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Self-Synchronization in Chaotic Systems

I.A. Kamil
Department of Electrical and Electronic Engineering, University of Ibadan, Ibadan, Nigeria

Abstract: In communication, the recovery of the information signal at the receiver end is only possible if carriers at the transmitting and receiving ends are synchronized both in frequency and phase. The notion of frequency and phase is in general not well-defined in chaotic systems and can thus not be used in characterizing synchronization in chaos communications. The first success in synchronization of two chaotic systems credited to Pecora and Caroll was termed self-synchronization. A chaotic system is self-synchronizing if it could be decomposed into at least two sub-systems; a drive sub-system (transmitter) and a stable response sub-system (receiver) that synchronize when coupled with a common signal. In this study, three chaotic systems: Chua's Circuit, Lorenz and Rossler System were modelled using Simulink in Matlab environment. Self-synchronization was carried out between two copies of each of the chaotic systems with variations in initial conditions. The trajectories of the drive and response signals obtained from each pair of chaotic system after running simulations clearly demonstrated the effect of self-synchronization.

Key words: Chaotic system, self-synchronization, chaos communication, information signal, frequency, Nigeria

INTRODUCTION

Tang *et al*. (1983) described synchronization as a phenomenon in which a small periodic signal (called synchronizing signal) with an accurate period is used to drive a system which can produce larger signal having a period not far from the driving signal in such a way that the larger signal locks on to the synchronizing signal's frequency (or to some multiple or sub-multiple of it). In general, two periodic systems are referred to as being synchronized if either their phases or frequencies are locked. Parlitz *et al*. (1999) reported that for chaotic systems, however the notion of frequency and phase are in general not well defined and can thus not be used in characterizing synchronization.

Communication with chaos-based systems is only possible if the system at the transmitter can synchronize with that at the receiver. This was the initial threat to the possibility of a chaos-based communication (Boccaletti *et al*., 1997). It was not until 1990 that it was discovered that by arranging parts of a chaotic system in a specific way, one could achieve identical chaotic behaviour even if the parts are isolated (Carroll, 1995; Pecora and Carroll, 1990). This heralded the era of chaotic secure communication in which a message signal is used to modulate a chaotic signal before transmission and a synchronizable and identical copy of the chaotic system is used for demodulation at the receiver.

Theory: The feasibility of synchronizing two chaotic systems was not obvious until 1990 since their trajectories starting arbitrarily close to each other diverge exponentially with time and quickly become uncorrelated (Carroll and Pecora, 1991; Parlitz *et al*., 1993; Parlitz and Ergezinger, 1994; Stojanovski *et al*., 1996; Yang and Chua, 1999). Pecora and Carroll (1990, 1991) introduced the idea of synchronizing two identical chaotic systems that start from different initial conditions. The idea involves linking the trajectory of one system to the same values in the other so that they remain in step with each other through the transmission of a signal (Boccaletti *et al*., 1997; Lu and He, 1996; Pecora and Carroll, 1990, 1991; Stojanovski *et al*., 1996).

The first success achieved in synchronization of two chaotic systems was credited to Pecora and Caroll (1990, 1991). This approach, termed self-sychronization has remained a dominant method in chaotic synchronization. A chaotic system is self-synchronizing if it could be decomposed into at least two sub-systems: a drive sub-system (transmitter) and a stable response sub-system (receiver) that synchronize when coupled with a common signal. It therefore follows that only 3-dimensional and higher order chaotic systems possess self-synchronizing property (Cuomo *et al*., 1993).

Consider an n-dimensional autonomous dynamical system described by Eq. 1:

$$\dot{u} = f(u) \quad (1)$$

The system is then divided arbitrarily as illustrated into two sub-systems:

$$\begin{aligned} \dot{v} &= g(v,w) \\ \dot{w} &= h(v,w) \end{aligned} \quad (2)$$

Where:

$v = (u_1,\ldots,u_m)$
$g = (f_1(u),\ldots,f_m(u))$
$w = (u_{m+1},\ldots,u_n)$
$h = (f_{m+1}(u),\ldots,f_n(u))$

