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Theoretical Model for Heat Storage Coefficient of Fruits as a Two Phase System

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Abstract: A theoretical model is presented to predict the effective Heat Storage Coefficient (HSC) of fruit system. The system is reduced to a two phase system: Solid phase and continuous fluid phase. The solid is considered as particles having spheroidal shape which are situated at the corners of cubic unit cell. The resistor model is developed to find effective heat storage coefficient from the values of HSC of the constituent phases: protein, fat, carbohydrate, ash and water and their volume fractions. The theoretical calculations of HSC for porous food samples carried out by the proposed model gives an average deviation of 12.8% from experimental values given in literature. A comparison with other models available in the literature has also been made. The theoretical HSC values determined from present model shows least deviation from experimental values.

Key words: Spheroidal particle, heat storage coefficient, fruits, two phase system and resistor model

INTRODUCTION

The thermal characteristics of fruit samples are very important in determining their ability to storage of heat. Theoretical modelling for these substances of agri-food bears industrial importance and is a challenging task for food technologist and physicists. It is required because of the increasing demand of food substances as processed and preserved and also in drying of perishable produce.

Thermal conductivity (K), thermal diffusivity (α) and specific heat (S) are the three parameters cited most often in the literature for describing the thermal behaviour of the substances. The heat storage coefficient or effusivity is another important thermophysical parameter for all kinds of heat transfer processes. Many workers including Babanov (1957), Jacob (1964), Nerpin and Chudnovskii (1970), Luc and Balageas (1981) have mentioned the HSC under various names. It is defined as

$$\beta = \frac{K}{\sqrt{\alpha}} = \sqrt{K \ \rho \ S}$$

Lichtenecker (1926) also presented a simple working empirical relation for porous mixture. In the literature (Ingersoll *et al*., 1969; Carslaw and Jaeger, 1959) one finds that the HSC of composites is an additive property and considering various components as resistors one can take a combination of these to predict effective HSC. This is a common practice adopted to predict effective thermal conductivity from the thermal conductivity of different phases for porous materials. Accepting the similarity, a geometry dependent resistor model has been proposed for heat storage coefficient of food materials.

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Verma *et al*. (1990) initiated experimental work and determined the HSC of metallic powders by using a plane heat source. Thermal heat storage coefficient or effusivity of drop size insulating liquid has been measured by pulse transient heat strip technique by Gustavsson *et al*. (2003). A new photo pyro-electric methodology suitable for HSC of high viscosity liquids is proposed by Balderas-Lopez (2003). This may be used for characterization for liquids of industrial importance viz vegetables oil. Measurements of HSC for powdered titania samples by photo acoustic technique is given by Hernandez -Ayala *et al*. (2005).

The theoretical models for the determination of HSC of porous materials are also available in literature. Shrotriya *et al*. (1991) have proposed a theoretical model for the prediction of HSC of loose granular substances and compared theoretical values of HSC obtained from the model with values obtained by experiments performed with plane heat source. They considered cubic particles in a cubic unit cell. Misra *et al*. (1994) proposed a resistor model to determine HSC of two phase systems, by assuming the grains of the medium as spherical in shape and by replacing porosity (Φ) by porosity correction factor (Fp). Heat storage characteristic of soil have also been investigated by Zhang *et al*. (2007). They used randomly mixed model to simulate the spatial structure of the multi-phase media and observed, the significant effect of the degree of saturation on heat storage coefficient.

However, it has been seen that these theoretical models are not suitable for food substances. Thus in the present study a theoretical model to predict the effective HSC of fruits is given. Since, the main constituents of the fruits are protein, fat, carbohydrate, ash and water. The system may be considered having two phases consisting of water as continuous phase and other constituents together as discontinuous solid phase. The arrangement of cubic array has been divided into unit cells. The solid phase is of spheroidal inclusion in a cubic unit cell and resistor model is applied to determine effective HSC of unit cell. Since the HSC of two phase systems also depends upon various factors such as HSC of constituent phases, porosity, shape factor, size of particles their distribution etc. and, incorporating all these factors in the prediction of HSC of two phase system is a complex affair. Therefore a porosity correction term has been introduced to account for HSC of real two phase systems. The theoretical values of HSCs obtained from this model are compared with values reported in literature and these values show a close agreement.

