ISSN 1996-3343

Asian Journal of Applied Sciences

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OPEN ACCESS Asian Journal of Applied Sciences

ISSN 1996-3343
DOI: 10.3923/ajaps.2018.38.45

Research Article
On Exponentiated Skewed Student T Error Distribution on Some Volatility Models: Evidence of Standard and Poor-500 Index Return

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Abstract
Background and Objective: Error distributions were found to be very useful in volatility modeling financial time series. To this end, several error distribution innovation were proposed for estimating the true parameters of volatility models in term of fitness and forecast as a results of excess leptokurtic presence in financial stock market trigger by economic crises and wars etc. The aim of this study was to propose new class of error distributions using class of exponentiated distribution method for fitness and forecasting performance of volatility models. **Materials and Methods:** A daily returns data from standard and poor 500 (S and P500) index return from the period of 2007-2017 were used to validate the new error distribution and Jarque-Bera (JB), ADF test, ARCH effect test were used to validate the assumption of volatility models, maximum likelihood (ML) methods were used to estimate the parameters of the volatility models under five error innovation distribution. **Results:** From the obtained results, it was observed that TGARCH and APARCH model outperformed in term of best fitness and forecasting performance under the proposed error innovation distribution. **Conclusion:** This study will enable a better understanding of error innovation distribution in improvement of volatility models.

Key words: Fitness and forecasting performance, volatility models, error innovation distributions and maximum likelihood

Citation: Samson Agboola, Hussaini Garba Dikko and Osebekwin Ebenezer Asiribo, 2018. On exponentiated skewed student T error distribution on some volatility models: Evidence of standard and poor-500 index return. Asian J. Applied Sci., 11: 38-45.

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Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Error distribution is one of the vital techniques in estimating the true parameters of any volatility models because volatility is affected by reaction from the stock market as a result of political disorder, wars and economic crises. Events of such could trigger variation to stock prices falling yielding to high leptokurtic and since normal distribution have this property of kurtosis, it were proposed and used as an error distribution were used in estimating volatility model[1]. However, this error distribution, the normal distribution proposed has gained more ground in the estimation of the volatility models, followed by the student t distribution proposed[1,2]. Furthermore, in order to estimate the parameters of these heteroscedastic models, various distribution of error innovation have been proposed. This is because error distribution plays significant role in estimating the parameters of the heteroscedastic model[3]. There are six forms of error distributions that have gained popularity as mention above in volatility modelling[4-6] namely normal distribution, Skewed normal distribution, Student-t distribution, Skewed student -t distribution, Generalized error distribution and Skewed Generalized error.

The previous literature has shown that for the past decade, no improvement or enhancement have been made on the existing error distributions. Most of the studies conducted within the last decade focus on the application of the existing distributions.

Therefore, this study tends to bridge this gap by developing a more vigorous error innovation distribution by improving on the flexibility of one of the existing distribution of error innovation by adopting the skewed student t proposed[7] with two degree of freedom as stated due to its unique properties and develop more flexible classes of the skewed student t error distribution using exponentiated methods that will give a better characterization of volatility behaviour of asset returns and also to find out if the having more shape parameter in an error distribution will give a more flexible results in the volatility models. This study aimed on volatility modelling using new class of error innovation distribution to estimate the parameter of some volatility models and evaluated the best performance in terms of fitness and forecasting.

MATERIALS AND METHODS

Data used to validate the new proposed error distribution were the data of standard and poor 500 index return from the period of 2007-2017.

Computation of return series from price: Let :

$$R_{st} = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (1)$$

where, P_t and P_{t-1} are the present and previous closing prices and R_{st} the continuously compounded return series which is the natural logarithm of the simple gross return.

Stationarity test: Stationarity of the return series of the augmented dickey-fuller (ADF) test is given as:

Let:

$$x_t = \phi_1 x_{t-1} \quad (2)$$

$$x_t - x_{t-1} = \phi_1 x_t - x_{t-1}$$

$$\Delta x_t = (\phi_i - 1)x_{t-1}$$

$$\Rightarrow \phi_1 - 1 = 0 \text{ or } \phi_1 = 1$$

Null hypothesis is H_0: $N_1 = 1$ and alternative hypothesis is: H_1: $\phi_1 < 1$:

$$\text{Test statistic }(\text{t-ratio}) = \frac{\phi_1^n - 1}{\text{std}(\phi_1^n)} = \frac{\sum_{t=1}^{T} P_{t-1} e_t}{\text{std}(\phi_1^n)} \quad (3)$$

