$

Taking into account the boundary conditions[17] turns the first term to zero. As a result, the equations of motion are:

$$\frac{\partial P_{\mu}^{\tau}}{\partial \tau} + \frac{\partial P_{\mu}^{\sigma}}{\partial \sigma} = 0 \tag{16}$$

It can be seen that the solution of this equation is a rather serious problem. The choice of gauge can greatly simplify the form of the presented equation.

Static gauge: Let us fix the lines of the constant τ, associating it with the time coordinate:

$$X^0 = ct \tag{17}$$

in some chosen Lorentzian system. In this gauge, the constant-time hyper plane:

$$t = t_0 \tag{18}$$

will cross the world sheet of motion of the string along the curve, which for observers in the chosen Lorentz system will be a string at time t_0. Thus, the curve of constant τ is the curve $\tau = t_0$. Extending this definition to all times t, it is assumed that for any point Q on the world sheet:

$$\tau(Q) = t(Q) \tag{19}$$

This choice of τ-parameterization is called static gauge. In this case, the set of coordinates of the string in this case can be written in the form:

$$X^{\mu}(\tau, \sigma) = \{c\tau, \vec{X}^{\mu}(\tau, \sigma)\} \tag{20}$$

where, the vector $\vec{X}$ is the spatial coordinates of the string. Then:

$$\frac{\partial X^\mu}{\partial\sigma}=\left(\frac{\partial X^0}{\partial\sigma},\frac{\partial\vec{X}}{\partial\sigma}\right)=\left(0,\frac{\partial\vec{X}}{\partial\sigma}\right) \quad (21)$$

$$\frac{\partial X^\mu}{\partial\tau}=\left(\frac{\partial X^0}{\partial t},\frac{\partial\vec{X}}{\partial t}\right)=\left(c,\frac{\partial\vec{X}}{\partial t}\right) \quad (22)$$

It can be seen that the static gauge separates the spatial and temporal components very neatly.

Action in terms of transverse velocity: It would be natural to determine the velocity of the string as:

$$\frac{\partial}{\partial t}\vec{X}^\mu(t,\sigma) \quad (23)$$

However, in this case it would depend on the parameter σ. A string is an object with an indefinite substructure. To talk about points on a string, you need some specific parameterization (see, for example, the s-parameterization below) but because of the reparameterization invariance it is clear that such a parameterization is ambiguous. Thus, in the absence of a string substructure, the component of its velocity along its motion can be described but it is not observable. When a one-dimensional object moves, the velocity of the points of that object is naturally tracked in the direction perpendicular to the object at that point. Therefore, the invariant characteristic is the transverse velocity.

In the simplest case, the distance *s* up to a point along the string from one of its ends is used as a parameter that monitors the position of this point on a string. In this case, the connection with an arbitrary parameter s is realized as:

$$ds=\left|d\vec{X}\right|=\left|\frac{\partial\vec{X}}{\partial\sigma}\right|d\sigma \quad (24)$$

and:

$$\frac{\partial\vec{X}}{\partial s}$$

is the unit vector tangent to the string. To find the component of any vector $\vec{\mu}$ perpendicular to the unit vector $\vec{n}$, the combination:

$$\vec{\mu}\text{-}(\vec{\mu}\vec{n})\vec{n} \quad (25)$$

should be computed. Thus, in the case of a string, its transverse velocity is expressed as:

$$\vec{v}_\perp=\frac{\partial\vec{X}}{\partial t}-\left(\frac{\partial\vec{X}}{\partial t}\frac{\partial\vec{X}}{\partial s}\right)\frac{\partial\vec{X}}{\partial s} \quad (26)$$

$$v_\perp^2=\left(\frac{\partial\vec{X}}{\partial t}\right)^2-\left(\frac{\partial\vec{X}}{\partial t}\frac{\partial\vec{X}}{\partial s}\right)^2 \quad (27)$$