THEORETICAL FORMULATION

In the following analysis we assumed a homogeneous medium with heat flux in the x-direction and the heat transfer is only by conduction. Let the solid inclusions be spheroids located at the corners of a cube of side 2b. Their distribution in 2D is shown in Fig. 1(a) and the 3D geometry of a unit cell is shown in Fig. 1(b).

Let the origin of the coordinate axis be located at the center of the spheroid having principal axes 2a, 2c and 2a (a < c). The unit cell can be divided into thin slices by planes perpendicular to the x-axis. Consider one such slice bounded by two planes at distances x and x + dx. The section shown in Fig. 1(c) is subdivided into four quadrants. One such section is shown in Fig. 1(d). Let us further divide the section by planes perpendicular to the z-axis. It will divide the section into rectangular bars. One such bar is shown in Fig. 1(e). Let the length of the bar be b and area of cross section dxdz. The shaded portion of the element in the Fig. 1(d) represents the solid phase and the non-shaded portion represents the fluid phase. The volume fraction of solid phase is $\frac{(ydxdz)}{(bdxdz)} = \frac{y}{b}$

and of the fluid phase is $\left\{\frac{(b-y)dxdz}{(bdxdz)}\right\} = \left(1-\frac{y}{b}\right)$

It is assumed that heat flux is incident normally on the face. Hence, heat storage coefficient of the bar is

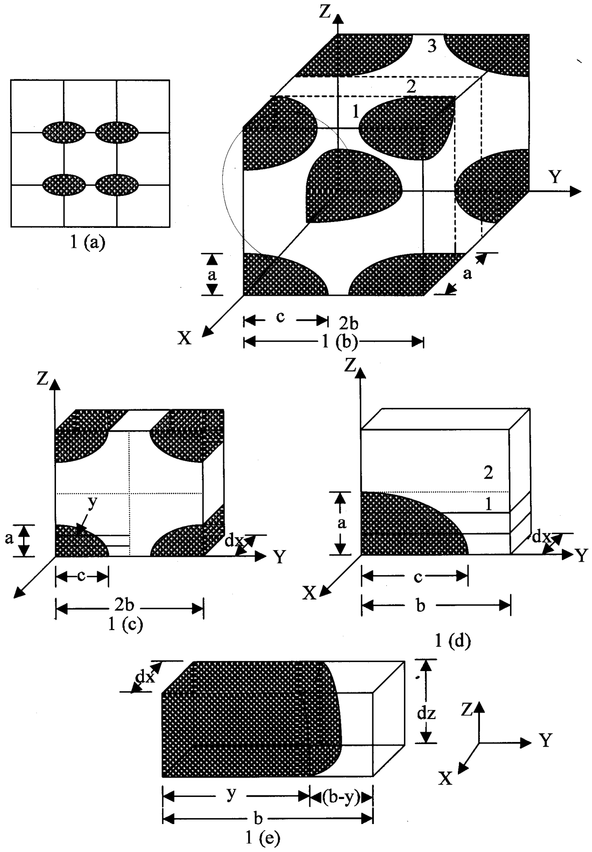

Fig. 1: The resistor model for two-phase system with spheroidal particles

$$\beta' = \beta_s(y/b) + \beta_f(1 - y/b) \tag{1}$$

Where, β_s and β_f are the heat storage coefficients of solid and fluid phase, respectively. In reference to the Fig. 1(d), the heat storage coefficient of the quadrant will be