Where:

$$\phi_1 = \frac{\sum_{t=1}^{T} P_{t-1} p_t}{\sum_{t=1}^{T} P_{t-1}^2} \text{ and } \hat{\sigma}^2 \; \frac{\sum_{t=1}^{T}\left(P_{t-1} - \hat{\phi}_1 p_{t-1}\right)^2}{T-1}$$

$P_0 = 0$, T is the sample size and ϕ_1 for each Insurance stock. The null hypothesis is rejected if the calculated value of t is greater than t critical value.

Test for ARCH effect:

$$r_t = \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1} \quad (4)$$

After obtaining the residuals e_t, the next step is regress the squared residual on a constant and its q lags as in the following equation:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \ldots + \alpha_q e_{t-q}^2 + r_t \quad (5)$$

The null hypothesis, that there is no ARCH effect up to order q can be formulated as:

$$H_0: \alpha_1 = ... = \alpha_q = 0 \quad (6)$$

Against the alternative:

$$H_a: \alpha \neq 0 \text{ for some } i \in [1,.., m] \quad (7)$$

Some volatility models: The ARCH (q) model formulates volatility as follows[1]:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_q \varepsilon_{t-q}^2 + r_t \quad (8)$$

where, α_i>0 for i = 0, 1, 2...q are the parameters of the models

The GARCH (p, q) model was stated as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_q \varepsilon_{t-q}^2 + \beta_i \varepsilon_{t-1}^2 + r_t \quad (9)$$

where, α_i>0 and β_i>0 for all i and j.

The EGARCH (p, q) model was proposed[8] formulate the volatility as follows:

$$\ln\left(\sigma_t^2\right) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left[\lambda \varepsilon_{t-i} + \gamma \left\{ \left| \varepsilon_{t-i} \right| - \sqrt{\frac{2}{\pi}} \right\} \right] + \sum_{i}^{q} \beta_i \ln\left(\sigma_{t-j}^2\right) \quad (10)$$

where, α_0, α_i, γ, β_j are the parameters of the model.

The Threshold GARCH model is similar to GJR-GARCH[9] stated as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \left(\alpha_i + \gamma_i + N_{t-i}\right) \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (11)$$

$$\alpha_0, \alpha_i, \gamma, \beta_j \geq 0$$

where, N_{t-i} is an indicator for negative ε_{t-i} that is N_{t-i} is 1 if ε_{t-i}<0 and 0.

Model selection: Akaike information criteria (AIC) the most commonly used model selection criteria. The formula is given as:

$$AIC = 2K\text{-}2\ln(L) \quad (12)$$

where, k is the number of parameters in the model and L is the maximized value of the likelihood function for the model.

Forecasting evaluation: Evaluating the performance of different forecasting models plays a very important role in choosing the most accurate models. The most widely used evaluation measure is root mean square error (RMSE) given as:

$$x \sim ESSTD\ (u, \lambda)$$

$$RMSE = \sqrt{\frac{\sum_{t=T+1}^{T+n} \left(\hat{\sigma}_t^2 - \sigma_t^2\right)^2}{n}} \quad (13)$$

where, n is the number of steps ahead, T is the sample size, $\hat{\sigma}_t$ and σ are the square root of the conditional forecasted volatility and the realized volatility, respectively.

Maximum likelihood estimator

Exponentiated skewed student-t distribution (ESSTD): The Exponentiated skewed student t-distribution as[10]:

$$z_t \sim ESSTD\ (u, \lambda)$$

Its PDF was given by:

$$g(x) = u \left\{ \frac{1}{2} \left(1 + \frac{x}{\sqrt{\lambda + x^2}} \right) \right\}^{u-1} \frac{\lambda}{2\left(\lambda + x^2\right)^{\frac{3}{2}}} \quad (14)$$

where, u>0, λ>0 and λ, is Skewed parameter and u is a shape parameters where the error distribution of the Exponentiated Skewed student-t distribution is given as:

$$g\left(z_t, \alpha, \lambda\right) = \alpha \left\{ \frac{1}{2} \left(1 + \frac{z_t}{\sqrt{\lambda + z_t^2}} \right) \right\}^{\alpha-1} \frac{\lambda}{2\left(\lambda + z_t^2\right)^{\frac{3}{2}}} \quad (15)$$

$$g\left(z_t, \alpha, \lambda\right) = \alpha \left\{ \frac{1}{2} \left(1 + \frac{z_t}{\sqrt{\lambda + z_t^2}} \right) \right\}^{\alpha-1} \frac{\lambda}{2\left(\lambda + z_t^2\right)^{\frac{3}{2}}} \quad (16)$$