In the static gauge follows:

$$(\dot{X})^2=-c^2+\left(\frac{\partial\vec{X}}{\partial t}\right)^2 \quad (28)$$

$$(X')^2=\left(\frac{\partial\vec{X}}{\partial\sigma}\right)^2 \quad (29)$$

$$\dot{X}\cdot X'=\frac{\partial\vec{X}}{\partial t}\cdot\frac{\partial\vec{X}}{\partial\sigma} \quad (30)$$

Then:

$$\begin{aligned}(\dot{X}\cdot X')^2-(\dot{X})^2(X')^2&=\left(\frac{\partial\vec{X}}{\partial t}\cdot\frac{\partial\vec{X}}{\partial\sigma}\right)^2+\left[c^2-\left(\frac{\partial\vec{X}}{\partial t}\right)^2\right]\left(\frac{\partial\vec{X}}{\partial\sigma}\right)^2\\&=\left(\frac{\partial s}{\partial\sigma}\right)^2\left[\left(\frac{\partial\vec{X}}{\partial t}\cdot\frac{\partial\vec{X}}{\partial s}\right)^2+c^2-\left(\frac{\partial\vec{X}}{\partial t}\right)^2\right]\end{aligned} \quad (31)$$

i.e.:

$$(\dot{X}\cdot X')^2-(\dot{X})^2(X')^2==\left(\frac{\partial s}{\partial\sigma}\right)^2\left[c^2-v_\perp^2\right] \quad (32)$$

or:

$$\sqrt{(\dot{X}\cdot X')^2-(\dot{X})^2(X')^2}=c\frac{ds}{d\sigma}\sqrt{1-\frac{v_\perp^2}{c^2}} \quad (33)$$

Then Eq. 11 and 12 take the form:

$$P^{\sigma\mu}=-\frac{T_0}{c}\frac{\left(\frac{\partial\vec{X}}{\partial\sigma}\cdot\frac{\partial\vec{X}}{\partial t}\right)\dot{X}^\mu-\left(-c^2+\left(\frac{\partial\vec{X}}{\partial t}\right)^2\right)X'^\mu}{c\frac{ds}{d\sigma}\sqrt{1-\frac{v_\perp^2}{c^2}}} \quad (34)$$

or:

$$P^{\sigma\mu}=-\frac{T_0}{c^2}\frac{\left(\frac{\partial\vec{X}}{\partial s}\cdot\frac{\partial\vec{X}}{\partial t}\right)\dot{X}^\mu+\left(c^2-\left(\frac{\partial\vec{X}}{\partial t}\right)^2\right)\frac{\partial X^\mu}{\partial s}}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \quad (35)$$

The component with μ = 0 is essentially simplified. Because $\dot{X}^0 = c$ and $\partial X^0/\partial s = c\partial t/\partial s = 0$, then:

$$P^{\sigma 0} = -\frac{T_0}{c}\frac{\left(\frac{\partial \vec{X}}{\partial s}\cdot\frac{\partial \vec{X}}{\partial t}\right)}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \quad (36)$$

A similar calculation for (Eq. 12) yields:

$$P^{\tau\mu} = -\frac{T_0}{c^2}\frac{\partial s}{\partial \sigma}\frac{\dot{X}^\mu - \left(\frac{\partial \vec{X}}{\partial s}\cdot\frac{\partial \vec{X}}{\partial t}\right)\frac{\partial X^\mu}{\partial s}}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \quad (37)$$

which, accordingly, gives:

$$P^{\tau 0} = \frac{T_0}{c}\frac{\partial s}{\partial \sigma}\frac{1}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \quad (38)$$

$$\vec{P}^{\tau} = -\frac{T_0}{c^2}\frac{\partial s}{\partial \sigma}\frac{\vec{v}_\perp}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \quad (39)$$

