ISSN 1996-3343

Asian Journal of Applied Sciences

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OPEN ACCESS Asian Journal of Applied Sciences

ISSN 1996-3343
DOI: 10.3923/ajaps.2018.213.220

Research Article
Flexural Motions of Non-uniform Beams Resting on Exponentially Decaying Foundation

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Abstract
Background and Objective: The dynamic studies of vibrating bodies resting on an elastic foundation carrying moving distributed load are of considerable importance. To this end, several methods have been proposed to address this class of dynamical problems. The aim of this study was to obtain a closed form solution to the new foundation model and analyze basically for exponentially decaying foundation for both constant magnitude and harmonic variable magnitude. **Materials and Methods:** The versatile solution technique called Garlerkin and integral transformation were used to obtain solution to the governing equation. Solutions obtained were calculated for various values of foundation modulli F_0 and spatial coordinates x. **Results:** Analyses show that the higher values of the foundation modulli decrease the transverse deflections of the non-uniform Bernoulli-Euler beam while transverse deflections of the beam decrease as the spatial coordinates x decreases under the actions of moving distributed loads. **Conclusion:** The paper presents closed form solution for the displacement response of non-uniform beam carrying distributed load and it was observed that the responses amplitude of the constant load is smaller than that of the variable magnitude load. This shows that resonance is reached earlier in harmonic variable magnitude problem.

Key words: Exponentially decaying foundation, foundation modulli, non-uniform beam, distributed loads, vibrating system differential equation

Citation: Adedowole Alimi and Segun Nathaniel Ogunyebi, 2018. Flexural motions of non-uniform beams resting on exponentially decaying foundation. Asian J. Applied Sci., 11: 213-220.

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Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

In the past few years, authors in the area of applied Mathematics and Engineering have considered problems of dynamic vibration of beams and plates resting on or without an elastic foundation[1-6]. Oftentimes, both analytical and numerical methods are widely used in solving the moving load problems on elastic structures. Examples of moving loads include running across the bridge, cars, trains, cranes etc., which causes elastic structures to vibrate intensively as they acted on them especially at high velocities.

Various researches have worked on the flexural vibration of elastic structures having uniform cross sections whether the inertia effects of the moving load is considered or not[7-13]. Among studies whose non-uniform structural members have been subjected to concentrated or distributed forces is the attempt of Wu and Dai[14]. They used the transfer matrix method to investigate the dynamic responses of multi-span non-uniform beams under moving load. In a later development, Oni[15] investigated the response of non-uniform beam resting on elastic foundation to several moving masses. The deflection of the non-uniform beam was calculated for several values of foundation modulli and shown graphically as a function of time. Dugush and Eisenberger[16] also considered the dynamic behavior of multi-span non-uniform team traversed by a moving load at constant and variable velocities. They used both modal analysis direct methods.

More recently, Omolofe *et al.*[17] studied the transverse motion of non-prismatic deep team under the actions of variable magnitude moving loads. Omolofe and Ogunyebi[18] considered the dynamic characteristics of a rotating Timoshenko beam subjected to a variable magnitude load traveling at varying speed.

This present study, therefore, focused on response of non-uniform beams resting on exponentially decaying foundation and under uniformly distributed load.

MATERIALS AND METHODS

Definition of the problem

Assuming non-uniform simply supported beam with length L under distributed load: The distributed loads M_i move across the beam starting at time t = 0 with constant velocity C_i. The equation of motion for the system is given by the fourth order partial differential Eq.[15]:

$$-\frac{\partial Q(x,t)}{\partial x} + \mu(x)\frac{\partial^2 y(x,t)}{\partial t^2} - N\frac{\partial^2 y(x,t)}{\partial x^2} + b(x)\frac{\partial y(x,t)}{\partial t} + F(x)y(x,t) - q(x,t) = 0 \quad (1)$$

$$Q(x,t) = \frac{\partial D(x,t)}{\partial x} \quad (2)$$

where, Q(x, t) is the shear force, q(x, t) is the constant moving distributed force acting on the beam, μ is the mass of the beam per unit length L, b is the material damping intensity, y(x, t) is the vertical response of the beam, D(x, t) is the flexural moment and t is time.