$$= \frac{(abdx)}{b^2dx}\beta'_{av} + \frac{(b-a)bdx}{b^2dx}\beta_f$$

Therefore

$$\beta'' = (a/b)\beta'_{av} + (1 - a/b)\beta_f \tag{2}$$

$$\beta'_{av} = (1/a)\int_0^a \beta'dz \tag{3}$$

Hence,

$$\beta'' = (1/b)\int_0^a \beta'dz + (1 - a/b)\beta_f \tag{4}$$

Since β'' varies as x changes from 0 to a, therefore, on averaging

$$\beta''_{av} = (1/a)\int_0^a \beta''dx \tag{5}$$

Combining (Eq. 2, 5) yields the following result

$$\beta''_{av} = (1/a)\int_0^a [(1/b)\int_0^a \beta'dz + (1-a/b)\beta_f]dx \tag{6}$$

Combining (Eq. 1, 6) yields the following result

$$\beta''_{av} = (1/a)\int_0^a [(1/b)\int_0^a \{\beta_s(y/b) + \beta_f(1-y/b)\}dz + (1-a/b)\beta_f]dx$$

Therefore

$$\beta''_{av} = \frac{(\beta_s - \beta_f)}{ab^2}\int_0^a \int_0^a ydxdz + \beta_f \tag{7}$$

For spheroidal particle we have

$$\frac{x^2}{a^2} + \frac{y^2}{c^2} + \frac{z^2}{a^2} = 1$$

Thus, from Eq. 7

$$\beta''_{av} = \frac{(\beta_s - \beta_f)}{ab^2}\int_0^a \int_0^a [c\sqrt{\{1-(x^2/a^2)-(z^2/a^2)\}}]dxdz + \beta_f$$

Therefore,

$$\beta''_{av} = \left\{\frac{(\beta_s - \beta_f)\pi ac}{6b^2}\right\} + \beta_f \tag{8}$$

As the quadrants are identical and parallel to heat flow direction, the heat storage coefficient of the complete section is

$$\beta'''_{av} = 4(\beta_s - \beta_f)\left(\frac{\pi ac}{6b^2}\right) + 4\beta_f \tag{9}$$

The sections 1, 2 and 3 in Fig. 1(b) form equivalent series resistors perpendicular to the direction of heat flow, therefore the effective heat storage coefficient β_e of the unit cell will be

$$\frac{1}{\beta_e} = \frac{(a/b)}{\beta'''_{av}} + \frac{(1-a/b)}{\beta_f}$$

or

$$\frac{1}{\beta_e} = \frac{(a/b)}{\{(\beta_s - \beta_f)\pi ac\}/(6b^2)} + \frac{(1 - a/b)}{\beta_f} \tag{10}$$

The unit cell contains one spheroid that lies inside. Hence fractional volume of the solid phase will be

$$\phi_1 = \{(\pi a^2 c)/(6b^3)\}. \tag{11}$$

And in the limiting condition c = b, we get

$$(a/b) = \{\sqrt{(6/\pi)}\}\phi_1^{1/2} \tag{12}$$

Thus, Eq. 10 may also be written as

$$\beta_e = \frac{\left[4(\beta_s - \beta_f)\sqrt{\pi/6}\phi_1^{1/2} + 4\beta_f\right]\beta_f}{\left[4(\beta_s - \beta_f)\sqrt{\pi/6}\phi^{1/2} - 4(\beta_s - \beta_f)\phi_1 + 4\beta_f - 3\sqrt{6/\pi}\phi_1^{1/2}\beta_f\right]} \tag{13}$$