σ-parameterization: The choice of parameterization, in which the parameter is the length of the string is natural but in the general case the scale of the motion along the string can be changed so that $\sigma \propto s$. When the string moves, the lines of the constant σ are always perpendicular (see the previous point) to the lines of constant t. With this parameterization of the string surface, the tangent $\partial\vec{X}/\partial\sigma$ to the strings and the tangent $\partial\vec{X}/\partial t$ to the lines of the constant σ are perpendicular to each other at each point:

$$\frac{\partial \vec{X}}{\partial \sigma}\cdot\frac{\partial \vec{X}}{\partial t} = 0 \quad (40)$$

Since the velocity $\partial\vec{X}/\partial t$ is perpendicular to the string, it coincides with:

$$\vec{v}_\perp = \frac{\partial \vec{X}}{\partial t}$$

for all points and not only for one of its end points, in which this takes place irrespective of the parameterization. Recalling that *s* is a length parameter along a string, it follows that:

$$\frac{\partial \vec{X}}{\partial s}\cdot\frac{\partial \vec{X}}{\partial t} = 0 \quad (41)$$

and:

$$P^{\tau\mu} = \frac{T_0}{c^2}\frac{\partial s}{\partial \sigma}\frac{\frac{\partial X^\mu}{\partial t}}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \quad (42)$$

$$P^{\sigma\mu} = -T_0\sqrt{1-\frac{v_\perp^2}{c^2}}\frac{\partial X^\mu}{\partial s} \quad (43)$$

The equations of motion (Eq. 16) for t = τ have the form:

$$\frac{\partial P^{\tau\mu}}{\partial t} = -\frac{\partial P^{\sigma\mu}}{\partial \sigma} \quad (44)$$

Let us consider the components with μ = 0. It follows from Eq. 43 that $P^{\sigma 0} = 0$. From Eq. 42, it follows:

$$P^{\tau 0} = \frac{T_0}{c}\frac{\frac{\partial s}{\partial \sigma}}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \quad (45)$$

i.e., taking into account (Eq. 44):

$$\frac{\partial P^{\tau 0}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{T_0\frac{\partial s}{\partial \sigma}}{c\sqrt{1-\frac{v_\perp^2}{c^2}}}\right) = 0 \quad (46)$$

Spatial components can be recovered from Eq. 42 and 43:

$$\vec{P}^{\tau} = \frac{T_0}{c^2}\frac{\frac{\partial s}{\partial \sigma}}{\sqrt{1-\frac{v_\perp^2}{c^2}}}\vec{v}_\perp \quad (47)$$

$$\vec{P}^{\sigma} = -T_0\sqrt{1-\frac{v_\perp^2}{c^2}}\frac{\partial \vec{X}}{\partial s} \quad (48)$$

Now from Eq. 44, follows:

$$\frac{\partial}{\partial \sigma}\left[T_0\sqrt{1-\frac{v_\perp^2}{c^2}}\frac{\partial \vec{X}}{\partial s}\right] = \frac{\partial}{\partial t}\left[\frac{T_0}{c^2}\frac{\frac{\partial s}{\partial \sigma}}{\sqrt{1-\frac{v_\perp^2}{c^2}}}\vec{v}_\perp\right] = \frac{T_0}{c^2}\frac{\frac{\partial s}{\partial \sigma}}{\sqrt{1-\frac{v_\perp^2}{c^2}}}\frac{\partial \vec{v}_\perp}{\partial t} \quad (49)$$

where, Eq. 46 was used in obtaining the last relation. Passing to the partial derivatives with respect to s:

$$\frac{\partial}{\partial s}\left[T_0\sqrt{1-\frac{v_\perp^2}{c^2}}\frac{\partial \vec{X}}{\partial s}\right]=\frac{T_0}{c^2}\frac{1}{\sqrt{1-\frac{v_\perp^2}{c^2}}}\frac{\partial \vec{v}_\perp}{\partial t} \tag{50}$$