If $\varepsilon_t = z_t \sigma_2$:

$$g\left(\varepsilon_t, \alpha, \lambda\right) = \alpha \left\{ \frac{1}{2} \left(1 + \frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda + \left(\frac{\varepsilon_t}{\sigma_t}\right)^2}} \right) \right\}^{\alpha-1} \frac{\lambda}{2\left(\lambda + \left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}} \left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}} \right) \quad (17)$$

Log-likelihood function of error innovation of Exponentiated Skewed student-t distribution:

$$L(\theta)=\prod_{i=1}^{n} g(\varepsilon_t,\ \alpha,\ \lambda)=L(\varepsilon_t,\ \alpha,\ \lambda)=\prod_{i=1}^{n} g(\varepsilon_t,\ \alpha,\ \lambda)$$

where, $\theta = (\alpha, \lambda, \sigma_t)$ and $\sigma_t^2=\omega+\sum_{i=1}^{n}\alpha_i\varepsilon_{t-i}^2+\sum_{i=1}^{n}\beta_i\sigma_{t-i}^2$:

$$=\prod_{i=1}^{n}\left[\alpha\left\{\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right\}^{\alpha-1}\frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right)\right] \quad (18)$$

$$=\alpha^n\lambda^n\prod_{i=1}^{n}\left[\left\{\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right\}^{\alpha-1}\frac{1}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right)\right] \quad (19)$$

Taking the log likelihood function of the above equation:

$$\begin{aligned}\text{InL}(x;\ u,\ \lambda)\sigma_t^2 &= n\log(\alpha)+nt\log(\lambda)-n\alpha\log 2+\\ &(\alpha-1)\sum_{i=1}^{n}\log\left(1+\frac{\varepsilon_t}{\sigma_t\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)-\\ &\frac{3}{2}\sum_{i=1}^{n}\log\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)-0.5n\log\left(\sigma_t^2\right)\end{aligned} \quad (20)$$

Exponentiated generalized skewed student-t distribution: The Exponentiated generalized skewed student t distribution as[11]:

$$g(z_t,\ \alpha,\ \nu,\ \lambda)=\alpha\nu\left[1-\frac{1}{2}\left(1+\frac{z_t}{\sqrt{\lambda+z_t^2}}\right)\right]^{\alpha-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{z_t}{\lambda+z_t^2}\right)\right]^{\alpha}\right\}^{\nu-1}\frac{\lambda}{2\left(\lambda+z_t^2\right)^{\frac{3}{2}}} \quad (21)$$

$$g(z_t,\ \alpha,\ \nu,\ \lambda)=\alpha\nu\left[1-\frac{1}{2}\left(1+\frac{z_t}{\sqrt{\lambda+z_t^2}}\right)\right]^{\alpha-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{z_t}{\lambda+z_t^2}\right)\right]^{\alpha}\right\}^{\nu-1}\frac{\lambda}{2\left(\lambda+z_t^2\right)^{\frac{3}{2}}} \quad (22)$$

If $\varepsilon t = z_t\sigma_2$:

$$g(z_t,\ \alpha,\ \nu,\ \lambda)=\alpha\nu\left[1-\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha}\right\}^{\nu-1}\frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right) \quad (23)$$

Log-likelihood function of error innovation of Exponentiated Generalized Skewed student-t distribution:

$$L(\theta)=\prod_{i=1}^{n} g(\varepsilon_t,\ \alpha,\ \nu,\ \lambda)=\prod_{i=1}^{n} g(\varepsilon_t,\ \alpha,\ \nu,\ \lambda) \quad (24)$$