Noting that the expression (13) is based on rigid geometry, which does not represent the true state of affairs of a real two-phase system. Thus, for practical utilization, we have to modify the expression (13) by incorporating some correction term. Tareev (1975) has shown that, during the flow of electric flux from one dielectric to another dielectric medium, the deviation of flux lines in any medium depends upon the ratio of the dielectric constants of the two media. By the same analogy we can have the concentration of thermal flux altered from its previous value as it passes through another medium and that the amount is a function of the heat storage coefficients of the constituent phases. Considering random packing of phases, non uniform shape of particles and the flow of heat flux lines not restricted to be parallel we here replace physical volume fraction of solid phase by porosity correction term F. F in general should be a function of the physical volume fraction of the solid phase and the ratio of the heat storage coefficients of the constituent phases. Therefore, expression (13) may be written as

$$\beta_e = \frac{\left[4(\beta_s - \beta_f)\sqrt{\pi/6}F + 4\beta_f\right]\beta_f}{\left[4(\beta_s - \beta_f)\sqrt{\pi/6}F - 4(\beta_s - \beta_f)F^2 + 4\beta_f - 3\sqrt{6/\pi}F\beta_f\right]} \tag{14}$$

Rearranging Eq. 14 we get

$$AF^2 + BF + C = 0 \tag{15}$$

Where:

$$A = [\beta_e(\beta_s - \beta_f)],$$

$$B = 4(\beta_s - \beta_f)\sqrt{\pi/6} - 4(\beta_s - \beta_f)\beta_e\sqrt{\pi/6} + 3\sqrt{6/\pi}\beta_e\beta_f$$

$$C = 4\beta_f^2 - 4\beta_e\beta_f$$

RESULTS AND DISCUSSION

We have tested the validity of theoretical model discussed above on two phase systems for which the characteristics of the constituent phases and the experimental values are given in literature. Thus the heat storage coefficients of the solid and fluid phases, porosity and the experimental results for

Table 1: Comparison of effective heat storage coefficient of two phase systems

Sample name	Vol.frac. (fluid)*	Vol.frac. (solid)*	Heat storage coefficient ($Wm^{-2}C^{-1}S^{1/2}$) Expt.	Our model	Lichnecker model	Misra modified model	AKS model
Apple	0.89180	0.1081	1117.1520	1091.530	1337.4879	1864.702	1546.4575
Apple, dried	0.42658	0.5734	706.8195	731.428	993.3553	2345.502	1901.5214
Appricots, dried	0.90800	0.0919	1130.6680	1118.170	1351.6821	1855.642	1614.3461
Beans, runner	0.93530	0.0647	971.0206	1332.150	1377.2248	1841.063	1699.7587
Carrot	0.91950	0.0805	1931.2370	1324.290	1363.6004	1849.461	1659.7928
Onions	0.93150	0.0685	1531.2930	1245.730	1373.2297	1843.061	1689.4808
Potatoes, salad	0.85600	0.1439	1328.5070	1192.330	1309.3779	1885.918	1366.2576
Squash	0.92030	0.0796	1513.5870	1431.410	1365.1393	1849.062	1664.9223

*Composition data from USDA (1996); ASHRAE Refrigeration Handbook (2002)

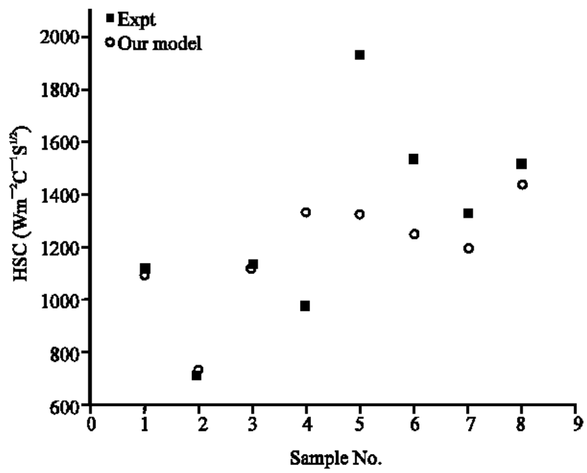

Fig. 2: Comparison of experimental and theoretical values of effective HSC

effective heat storage coefficients have been considered as are given in literature. The solid phase consists of protein, carbohydrate, fat and ash. The effective heat storage coefficient of solid phase is calculated from parallel resistor model because series resistor model results show more deviation from experimentally measured values (Rahman *et al.*, 1991). The theoretical values of the heat storage coefficients have been calculated using Eq. 13.These are compared with experimentally known values which are determined by empirical relations (Appendix).These are based on extensive experimental data. Since the deviation between these experimental and theoretical values is appreciable , therefore formation factor has been introduced in porosity. The correction factor introduced for each sample has been computed using Eq. 15 and plotted as a function of $\phi_s^{1/2} \exp\left(\frac{\beta_s}{\beta_f}\right)$. The curve fitting technique gives the following formation factor for food samples