The flexural moment acting on the beam across section is related to the vertical response as:

$$D(x,t) = -EI(x)\frac{\partial^2 y(x,t)}{\partial x^2} \quad (3)$$

where, EI (x) the flexural rigidity of the beam, E is the young modulus.

The distribution of the non-uniform characteristics may be assumed as power functions. The parameters α and n are used to approximate the actual non-uniformity of the beam given as:

$$I(x) = I_o(1+\alpha x)^{n+2} \quad (4a)$$

$$\mu(x) = \mu_o(1+\alpha x)^{n} \quad (4b)$$

$$b(x) = b_o(1+\alpha x)^{n} \quad (4c)$$

where, I(x) is the variable moment of inertia of the be beam, I_o, μ_o and b_o are the beam characteristics at x = 0. The boundary conditions depend on the constraints at the beam ends, however for a simply supported beam whose length is L, the vertical displacement at the beam ends are given as:

$$y(0,t) = y(L,t) = 0 \quad (5a)$$

$$y''(0,t) = y''(L,t) = 0 \quad (5b)$$

where dash means derivative with respect to x.

It is assumed that the initial conditions are:

$$y(x,0) = 0 = \frac{\partial^2 y(x,0)}{\partial t^2} \tag{6}$$

Analysis of non-uniform beam with exponentially decaying foundation to constant magnitude moving distributed forces: In this study, define the exponential decaying elastic foundation F(x) as:

$$F(x) = F_o e^{-\lambda x} \tag{7}$$

where, λ is a constant and F_o is the elastic foundation constant Furthermore, the constant vertical excitation acting on the beam is chosen as:

$$q(x,t) = PH(x - c_i t) \tag{8}$$

The distributed load is assumed to be of mass M and the time t is assumed to be limited to that interval of time within the mass on the beam, that is:

$$0 \leq f(t) \leq L \tag{9}$$

and H(x-ct) is the Heaviside function defined as:

$$H(x - ct) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases} \tag{10a}$$

with the properties:

$$\frac{d}{dx}\{H(x-ct)\} = \delta(x-ct)$$

$$H(x-ct)f(x) = \begin{cases} 0, & x \leq ct \\ f(x), & x \geq ct \end{cases} \tag{10b}$$

where, δ(x-ct) represents the Dirac delta function and H(x-ct) is a typical engineering function made to measure engineering applications which often involved functions that are either "off" or "on". c_i is the velocity of the ith particle of the system, t is the travelling time substituting Eq. 2-6 and 7 into Eq. 1:

Taking n = 1 for simplicity yields:

$$EI_o(x)\left[\frac{\partial^2}{\partial x^2}\left((1+\alpha x)^3 \frac{\partial^2 y(x,t)}{\partial x^2}\right)\right]$$

$$+\mu_o(1+\alpha x)\frac{\partial^2 y(x,t)}{\partial t^2} + b_0(1+\alpha x)\frac{\partial y(x,t)}{\partial t} + F_0 e^{-\lambda x} y(x,t) = PH(x - c_i t) \tag{11}$$

To the authors best of knowledge, a closed form solution to the fourth order Partial Deferential Eq. 2 governing the motion of the slender beam under the actions of moving force, does not exist. It is desirable to obtain some vital information about the vibrating system.

Approximate analytical solution: To solve the beam problem above in Eq. 11, it shall use the versatile solution technique called Galerkin's method often used in solving diverse problems involving mechanical vibrations[17] . The equation of the motion of an element of the beam is generally symbolically written in the form:

$$\Gamma y(x,t) - q(x,t) = 0 \tag{12}$$

where, Γ is the differential operator with variable coefficients, y(x, t) is the beam displacement, q(x, t) is the load acting on the beam, x and t are spatial coordinates and time, respectively. The solutions of the system of Eq. 11 is expressed as:

$$y(x,t) = \sum_{i=1}^{n} W_i(t) Q_i(x) \tag{13}$$

where, W_i (t) are coordinates in modal space and Q_i (x)are the normal modes of free vibration written as:

$$Q_i(x) = \sin\theta_i x + A_i \cos\theta_i x + B_i \sinh\theta_i x + C_i \cosh\theta_i x \tag{14}$$

where the constant, A_i, B_i and C_i define the space and amplitude of the beam vibration. Their values depend on the boundary condition associated with the structure. Thus, for a simply supported beam, it can be shown that:

$$A_i = B_i = C_i = 0 \text{ and } \theta_i = \frac{i\pi}{L} \tag{15}$$

Thus, for a beam with simple supports at both ends, Eq. 14 takes the form:

$$Q_i(x) = \sin\frac{i\pi x}{L} \tag{16}$$

Thus in view of Eq. 16 the transverse displacement response of a simply supported elastic beam, using an assumed mode method can be written as:

$$y(x,t)=\sum_{i=1}^{n}W_i(t)\sin\frac{k\pi x}{L} \tag{17}$$

Substituting Eq. 17 into the governing Eq. 11 and after some simplifications and arrangements one obtains:

$$\begin{aligned}&EI_o(x)\left[\frac{\partial^2}{\partial x^2}\left(\left(1+3\alpha x+3\alpha^2x^2+\alpha^3x^3\right)\frac{\partial^4}{\partial x^4}\sum_{i=1}^{n}W_i(t)\sin\frac{i\pi x}{L}\right)\right.\\&+\left(3\alpha+6\alpha^2x+3\alpha^3x^2\right)\frac{\partial^3}{\partial x^3}\sum_{i=1}^{n}W_i(t)\sin\frac{i\pi x}{L}+\left(6\alpha^2+6\alpha^3x\right)\frac{\partial^2}{\partial x^2}\sum_{i=1}^{n}W_i(t)\sin\frac{i\pi x}{L}+\\&\left.+\left(1+\alpha x\right)\frac{\partial}{\partial x}\sum_{i=1}^{n}W_i(t)\sin\frac{i\pi x}{L}\right]+\mu_o\left(1+\alpha x\right)\frac{\partial^2 y(x,t)}{\partial t^2}\sum_{i=1}^{n}W_i(t)\sin\frac{i\pi x}{L}\\&+b_0\left(1+\alpha x\right)\frac{\partial}{\partial t}\sum_{i=1}^{n}W_i(t)\sin\frac{i\pi x}{L}+F_0e^{-\lambda x}\sum_{i=1}^{n}W_i(t)\sin\frac{i\pi x}{L}=PH(x-c_it)\end{aligned} \tag{18}$$

To determine, W_i the expressions on the left hand sides of Eq. 18 are required to be orthogonal to the function $\sin\frac{k\pi x}{L}$. Thus:

$$\begin{aligned}&\int_0^L\sum_{i=1}^{n}\left[EI_o\left\{G_1(x)\left(\frac{i\pi}{L}\right)^4\sin\frac{i\pi x}{L}-G_2(x)\left(\frac{i\pi}{L}\right)^3\cos\frac{i\pi x}{L}-G_3(x)\left(\frac{i\pi}{L}\right)^2\sin\frac{i\pi x}{L}\right\}W_i(t)\right.\\&\left.+\mu_oG_4(x)\ddot{W}_i(t)\sin\frac{i\pi x}{L}+F_0e^{-\lambda x}W_i(t)\sin\frac{i\pi x}{L}\right]\sin\frac{k\pi x}{L}=\int_0^L PH(x-c_it)x\sin\frac{k\pi x}{L}dx\end{aligned} \tag{19}$$

Where:

$$G_1(x)=\left(1+3\alpha x+3\alpha^2x^2+\alpha^3x^3\right) \tag{20}$$

$$G_2(x)=\left(3\alpha+6\alpha^2x+3\alpha^3x^2\right) \tag{21}$$

$$G_3(x)=\left(6\alpha^2+6\alpha^3x\right) \tag{22}$$

$$G_4(x)=\left(1+\alpha x\right) \tag{23}$$

Further rearrangements and simplifications of Eq. 19 we obtain:

$$a_1(i,k)\ddot{W}_i(t)+a_2(i,k)\dot{W}_i(t)+a_3(i,k)W_i(t)=-P\left(\frac{k\pi}{L}\right)\cos\frac{k\pi ct}{L} \tag{24}$$