Where:

$$\theta=(\alpha,\ \nu,\ \lambda,\ \sigma_t)\ \text{and}\ \sigma_t^2=\omega+\sum_{i=1}^{n}\alpha_i\varepsilon_{t-i}^2+\sum_{i=1}^{n}\beta_i\varepsilon_{t-i}^2$$

$$=\prod_{i=1}^{n}\left[\alpha\nu\left[1-\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha}\right\}^{\nu-1}\frac{\lambda}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right)\right] \quad (25)$$

$$=\alpha^n\nu^n\lambda^n\prod_{i=1}^{n}\left[\left[1-\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha-1}\left\{1-\left[1-\frac{1}{2}\left(1+\frac{\frac{\varepsilon_t}{\sigma_t}}{\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2}}\right)\right]^{\alpha}\right\}^{\nu-1}\frac{1}{2\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right)^{\frac{3}{2}}}\left(\frac{1}{\left(\sigma_t^2\right)^{\frac{1}{2}}}\right)\right] \quad (26)$$

Taking the log likelihood function of the above equation:

$$
\begin{aligned}
\text{InL}\,(x;\, u,\, v,\, \lambda) &= n\log(\alpha) + n\log(v) + n\log(\lambda) - n\log(2) \\
&+ (\alpha-1)\sum_{i=1}^{n}\log\left(1-\frac{1}{2}\left(1+\frac{\varepsilon_t}{\sigma_t\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma}\right)^2}}\right)\right) \\
&+ (v-1)\sum_{i=1}^{n}\log\left\{1-\left[1-\frac{1}{2}\left(1+\frac{\varepsilon_t}{\sigma_t\sqrt{\lambda+\left(\frac{\varepsilon_t}{\sigma}\right)^2}}\right)\right]^u\right\} \\
&- \frac{3}{2}\sum_{i=1}^{n}\log\left(\lambda+\left(\frac{\varepsilon_t}{\sigma_t}\right)^2\right) - 0.5n\log\left(\sigma_t^2\right)
\end{aligned}
\quad (27)
$$

Standardized Skewed student t-distribution:

$$
f(z_t,\, \mu,\, \sigma,\, v,\, \lambda) = \begin{cases} bc\left[1+\frac{1}{v-2}\left(\frac{b\left(\frac{z_t-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right]^{-\frac{v+1}{2}}, & z_t < -\frac{a}{b} \\ bc\left[1+\frac{1}{v-2}\left(\frac{b\left(\frac{z_t-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right]^{-\frac{v+1}{2}}, & z_t \geq -\frac{a}{b} \end{cases} \quad (28)
$$

where, v is the shape parameter with $2<v<\infty$ and λ is the Skewedness parameters with $-1<\lambda<1$, μ and σ^2 are the mean and variance of the Skewed student t-distribution:

$$
a = 4\lambda c\left(\frac{v-2}{v-1}\right),\ b = 1+3\lambda^2-a^2,\ c = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\Gamma\left(\frac{V}{2}\right)}}
$$

Standardized skewed generalized error distribution:

$$
f(z_t / v,\, \varepsilon,\, \theta,\, \delta) = \frac{v}{2\theta\Gamma\left(\frac{1}{v}\right)}\exp\left[-\frac{|z_t-\delta|^v}{\left[1+\text{sign}(z_t-\delta)\varepsilon\right]^v\theta^v}\right] \quad (29)
$$

$$
\theta>0,\ -\infty<z_t<\infty,\ v>0,\ -1<\varepsilon<1,\ -\infty<z_t<\infty
$$

Where:

$$
\theta = \Gamma\left(\frac{1}{v}\right)^{0.5}\Gamma\left(\frac{3}{V}\right)^{-0.5} S(\varepsilon)^{-1}
$$

$$
\delta = 2\varepsilon S\,(\varepsilon)^{-1}
$$

$$
S(\varepsilon) = \sqrt{1+3\varepsilon^2-4A^2\varepsilon^2}
$$

$$
A = \Gamma\left(\frac{2}{v}\right)\Gamma\left(\frac{1}{v}\right)^{-0.5}\Gamma\left(\frac{3}{v}\right)^{-0.5}
$$

where, $v>0$ is the shape parameter, ε is a Skewedness parameter with $-1<\varepsilon<1$.

Skewed normal distribution:

$$
f(z_t) = \frac{1}{\sigma\pi}e^{-\frac{(z_t-\varepsilon)^2}{2\sigma^2}}\int_{-\infty}^{\alpha}\frac{z_t-\varepsilon}{\sigma}e^{-\frac{t^2}{2}}dt,\ -\infty<z_t<\alpha \quad (30)
$$

where, ε is the location, σ is the scale and α denotes the shape parameter.