$$F = C_1 \phi_s^{1/2} \exp\left(\frac{\beta_s}{\beta_f}\right) + C_2 \tag{16}$$

Where, constant C_1 and C_2 are 0.184, 2.116, respectively. On applying above equation as the porosity correction in Eq. 14 we have calculated the values of heat storage coefficient for a number of samples (Table 1). Figure 2 shows a comparison of the experimental results of heat storage coefficient and calculated values from Eq. 14. It is seen from this plot that experimental values and the proposed

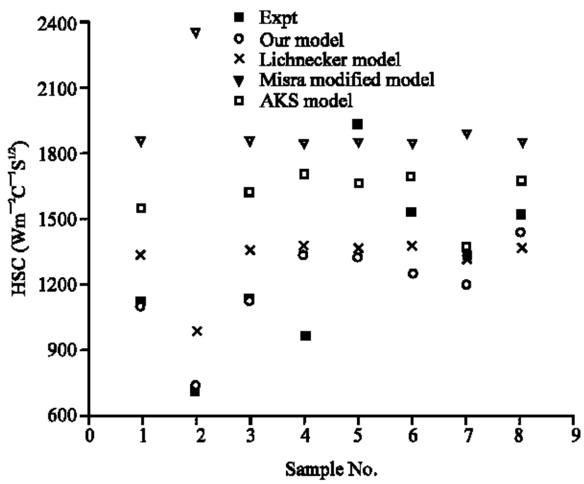

Fig. 3: Comparison of experimental and theoretical values of effective HSC

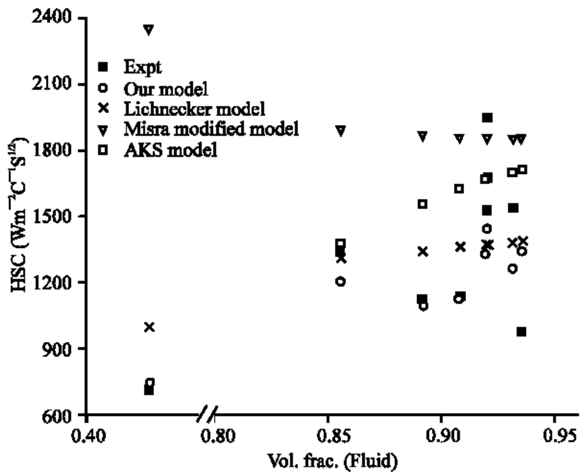

Fig. 4: Comparison of experimental and theoretical values of effective HSC as a function of volume fraction of fluid

spheroidal model values show an average deviation of 12.8%. Thus, the spheroidal model with porosity correction can be used successfully to predict the heat storage coefficients of similar systems when heat storage coefficients of their constituents phases and the porosity values are known.

Since the samples under study are porous therefore, a comparison with other models for effective heat storage coefficients for porous materials have also been made. Thus, HSC using Shrotriya *et al*. (1991), Misra *et al*. (1994) and Lichtenecker model (1926) has been determined. Fig. 3 shows comparison of experimental values of some food samples with these models. The average deviation in HSC for food materials is 21.57, 43.14, 24.39%, for Lichtenecker (1926), Misra *et al*. (1994) and Shrotriya (1991) models, respectively. However, the proposed model shows only 12.87% deviation. Thus, the present model gives better results for food samples than the other models. Figure 4 and 5 show a comparative variation of effective heat storage coefficient as a function of