Where:

$$a_3(i,k)=EI_o\left[H_1-H_2-H_3\right]+H_4 \tag{25}$$

$$a_1(i,k)=\mu_o\left[I_1+\alpha I_2\right] \tag{26}$$

$$a_2(r,k)=b_o\frac{I_1}{2} \tag{27}$$

$$H_1=\left(\frac{r\Pi}{L}\right)^4\left[I_1+3\alpha I_2+3\alpha^2I_3+\alpha^3I_4\right] \tag{28}$$

$$H_2=\left(\frac{r\Pi}{L}\right)^3\left[3\alpha I_5+6\alpha^2I_6+3\alpha^3I_7\right] \tag{29}$$

$$H_3=\left(\frac{r\Pi}{L}\right)^2\left[6\alpha^2I_1+6\alpha I^3I_2\right] \tag{30}$$

The integrals I_i are as follow:

$$\begin{aligned}&I_1=\int_0^L\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx,\quad I_2=\int_0^L x\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx\\&I_3=\int_0^L x^2\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx,\ I_4=\int_0^L x^3\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx\\&I_5=\int_0^L\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx,\quad I_6=\int_0^L x\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx\\&I_7=\int_0^L x^2\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx\end{aligned} \tag{31}$$

Equation 24 is the second order ordinary differential equation with constant coefficient to a transformation. In what follow it subject the system of ordinary differential Eq. 24 to a Laplace transform defined as:

$$(\sim)\int_o^\infty(\cdot)e^{-st}dt \tag{32}$$

In conjunction with the initial conditions define in Eq. 8, yields the following algebraic equation:

$$\left[a_1(r,k)s^2+a_2(r,k)S+a_3(r,k)\right]W_r(s)=P_0\frac{S}{S^2+\theta^2} \tag{33}$$

Where:

$$P_0=\left(\frac{k\pi}{L}\right),\ \theta=\left(\frac{k\pi ct}{L}\right) \tag{34}$$

Subjecting Eq. 33 for further simplification yields:

$$W_i(S)=\frac{P_o}{(d_1-d_2)}\left(\frac{S}{S^2+\theta^2}\cdot\frac{1}{S-d_1}-\frac{S}{S^2+\theta^2}\cdot\frac{1}{S-d_2}\right) \quad (35)$$

Where:

$$d_1=\frac{-a_2+\sqrt{a_2^2-4a_1a_3}}{2a_1} \quad (36)$$

$$d_2=\frac{-a_2-\sqrt{a_2^2-4a_1a_3}}{2a_1} \quad (37)$$

To obtain the Laplace inversion of Eq. 35, it shall adopt the following representations:

$$g(s)=\frac{S}{S^2+\theta^2}, f_1(s)=\frac{1}{S-d_1} \text{ and } f_2(s)=\frac{1}{S-d_2} \quad (38)$$

So that the Laplace inversion of each term of the RHS (35) is the convolution of f_i's and g defined by as:

$$f_s g=\int_0^t f_i(t-u)g(u)du, \qquad i=1,2 \quad (39)$$

Thus the Laplace inversion of Eq. 35 is given by:

$$W_i(t)=P_k\left[\frac{e^{d_1t}}{d_1}I_8-\frac{e^{d_2t}}{d_2}I_9\right] \quad (40)$$

Where:

$$P_k=\frac{P_0}{(d_1-d_2)}, I_8=\int_0^t e^{-d_1u}\cos\theta u du \text{ and } I_9=\int_0^t e^{-d_2u}\cos\theta u du \quad (41)$$

Evaluating integrals in Eq. 41 we have:

$$\begin{aligned} I_8&=\frac{-\theta e^{d_1t}\sin\theta t+d_1-d_1e^{d_1t}\cos\theta t}{(\theta^2+d_1^2)}\\ I_9&=\frac{-\theta e^{d_2t}\sin\theta t+d_2-d_2e^{d_2t}\cos\theta t}{(\theta^2+d_2^2)}\end{aligned} \quad (42)$$