Now parameterization of the string is selected so that each of its segments with the same parameter of length s carries the same amount of energy (Eq. 55):

$$\frac{1}{c^2}\frac{\partial^2 \vec{X}}{\partial t^2}=\frac{\sqrt{1-\frac{v_\perp^2}{c^2}}}{\frac{\partial s}{\partial \sigma}}\frac{\partial}{\partial \sigma}\left[\frac{\sqrt{1-\frac{v_\perp^2}{c^2}}}{\frac{\partial s}{\partial \sigma}}\frac{\partial \vec{X}}{\partial \sigma}\right] \tag{51}$$

Let us define the quantity $A(\sigma)$:

$$A(\sigma)=\frac{\frac{\partial s}{\partial \sigma}}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \tag{52}$$

It follows from Eq. 46 that A (σ) does not depend on time. Let choose σ so that $A\equiv c/\tilde{c}$, where, $\tilde{c}$ is the speed of a wave described by a string. Then from Eq. 51 follows:

$$\frac{1}{\tilde{c}^2}\frac{\partial^2 \vec{X}}{\partial t^2}=\frac{\partial^2 \vec{X}}{\partial \sigma^2} \tag{53}$$

It can be seen that by choosing $\tilde{c}$ for A the propagation velocity of the wave in the resulting wave equation can be arbitrary (the analysis ends with the condition A = 1)[17]. On the one hand, it may seem that there is something artificial in this arbitrariness. On the other hand, this can be considered as the circumstance that a particle moving with arbitrary velocities can be described by wave.

HEISENBERG UNCERTAINTY RELATIONS IN NON-LOCAL APPROACH

Back to Eq. 46, to understand the physical meaning of this result, consider a small piece of string connected to $d\sigma$, which is a small fixed number. The movement of this particular piece of string is well defined now, after the lines of the constant σ have been fixed. Since ds denotes the length of a piece of string $d\sigma$, the value of ds can depend on time. If the expression inside the derivative in Eq. 46 is multiplied by a constant value $d\sigma$, then it can be seen that expression:

$$\frac{T_0 ds}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \tag{54}$$

does not depend on time. Expression (Eq. 54) is the relativistic energy associated with a piece of string. The rest energy in the above expression is T_0ds and the relativistic factor in the denominator turns (Eq. 54) into total energy. Therefore, expression (Eq. 46) asserts that the energy in each piece of string $d\sigma$ is conserved.

To find s leading to $A\equiv c/\tilde{c}$, let assign $\sigma = 0$ to one of the endpoints of the open string and continue path along the string, assigning to each piece ds of the string the interval $d\sigma$ given by the expression:

$$\frac{c}{\tilde{c}}d\sigma=\frac{ds}{\sqrt{1-\frac{v_\perp^2}{c^2}}}=\frac{dE}{T_0} \tag{55}$$

The first equality is equivalent to the requirement $A\equiv c/\tilde{c}$ and the second equality follows from the identification (Eq. 54) with energy dE, carried by a small piece of string. Equation 55 can be integrated from the end point $\sigma = 0$ up to the point Q, then:

$$\frac{c}{\tilde{c}}\sigma(Q)=\frac{E(Q)}{T_0} \tag{56}$$

The coordinate σ(Q) assigned to the point Q is equal to the energy E(Q), which is carried by a piece of string stretched from the selected end point to the point Q divided by the tension. From the above expression it also follows that:

$$\frac{c}{\tilde{c}}\sigma_1=\frac{E}{T_0} \tag{57}$$

where, $\sigma\in[0;\sigma_1]$ and E is the total energy of the string.

The choice of σ made for all strings is compatible with the orthogonality condition (Eq. 40). In fact, the lines of the constant σ did not change, only the values assigned to them have changed. The lines of the constant σ mean that the energy in the piece of the string on $[0;\sigma]$ is constant, such lines are perpendicular to the string.