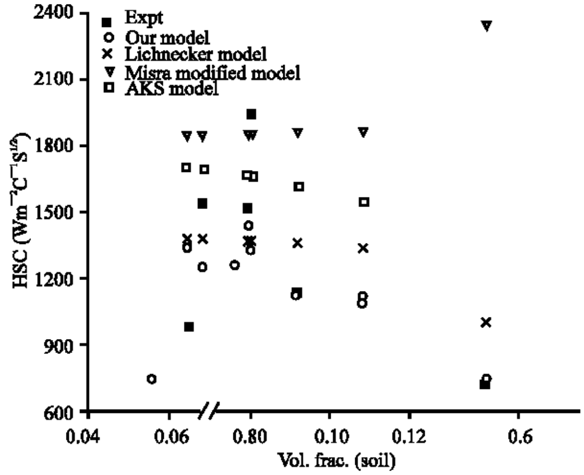

Fig. 5: Comparison of experimental and theoretical values of effective HSC as a function of volume fraction of solid

volume fraction of fluid and solid, respectively and calculated from different models. The results using present model again show least deviation from the experimental values.

CONCLUSIONS

The effective heat storage coefficient of food systems may be determined with empirical correction to porosity in the theoretical model. The porosity correction term in the spheroidal model for prediction of heat storage coefficient is found to be dependent on the ratio of the HSC of the constituent phases of the system. And, using the parallel resistor model and the HSC of constituent phases, the solid phase HSC may be known. The proposed spheroidal model with porosity correction shows an average deviation of 12.8% from the experimental values. Thus, the values of HSC predicted by the present model are close to experimental results than obtained from other models cited in the literature. Thus, using this theoretical model one can find out the HSC of fruit samples.

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NOMENCLATURE

a = Semi-minor axis length (m)
b = Side of the cube (m)
c = Semi-major axis length (m)
C = Empirical constants
F = Formation factor
K = Thermal conductivity ($Wm^{-1}K^{-1}$)
S = Specific heat ($kJ\ kg^{-1}K^{-1}$)
ϕ = Volume fraction
α = Thermal diffusivity ($m^2\ sec^{-1}$)
β = Heat storage coefficient ($Wm^{-2}C^{-1}sec^{1/2}$)
ρ = Density ($kg\ m^{-3}$)

SUBSCRIPTS

av = average
e = effective
f = fluid
s = solid
1, 2 respective values

APPENDIX

Thermal property model for food components

Thermal properties	Food component	Thermal property model
Thermal conductivity (W mK^{-1})	Protein	$K = 1.788\times10^{-1}+1.1958\times10^{-3}\,t-2.7178\times10^{-6}t^2$
	Fat	$K = 1.8071\times10^{-1}+2.7604\times10^{-3}\,t-1.7749\times10^{-6}t^2$
	Carbohydrate	$K = 2.0141\times10^{-1}+1.3847\times10^{-3}\,t-4.3312\times10^{-6}t^2$
	Ash	$K = 3.2962\times10^{-1}+1.4011\times10^{-3}\,t-2.9069\times10^{-6}t^2$
	Water	$K = 5.7109\times10^{-1}+1.7625\times10^{-3}\,t-1.3129\times10^{-6}t^2$
Thermal diffusivity (m^2 sec^{-1})	Protein	$\alpha = 6.8717\times10^{-8}+4.7578\times10^{-10}t-1.464\times10^{-12}t^2$
	Fat	$\alpha = 9.8777\times10^{-8}+1.2569\times10^{-10}t-3.8286\times10^{-14}t^2$
	Carbohydrate	$\alpha = 8.0842\times10^{-8}+5.3052\times10^{-10}t-2.3218\times10^{-12}t^2$
	Ash	$\alpha = 1.2461\times10^{-7}+3.7321\times10^{-10}t-1.2244\times10^{-12}t^2$
	Water	$\alpha = 13.168\times10^{-7}+6.2477\times10^{-10}t-2.4022\times10^{-12}t^2$

Source: Choi and Okas (1986)

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