Further simplification of Eq. 40 yields:

$$\begin{aligned} W_i(t)=P_k\Bigg[&\frac{e^{d_1t}}{d_1(\theta^2+d_1^2)}\left(d_1-\theta e^{d_1t}\sin\theta t-d_1e^{d_1t}\cos\theta t\right)\\ &-\frac{e^{d_2t}}{d_2(\theta^2+d_2^2)}\left(d_2-\theta e^{d_2t}\sin\theta t-d_2e^{d_2t}\cos\theta t\right)\Bigg]\end{aligned} \quad (43)$$

Substituting Eq. 43 into Eq. 13 which on inversion yields:

$$\begin{aligned} y(x,t)=\sum_{i=1}^{n}P_k\Bigg[&\frac{e^{d_1t}}{d_1(\theta^2+d_1^2)}\left(d_1-\theta e^{d_1t}\sin\theta t-d_1e^{d_1t}\cos\theta t\right)\\ &-\frac{e^{d_2t}}{d_2(\theta^2+d_2^2)}\left(d_2-\theta e^{d_2t}\sin\theta t-d_2e^{d_2t}\cos\theta t\right)\Bigg]\sin\frac{i\pi x}{L}\end{aligned} \quad (44)$$

Equation 44 represents the transverse displacement response of the non-uniform Bernoulli beam resting on exponentially decaying foundation under the action of fast moving distributed forces.

Response of non-uniform beam to harmonic variable magnitude moving distributed loads: The dynamic behavior of structurally damped non-uniform beam when subjected to harmonic variable magnitude moving load is investigated. Thus, the load P(x,t) is given as:

$$q(x,t)=P\sin\Omega tH(x-c_mt) \quad (45)$$

where, Ω is the circular frequency of the harmonic force and all parameters are as defined previously substituting Eq. 45 in 1, vibration of the beam is then described by the equation:

$$\begin{aligned}&-\frac{\partial Q(x,t)}{\partial x}+\mu(x)\frac{\partial^2 y(x,t)}{\partial t^2}+b(x)\frac{\partial y(x,t)}{\partial t}+\\ &F(x)y(x,t)=P\sin\Omega tH(x-c_mt)\end{aligned} \quad (46)$$

Equation 46 is the governing equation describing the motion of non-uniform beams subjected to last moving loads of varying magnitude. Like in the previous section as closed-form solution to Eq. 46 is sought.

To this effect, use is made of an assumed mode method already alluded to and by this method the transverse detection $y_b(x, t)$ of non-uniform beam under the action of variable magnitude moving load can be written as:

$$y_b(x,t)=\sum_{m=1}^{n}W_m(t)D_m(x) \quad (47)$$

where, $W_m(t)$ coordinates in modal are space and $D_m(x)$ are the normal modes of free vibration.

Thus, for a simply supported beam Eq. 47 becomes:

$$y_b(x,t)=\sum_{m=1}^{\infty}W_m(t)\sin\frac{m\pi x}{L} \quad (48)$$

In view of Eq. 48 and following the same arguments as in the previous section and after some simplifications and rearrangements Eq. 46 becomes:

$$\sum_{m=1}^{\infty}\begin{pmatrix} a_1(m,k)\ddot{W}_m(t)+a_2(m,k)\dot{W}_m(t)+ \\ a_3(m,k)W_m(t)\end{pmatrix} = P_o \sin\Omega t \cos\frac{k\pi ct}{L} \quad (49)$$

where, c_m is the velocity of the mth particle of the system and other parameter are as defined previously. Without loss of generality, considering only the mth particle of the dynamical system yields:

$$a_1(m,k)\ddot{W}_m(t)+a_2(m,k)\dot{W}_m(t)+a_3(m,k)W_m(t) = P_o \sin\Omega t\cos\theta \quad (50)$$