RESULTS AND DISCUSSION

Empirical results: An empirical analysis of the S and P500 index returns were carried out on returns series. The obtained results as shown in Table 1 showed that the mean return series were negative, positive Skewed and high kurtosis for S and P500 index returns. The result of Jarque-Bera statistic revealed that the return series for S and P500 index returns was not normally distributed as the p-values were less than 1 and 5%.

Stationarity test: A test of stationarity were carried out using the augmented dickey- fuller (ADF) test. The results obtained for S and P500 index returns showed that the Augmented

Table 1: Descriptive statistics of standard and poor 500 (S and P500) index returns

Statistics	Returns of S and P500
Mean	-0.0002
Median	-0.0004
Std. Dev.	0.0129
Skewedness	0.3357
Kurtosis	13.485
Jarque-bera	12008.370
Probability	0.001
Observations	2610.000

Dickey-Fuller test statistic were all less than their critical values at 1% as shown in Table 2. Hence, there were no unit root. The return series were all stationary. Therefore, there were no need for transformation.

Autoregressive Conditional Heteroscedasticity (ARCH) effect test: Table 3 showed the result of the ARCH effect test at different lag with T-2m-1, where T is the total sample size of the return and m is the lag of the series. The results of F Statistic were tested at different lag to validate the presence of heteroscedasticity in the return series. Therefore, the statistic shows that there were all significant at 1% for the S and P500 index returns which showed that there was ARCH Effect in the return series using Lagrange Multiplier test.

Table 2: Augmented dickey-fuller (ADF) test of S and P500 index returns

Stocks	ADF test statistic	Comment
S and P500	-12.36540	Stationary at level without transformation

1% critical = -3.432219

Table 3: Lagrange multiplier test of the presence of ARCH effect

	ARCH Effect	F-statistic	p-value
S and P500	ARCH 1-2 test	F (2,2390) = 243.93	0.001
index returns	ARCH 1-5 test	F (5,2384) = 150.66	0.001
	ARCH 1-10 test	F (10,2374) = 93.623	0.001

Estimates of the parameters of GARCH models and its extension based on standard and poor 500 (S and P500) Index returns: Table 4 and 5 presented the parameter estimates of GARCH model and its extension estimated at five error distributions such as Skewed normal, Skewed Student-distribution and Skewed generalized error distribution, Exponentiated skewed student t and the proposed Exponentiated generalized skewed student t distribution using returns from S and P500. Table 4 showed the estimate using the new proposed error distribution with ESSTD and Table 5 showed the estimate using the existing error distributions. The result showed that the returns exhibited volatility clustering. This was concluded because the GARCH term were significant in most of the models considered ($p<0.05$) and ($p<0.01$) which means that small changes in volatility of both returns tends to be followed by large changes in volatility while small changes in volatility tends to be followed by small changes in volatility. In terms of leverage effect which measured whether there was a negative relationship between asset returns and volatility were found to be significant in GARCH, GJR-GARCH, EGARCH, TGARCH and APARCH models estimated at the five distributions of error innovation ($p<0.05$).

Table 4: Parameter estimation of GARCH and its extension on S and P500 index returns

Model	Error	ω (p-value)	α_1 (p-value)	β_1 (p-value)	γ_1 (p-value)	δ (p-value)	Skewed (p-value)	Shape (p-value)
GARCH (1,1)	SSTD	1.781×10^{-06}***	1.301×10^{-01}***	8.671×10^{-01}			1.099***	5.018***
	SNORM	2.254×10^{-06}***	1.169×10^{-01}***	8.656×10^{-01}			1.172**	
	SGED	2.091×10^{06}***	1.259×10^{-01}***	8.624×10^{-01}***			1.052***	1.173***
GJR-GARCH (1,1)	SSTD	0.00002***	0.2492***	0.8845***	-0.2861**		1.1651**	5.5185**
	SNORM	0.00002***	0.2007***	0.8944***	-0.2300***		1.2036***	
	SGED	0.000002***	0.2360***	0.8827***	-0.2689***		1.1244***	1.2603***
EGARCH (1,1)	SSTD	-0.21282***	0.21282**	0.97706**	-0.12412**		1.1764**	5.5422**
	SNORM	-0.23031***	0.17975**	0.97431**	0.11866**		1.22533**	
	SGED	-0.24797**	0.20479***	0.97327***	0.12999**		1.14487**	1.27319***
TGARCH (1,1)	SSTD	0.000002	0.13037	0.86715**			1.0991**	5.0139
	SNORM	0.000002	0.11690*	0.86596***			1.17207**	
	SGED	0.000002	0.12573	0.86271***			1.0524***	1.17286***
APARCH (1,1)	SSTD	0.00002	0.05980	0.86703	-0.9996*	2.000***	1.1489**	5.7842**
	SNORM	0.000002	0.05116	0.8780***	-0.9959**	2.000**	1.2005**	
	SGED	0.000002	0.06014	0.8651**	0.9984*	2.000**	1.1141**	1.2524**