Let us return to the expression (Eq. 56):

$$\frac{c}{\tilde{c}}T_0\sigma=E \tag{58}$$

The meaning of T_0 is clear on examination of static string[17]. On the one hand T_0/c^2 is a rest mass per unit length of string:

$$d\mu_0 = \frac{T_0}{c^2} ds \quad (59)$$

on the another, it is the potential energy per unit length of string:

$$dU = T_0 ds \quad (60)$$

According to Newton's second law at each point of the string, the derivative of the momentum of some of its part in time is determined by the tension of the string:

$$T_0 = \frac{dU}{ds} = -F_s = -\frac{dp_s}{dt} \quad (61)$$

where, F_s and p_s are projections of force acting on section ds and momentum of section ds on the direction of this section. Relation (Eq. 56) can be rewritten as:

$$\frac{c}{\tilde{c}} T_0 \sigma = \frac{c}{\tilde{c}} \left| \sum_i \vec{T}_{0i} d\vec{\sigma}_i \right| = \frac{c}{\tilde{c}} \left| \sum_i \vec{F}_i d\vec{\sigma}_i \right| = \frac{c}{\tilde{c}} \left| \sum_i \frac{d\vec{p}_i}{dt} d\vec{\sigma}_i \right| = E \quad (62)$$

where summation is on the all sections of string.

Uncertainty relations on the spatial components of the coordinates and momenta: In the wave description of a particle, its energy and wave frequency are related according to the corpuscular-wave dualism by the ratio:

$$E = h\nu \quad (63)$$

Taking into account Eq. 55 and 62:

$$\frac{c}{\tilde{c}} \left| \sum_i \frac{d\vec{p}_i}{dt} d\vec{\sigma}_i \right| = \frac{c}{\tilde{c}} \left| \sum_i \frac{d\vec{p}_i}{dt} \frac{d\vec{s}_i}{\sqrt{1-\frac{v_{\perp i}^2}{c^2}}} \right| = \frac{c}{\tilde{c}} \left| \int_{\Gamma_s} d\vec{s} \frac{d\vec{p}/dt}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \right| = h\nu \quad (64)$$

where, the integration takes place along the contour Γ_s, which is a string. Integration with respect to time of this relation leads to:

$$\left| \int_{\Gamma_s} d\vec{s} \int_{\Gamma_p} \frac{d\vec{p}}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \right| = \frac{\tilde{c}}{c} \int dt h\nu \quad (65)$$

where, the second integration on the left-hand side occurs along the contour Γ_p in the momentum space, which is a curve of the variation of the string sections momenta.

A string can describe a particle with certain energy if the spectrum of the string contains the given energy. The frequency ν on the right-hand side characterizes some oscillatory process, therefore, when under general superposition of arbitrary motions in the system, for compensation of large contributions to changes in coordinates and momenta due to the amplitude values of the quantities participating in the oscillations it is necessary to choose an integration time equal to:

$$T = 1/\nu \quad (66)$$

In the non-local approach, the objects under consideration are strings but it is not yet clear what they contain in terms of the constituent components. The only thing that while it is possible to rely is the fact that the speed of interaction propagation along a string is naturally to take as the speed of light. Taking into account, that string itself describes the wave behavior of a particle with propagation velocity $\tilde{c}$, the time T should be increased to:

$$\tilde{T} = T\frac{c}{\tilde{c}} \quad (67)$$

for compensation of large contributions to changes in coordinates and momenta due to the amplitude values of the quantities participating in the oscillations corresponding to the wave processes with propagation velocity $\tilde{c}$. Integrating the presented expression in accordance with the time variation within the time $\tilde{T}$, the relation:

$$\left| \int_{\Gamma_s} d\vec{s} \int_{\Gamma_p} \frac{d\vec{p}}{\sqrt{1-\frac{v_\perp^2}{c^2}}} \right| = h \quad (68)$$

can be obtained. This relation can now be rewritten as:

$$\left| \Delta\vec{p}\Delta\vec{s} \right| = \alpha \left\langle \sqrt{1-\frac{v_\perp^2}{c^2}} \right\rangle h \quad (69)$$

where, $\Delta\vec{p}$ and $\Delta\vec{s}$ are, respectively, the characteristic ranges of the momentum and the coordinates spread of the various sections of the string, angular brackets indicate the mean value of the value in brackets, α is a certain coefficient of the order of unity arising as a result of averaging. Let us decompose the vectors $\Delta\vec{p}$ and $\Delta\vec{s}$:

$$\Delta\vec{p} = \sum_{\mu=1}^{N-1} \Delta p_\mu \vec{e}_\mu \quad (70)$$

$$\Delta\vec{s} = \sum_{\mu=1}^{N-1} \Delta s_\mu \vec{e}_\mu \quad (71)$$

with respect to some orthonormal system $\vec{e}_\mu$, where (N-1) is the spatial dimension of the space under consideration (of full dimension N), then:

$$\left| \sum_{\mu=1}^{N-1} \Delta p_\mu \Delta s_\mu \right| = \alpha \left\langle \sqrt{1 - \frac{v_\perp^2}{c^2}} \right\rangle h \quad (72)$$

If the string is not strongly deformed during its movement, then:

$$\left| \sum_{\mu=1}^{N-1} \Delta p_\mu \Delta s_\mu \right| \approx \left\langle \sqrt{1 - \frac{v_\perp^2}{c^2}} \right\rangle h \quad (73)$$

If, moreover, it is non-relativistic, then:

$$\left| \sum_{\mu=1}^{N-1} \Delta p_\mu \Delta s_\mu \right| = h \quad (74)$$

Additional (compactificated) dimensions should be different from those observed. But this difference is not clear, because any data on the actual structure of the additional dimensions are absent. Therefore, it will be assumed that none of the spatial components is distinguished with respect to the others. Not distinguished in the sense of its contribution to the presented sum. Of course, explicit and implicit dimensions are different. Some of them are observable in everyday experience, while others are not. Then the relation under consideration becomes:

$$\Delta p_\mu \Delta s_\mu \approx \frac{h}{N-1} \quad (75)$$

At the moment it is assumed that the dimension of our space[15-17] is 11. Then the coefficient on the right-hand side is equal to:

$$\frac{h}{10} \quad (76)$$

and is in good agreement with the coefficient adopted by far in the overwhelming number of monographs on theoretical physics:

$$\frac{\hbar}{2} = \frac{h}{4\pi} \quad (77)$$

In Boichenko[13] it was noted that a multidimensional space is a kind of bridge connecting the phenomena of the micro and macro-world. The non-locality of our world leads in non-local description to multidimensional spaces, the possibility of changing the dimension-to clarify the nature of the cosmological constant. However, the specific size of the dimension did not appear in this case at all. In considering the nature of the Heisenberg uncertainty relations in the nonlocal approach, it turns out that the correct coefficient in this relation is obtained only for a certain dimensionality of space, which coincides with the dimension obtained when considering the internal logic of constructing a nonlocal theory.

Uncertainty relation on energy and time: From expression:

$$\frac{E^2}{c^2} - p^2 = m^2c^2 \quad (78)$$

where, E is the energy, p is the momentum, m is the rest mass of the object under consideration, which is an invariant for any system (in this case, strings) in inertial reference systems, the relationship between the change of momentum and energy in the system:

$$2\vec{p}\Delta\vec{p} = \frac{2E\Delta E}{c^2} \quad (79)$$

is obtained or:

$$p\Delta p_\mu = \frac{E\Delta E}{c^2} \quad (80)$$

where, μ is the coordinate of the vector Δp_μ along the direction of the vector p. Then:

$$\Delta p_\mu = \frac{\Delta E}{c^2}\frac{E}{p} = \frac{\Delta E}{c}\sqrt{1 + \frac{m^2c^2}{p^2}} = \frac{\Delta E}{c}\sqrt{1 + \frac{1-\beta^2}{\beta^2}} = \frac{\Delta E}{c}\sqrt{\frac{1}{\beta^2}} = \frac{\Delta E}{\tilde{c}} \quad (81)$$

where, the relativistic expression for the momentum was used:

$$p = \frac{m\tilde{c}}{\sqrt{1-\beta^2}} \qquad \beta = \frac{\tilde{c}}{c} \tag{82}$$