Equation 50 is analogous to Eq. 24, thus subjection Eq. 50 to Laplace transform in conjunction with the boundary conditions and using convolution theory we obtain:

$$\begin{aligned} W_m(t) = P_k\Bigg[& \frac{e^{d_1t}\eta_1}{d_1(\eta_1^2+d_1^2)}\left(1-\frac{d_1}{\eta_1}e^{d_1t}\sin\eta_1 t - e^{d_1t}\cos\eta_1 t\right) \\ & + \frac{e^{d_1t}\eta_2}{d_1(\eta_2^2+d_1^2)}\left(1-\frac{d_1}{\eta_2}e^{d_1t}\sin\eta_2 t - e^{d_1t}\cos\eta_2 t\right) \\ & - \frac{e^{d_2t}\eta_1}{d_2(\eta_1^2+d_2^2)}\left(1-\frac{d_2}{\eta_1}e^{d_2t}\sin\eta_1 t - e^{d_2t}\cos\eta_1 t\right) \\ & - \frac{e^{d_2t}\eta_2}{d_2(\eta_2^2+d_2^2)}\left(1-\frac{d_2}{\eta_2}e^{d_2t}\sin\eta_2 t - e^{d_1t}\cos\eta_2 t\right)\Bigg] \end{aligned} \quad (51)$$

which on inversion yields:

$$\begin{aligned} y_b(x,t) = \sum_{m=1}^{\infty} P_k\Bigg[& \frac{e^{d_1t}\eta_1}{d_1(\eta_1^2+d_1^2)}\left(1-\frac{d_1}{\eta_1}e^{d_1t}\sin\eta_1 t - e^{d_1t}\cos\eta_1 t\right) \\ & + \frac{e^{d_1t}\eta_2}{d_1(\eta_2^2+d_1^2)}\left(1-\frac{d_1}{\eta_2}e^{d_1t}\sin\eta_2 t - e^{d_1t}\cos\eta_2 t\right) \\ & - \frac{e^{d_2t}\eta_1}{d_2(\eta_1^2+d_2^2)}\left(1-\frac{d_2}{\eta_1}e^{d_2t}\sin\eta_1 t - e^{d_2t}\cos\eta_1 t\right) \\ & - \frac{e^{d_2t}\eta_2}{d_2(\eta_2^2+d_2^2)}\left(1-\frac{d_2}{\eta_2}e^{d_2t}\sin\eta_2 t - e^{d_1t}\cos\eta_2 t\right)\Bigg]\sin\frac{m\pi x}{L} \end{aligned} \quad (52)$$

Equation 52 represents the transverse displacement response of non-uniform beam resting on exponentially decaying foundation under actions of harmonic variable magnitude moving distributed loads.

RESULTS

In dynamics of structure, the transverse displacement of an elastic beam may increase without bound. Therefore, one is interested in resonance conditions. Equation 51 clearly depicts that the non-uniform beam resting on exponentially decaying foundation traverse by a constant moving load will grow without bound whenever:

$$d_1 = d_2, d_1^2 = -\theta^2 \text{ or } d_2^2 = -\theta^2 \quad (53)$$

and the velocity at which this occurs, known as critical velocity is given by the relation:

$$c_i^2 = \left(a_a\left(a_2^2 - 4a_1a_2\right)^{\frac{1}{2}} + 2a_1a_3 - a_2^2\frac{1}{2}\left(\frac{L}{a_1k\pi}\right)^2\right) \quad (54)$$

while Eq. 51 shows that the same beam under the action of harmonic variable magnitude moving loads will experience resonance effects whenever:

$$d_1 = d_2, d_1^2 = -\eta_1^2 \quad \text{or } d_2^2 = -\eta_1^2 \quad (55)$$

and the velocity at which this occurs, known as critical velocity is given by the relation:

$$c_i^2 = \left(a_a\left(a_2^2 - 4a_1a_2\right)^{\frac{1}{2}} + 2a_1a_3 - a_2^2\frac{1}{2}\left(\frac{L}{a_1k\pi}\right)^2\right) \quad (56)$$

Therefore, it is evident from Eq. 54 and 56 that the critical velocity of non-uniform beam resting exponentially decaying foundation and under the actions of constant magnitude moving load is greater than that of the same beam under the action variable magnitude moving load. Hence, resonance is reached earlier in the latter.