Table 5: Estimates of the parameters of GARCH models and its extension based on Standard and Poor 500 (S and P500) index returns using new classes of error distributions

Model	Error	ω	α_1	β_1	γ_1	δ	Skewed	Shape(u)	Shape (v)
GARCH (1,1)	ESSTD	3.817×10^{-07}**	1.225×10^{-02}	-6.788×10^{-03}			6.736×10^{-02}	4.3610**	
GJR-GARCH (1,1)	ESSTD	0.06105**	0.00875	0.001961**	0.1000*		0.5530	4.4560**	
EGARCH (1,1)	ESSTD	0.50076**	0.10069**	0.08105	0.06155*		0.2698**	1.2651*	
TGARCH (1,1)	ESSTD	0.00946*	0.00451**	0.1306	-0.0725		1.7040	1.3170*	
APARCH (1,1)	ESSTD	3.792×10^{-07}	0.10000**	0.1021**	0.0995	0.08976	0.03855	7.6382**	
GARCH (1,1)	EGSSTD	0.0099**	0.1000	0.09240**			3.862×10^{-7}	7.1781**	6.1052**
GJR-GARCH (1,1)	EGSSTD	0.0099**	0.1000	0.09240**			3.862×10^{-7}	7.1781**	6.1052**
EGARCH (1,1)	EGSSTD	0.01312*	0.00100**	0.09931	0.09262		0.4218	7.2492**	6.1748**
TGARCH (1,1)	EGSSTD	-0.007444	0.0413**	0.12989	0.13720**		0.14319	1.2214**	1.7869*
APARCH (1,1)	EGSSTD	3.251×10^{-07}	1.0011	0.0994**	0.1021*	1.06782**	0.5547**	1.0010*	9.6450**

*at 5%, **at 1% and ***at 10% significant

Table 6: Shows the result of the fitness and model selection based on log-likelihood and akaike information criteria (AIC) of S and P500 index returns

Models	Error	Log-Likelihood	AIC
GARCH (1,1)	SSTD	8456.201	-6.4752
	SNORM	8393.955	-6.4283
	SGED	8476.708	-6.4909
	ESSTD*	148400.191*	-13.3623*
	EGSSTD	26400.8812	-8.3623
GJR-GARCH (1,1)	SSTD	8521.264	-6.5243
	SNORM	8464.151	-6.4813
	SGED	8531.827	-6.5324
	ESSTD*	447704.856*	-14.0237*
	EGSSTD	26403.881	-6.3625
EGARCH (1,1)	SSTD	8530.299	-6.5313
	SNORM	8472.271	-6.4876
	SGED	8538.083	-6.5372
	ESSTD*	47009.9981*	-9.5162*
	EGSSTD	26011.9779	-6.3326
TGARCH (1,1)	SSTD	8456.228	-6.4753
	SNORM	8393.938	-6.4283
	SGED	8476.71	-6.4910
	ESSTD*	42954.5434*	-9.3357*
	EGSSTD	46301.7736	-7.4858
APARCH (1,1)	SSTD	8515.994	-6.5203
	SNORM	8459.808	-6.4780
	SGED	8528.098	-6.5296
	ESSTD	28485.12	10.25714
	EGSSTD*	1372001.343*	-12.2635*

*Values are the highest value of likelihood function and the least value of AIC

Table 7: Forecasting evaluation of GARCH models and its extension based on standard and poor 500 (S and P500) index returns