Multiplying the left and right sides of the equation by Δs_μ, it can be obtained (with respect to μ there is no summation):

$$\Delta s_\mu \Delta p_\mu = \frac{\Delta E \Delta s_\mu}{\tilde{c}} = \Delta E \Delta t \tag{83}$$

Finally, taking into account (Eq. 75):

$$\Delta E \Delta t \approx \frac{h}{N-1} \tag{84}$$

So, there was proved the.

Statement: In space-time structures with one-dimensional time in the simplest non-local description of particles (string theory), the Heisenberg uncertainty relations have the form:

$$\Delta s_\mu \Delta p_\mu \geq \delta_\mu \left\langle \sqrt{1-\frac{v_\perp^2}{c^2}} \right\rangle h \tag{85}$$

$$\Delta E_\mu \Delta t_\mu \geq \delta_{\mu p} \left\langle \sqrt{1-\frac{v_\perp^2}{c^2}} \right\rangle h \tag{86}$$

where, Δs_μ, Δp_μ, ΔE, Δt are the uncertainties of the μ-th coordinate and momentum of the particle, the energy and the time of registration, δ_μ, $\delta_{\mu p}$ are a certain coefficients arising as a result of averaging and depending on the structure of space-time, index μp corresponds to the direction along the momentum axis, h is Planck constant.

In spaces in which explicit and hidden measurements make the same contribution to the sum:

$$\left| \sum_{\mu=1}^{N-1} \Delta p_\mu \Delta s_\mu \right|$$

$$\delta_{\mu p} = \delta_\mu = \delta = 1/N_{sp} \tag{87}$$

$$N_{sp} = N\text{-}1 \tag{88}$$

where, N_{sp}, N are the dimension of space and the full dimension of space-time, respectively.

CONCLUSION

A derivation of the Heisenberg uncertainty relations in the non-local approach of string theory based on the action in the form of Nambu-Goto was proposed.

In the uncertainty relations, Planck's constant contains a coefficient that depends on the dimensionality of space.

Given the equivalence of the contribution of explicit and compactificated dimensions to these relations, this coefficient practically becomes equal to the coefficient used in the vast majority of existing monographs and papers on quantum mechanics.

Thus, both the corpuscular-wave dualism and the Heisenberg uncertainty relations receive their natural explanation in the non-local approach. In the local approach the nature of Heisenberg uncertainty relations is hidden behind the nature of the Schrödinger equation, which was actually postulated.

SIGNIFICANCE STATEMENT

At the moment, there is no derivation of Heisenberg's uncertainty relations in the non-local approach in the world literature. The justification of the uncertainty relations in the local approach is based on the algebra of quantum mechanical operators, which is postulated together with the Schrödinger equation. The novelty consists not only in deriving the Heisenberg uncertainty relations in the nonlocal approach. In this approach, we get something more. The right-hand side of the Heisenberg uncertainty relations contains a factor for h (Plank constant), depending on the dimension of the space. The local approach did not allow one to see the dependence of the Heisenberg uncertainty form on the dimensionality of the space. The local approach operates with three-dimensional space. The multidimensionality of space is another aspect introduced into the theory in the nonlocal approach (in string theory). Although a local approach also outlined a multi-dimensional space (starting with the Kaluza-Klein theories) but this was consistently implemented only in string theory, since string theory can not exist in a space of arbitrary dimension.

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