In order to illustrate the foregoing analysis, the non-uniform beam of length 12.20 m considered. Furthermore, flexural rigidity EI is 6.068×10^6 $m^{-3}/5^2$, α = 0.025 and the moving load is assumed to travel at the constant velocity of 8.123 m sec^{-1}. The transverse deflections of the beam are calculated and plotted against time for various values of foundation constant (modulli) and spatial coordinates x. Values of K b/w 0 N m^{-3} and 500000 N m^{-1}.

Figure 1 displayed the deflection profile of the simply supported beam under the action of traveling distributed forces when traveling loads are of constant magnitude. For various values of foundation modulli Fo, the figure shows that as Fo increases the deflection of the non-uniform beam decreases. For various time t, the displacement of the beam

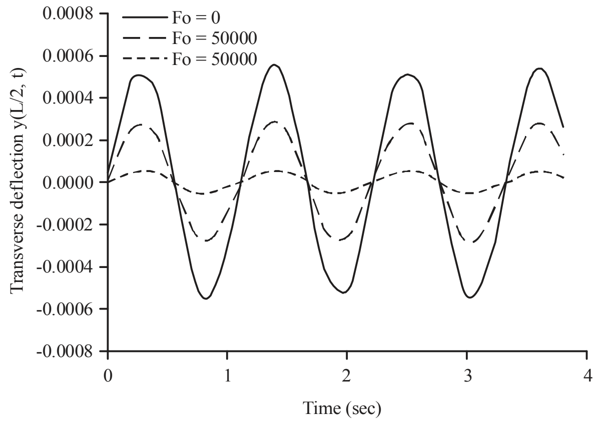

Fig. 1: Displacement response of a simply supported structurally damped thin beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads for various values of foundation modulus K_0

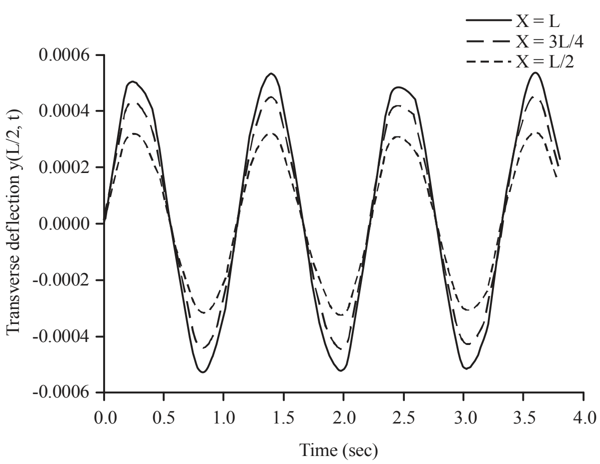

Fig. 2: Transverse displacement response of a simply supported structurally damped thin beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads for various values of spatial coordinates x and for fixed value of foundation modulus $F_0 = 4000$

for fixed K for various values of spatial coordinates x are shown in Fig. 2. In Fig. 3, the deflection profile for various values of foundation modulli Fo was given when the simply supported beam is traversed by traveling load are of variable harmonic magnitude. Figure 4 shows the displacement response of the structure for fixed k and various values of spatial coordinate x. It is deduced from these figures that the interval at which structure is supported affect the response amplitude of the beam significantly.

Finally, Fig. 5 depicts the comparison of the traversed displacement of constant and harmonic variable moving loads

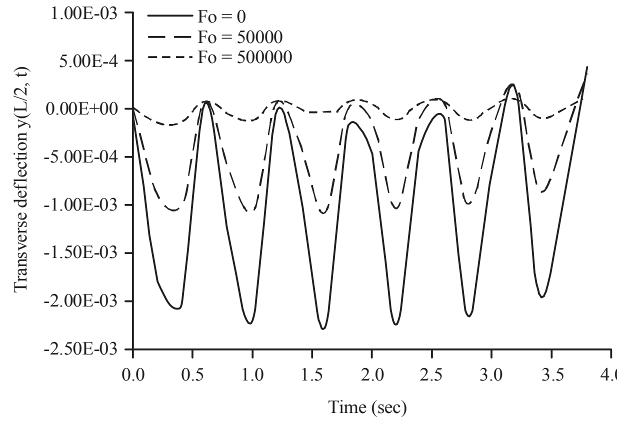

Fig. 3: Deflection profile of a simply supported structurally damped thin beam resting on exponentially decaying foundation and subjected to harmonic variable magnitude moving loads for various values of foundation modulus K_0