Model	Error	RMSE
GARCH (1,1)	SSTD	1.08230
	SNORM	0.52948
	SGED	0.41350
	ESSTD*	0.00651*
	EGSSTD	0.000334
GJR-GARCH (1,1)	SSTD*	0.02316*
	SNORM	0.25949
	SGED	0.11459
	ESSTD*	0.000001*
	EGSSTD	0.000211
EGARCH (1,1)	SSTD	0.014635
	SNORM	0.31093
	SGED	0.09639
	ESSTD*	0.005432*
	EGSSTD	0.02180
TGARCH (1,1)	SSTD	1.082566
	SNORM	0.5224235
	SGED	0.4016423
	ESSTD*	0.048765*
	EGSSTD	0.127034
APARCH (1,1)	SSTD	0.00974
	SNORM	0.1412954
	SGED	0.00515867
	ESSTD	0.081130
	EGSSTD*	0.0035911*

RMSE: Root mean square error, *Values are the least root mean square error (RMSE)

Fitness and model selection of GARCH models and its extension based on standard and poor 500 (S and P500) index returns: The performance of some selected volatility models were estimated using five error distributions such as Exponentiated skewed student t, Skewed student-t distribution, skewed normal and skewed generalized error distribution were compared with that of the proposed distributions. Table 6 showed the result of the fitness and model selection based on log-likelihood and akaike information criteria (AIC) of GARCH, GJR-GARCH, EGARCH, TGARCH and APARCH models. The ESSTD performed better than the remaining distributions of error innovation on GARCH (1,1), GJR-GARCH (1,1), EGARCH (1,1) and TGARCH (1,1) while EGSSTD outperformed others error distribution on APARCH (1,1) model. The Exponentiated Skewed Student-t distribution (ESSTD) were found to outperform on four volatility models as revealed by its largest log-likelihood and least value of akaike information criteria (AIC) for the S and P 500 index returns while Exponentiated Generalized Skewed student t error distribution (EGSSTD) outperformed SSTD, SNORM, SGED and ESSTD error distribution on APARCH model. Furthermore, the results of the AIC based on models selection shows that GJR-GARCH (1,1) with ESSTD performed better than the other models with the least AIC value of -14.0237 followed by GARCH (1,1) with ESSTD and APARCH (1,1) with EGSSTD error distribution models. This results pointed out the significant for further studies in the area of error innovation distribution because the finding contradict support of finding[12-15] that the APARCH models follow a skewed student t error distribution in term of best fitness.

Forecasting evaluation performance of estimated GARCH model and its extension on S and P500 index returns: Table 7 showed the forecasting performance of the estimated models using root mean square error (RMSE). Model with the smallest RMSE were considered to be most suitable for forecasting evaluation of GARCH model and its extension. From the results obtained showed that GARCH (1,1), GJR-GARCH (1,1), EGARCH (1,1) and APARCH (1,1) models under five error distribution, the ESSTD error distribution outperformed other error distribution in the estimation of GARCH model and its extension while EGSSTD error distribution outperformed other error distributions such as the Skewed normal, Skewed student-t, Skewed generalized error distributions and Exponentiated Skewed student t distribution. Hence, from the results obtained showed that GARCH, GJR-GARCH, EGARCH, TGARCH and APARCH models forecast evaluated at Exponentiated skewed student t distribution than the remaining distributions of error innovation. Also based on models forecasting performance, APARCH (1,1) outperformed with the proposed EGSSTD error distribution followed by GARCH (1,1) with ESSTD, GJR-GARCH (1,1) with ESSTD and EGARCH (1,1) with ESSTD error distribution model. Therefore, results shows that models that outperformed in terms of fitness and outperformed in terms of forecasting performance supporting the finding[16,17] that models that are best fit are also best in forecast while contradicting the finding[18,19] were results of best fit models does not out performance in terms of forecast.

CONCLUSION

This study reported for the first time a comparative on error innovation distribution and development of a new error innovation distribution. Obtained results pointed out the significant for further studies in the area of error innovation distribution because the finding does not support others finding that the APARCH models follow a skewed student t error distribution in term of best fitness and forecasting performance. From the new error innovation distribution, shows significant finding on APARCH models in best fitness and forecasting performance accounting for stylized facts in the financial time series.

SIGNIFICANT STATEMENT

The present study reported for the first time a comparative study of error innovation distribution in modelling volatility models. This study discover the react of political disorder, wars and economic shock relating to high kurtosis in the stock market and how the new proposed error innovation distribution handle it with two degree of freedom. This study will help researcher and investor to make proper used of this finding in investment decision making and further opening to the academic.

ACKNOWLEDGMENTS

This study acknowledge the support of standard and poor 500 stock market for providing daily return data to test and validate the new proposed error distributions.

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