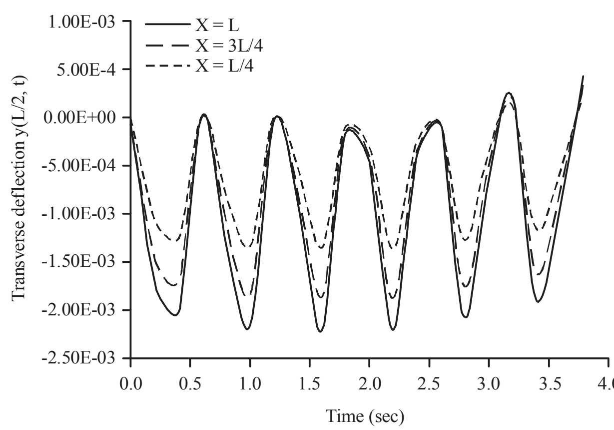

Fig. 4: Transverse displacement response of a simply supported structurally damped thin beam resting on exponentially decaying foundation and subjected to harmonic variable magnitude moving loads for various values of spatial coordinates and for fixed value of foundation modulus $F_0 = 4000$

for fixed value of foundation modulus Fo = 4000. Clearly, the response amplitudes of variable magnitude, moving load is higher than that of the constant magnitude moving load.

DISCUSSION

In this study, analytical solutions were obtained for the beam's governing equation. Numerical analysis is also carried out and results show the critical speed for the system traversed by constant moving loads is higher than that under the influence of variable moving loads.

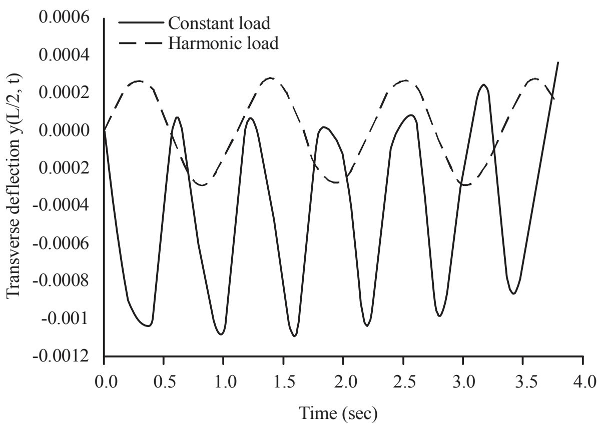

Fig. 5: Comparison of the response of the simply supported structurally damped thin beam to constant and variable magnitude moving distributed load for foundation modulus $K_0 = 4000$

Hence, justification was achieved when this study was correlated with results obtained in Prokic *et al.*[2], Li *etal.*[3], Oni[7], Gbadeyan and Oni [9], Dugush and Eisenberger[16], Saravi *et al.*[4] and Omolofe *et al.*[17].

CONCLUSION

The results obtained in this present study give some contribution to the theory of vibration in solid mechanics especially moving load problems for exponentially decaying system. Here, the response amplitude of the structural member decreases with an increase in the foundation constant. Essentially, this new findings are likely to find certain applications in solutions of Beam and Plate problems for all variants of boundary conditions.

SIGNIFICANCE STATEMENT

This present study reported for the first time a closed form solution to the new structural models incorporating the dynamic effects of constant and variable harmonic magnitude. From the study, results obtained enabled the validation, in comparison, that the responses of constant moving distributed load is smaller than that of the variable magnitude load for the non-prismatic beam considered. This shows that resonance is reached earlier in variable magnitude load problem. Hence, the study can be used in real design problems by construction and transport engineers.

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