A first response, system may be created by duplicating a new sub-system w identical to the w' system, substituting the set of variables v for the corresponding v' in the function h and augmenting Eq. 2 with this new system, giving:

$$\begin{aligned} \dot{v} &= g(v, w) \\ \dot{w} &= h(v, w) \\ \dot{w}' &= h(v, w') \end{aligned} \quad (3)$$

It is possible to create a second response sub-system by reproducing v sub-system and driving it with w' variable giving:

$$\begin{aligned} \dot{v} &= g(v, w) \\ \dot{w} &= h(v, w) \\ \dot{w}' &= h(v, w') \\ \dot{v}'' &= g(v'', w') \end{aligned} \quad (4)$$

If all Lyapunov exponents of the w' and v" sub-systems (as they are driven) are <0 then (w'-w) →0 and (v"-v) →0 as t→∞, i.e., synchronization is achieved. Since, the evolution of the Chaotic Synchronization Theory, many researchers have demonstrated the possibility of employing the synchronization properties of chaos chaotic sub-systems for secure communications (Kennedy *et al*., 2000; Kocarev *et al*., 1992; Parlitz *et al*., 1993; Parlitz and Ergezinger, 1994).

MATERIALS AND METHODS

Self-synchronization in Chua's circuit: The Chua's System used as the transmitter is given as:

$$\begin{aligned} \dot{v}_{c1} &= \frac{G}{C_1}(v_{c2} - v_{c1}) - \frac{1}{C_1}h(v_{c1}) \\ \dot{v}_{c2} &= \frac{G}{C_2}(v_{c1} - v_{c2}) + \frac{1}{C_2}i_L \\ \dot{i}_L &= -\frac{1}{L}v_{c2} \end{aligned} \quad (5)$$

There are two possible way of decomposing Chua's circuit using self-synchronization scheme. The 1st is by using v_{c1} as the drive signal and the 2nd is by using v_{c2} as the drive signal.

In the first approach, the receiver is made up of two stable sub-systems decomposed from the original system using Pecora and Carrol Scheme.

The 1st sub-system (v'_{c2}, i'_L) (referred to as the first response sub-system is driven by from the transmitter to give an output v'_{c2}:

$$\begin{aligned} \dot{v}'_{c2} &= \frac{G}{C_2}(v_{c1} - v'_{c2}) + \frac{1}{C_2}i'_L \\ \dot{i}'_L &= -\frac{1}{L}v'_{c2} \end{aligned} \quad (6)$$

The 2nd sub-system (v'_{c1}) referred to as the second response sub-system is driven by v'_{c2} to give v'_{c1} as output:

$$\dot{v}'_{c1} = \frac{G}{C_1}(v'_{c2} - v'_{c1}) - \frac{1}{C_1}h(v'_{c1}) \quad (7)$$

Since the Lyapunov exponents of the response systems is negative, $v_{c1} \approx v'_{c1}$ and synchronization is thus achieved. The receiver is therefore given by:

$$\begin{aligned} \dot{v}'_{c1} &= \frac{G}{C_1}(v'_{c2} - v'_{c1}) - \frac{1}{C_1}h(v'_{c1}) \\ \dot{v}'_{c2} &= \frac{G}{C_2}(v_{c1} - v'_{c2}) + \frac{1}{C_2}i'_L \\ \dot{i}'_L &= -\frac{1}{L}v'_{c2} \end{aligned} \quad (8)$$

The transmitter and the receiver systems were modelled with Simulink as shown in Fig. 1. For the transmitter, the initial conditions were v_{c1} (0) = 0.001, v_{c2} (0) = -0.05, i_L = -0.02 and for the receiver, the initial conditions were v'_{c1} (0) = 1.0, v'_{c2} (0) = -0.05 = i'_L (0) -0.02. A parameter variation of 0.1 was also introduced between the transmitter and receiver systems. The time series and synchronization for the composite system were displayed on XY graph and the scope of the model.

In the second approach using v_{c2} as the drive signal, the first response system is the (v'_{c1}) stable sub-system driven by v_{c2} to give v'_{c1} as output and it is given by:

$$\dot{v}'_{c1} = \frac{G}{C_1}(v_{c2} - v'_{c1}) - \frac{1}{C_1}h(v'_{c1}) \quad (9)$$

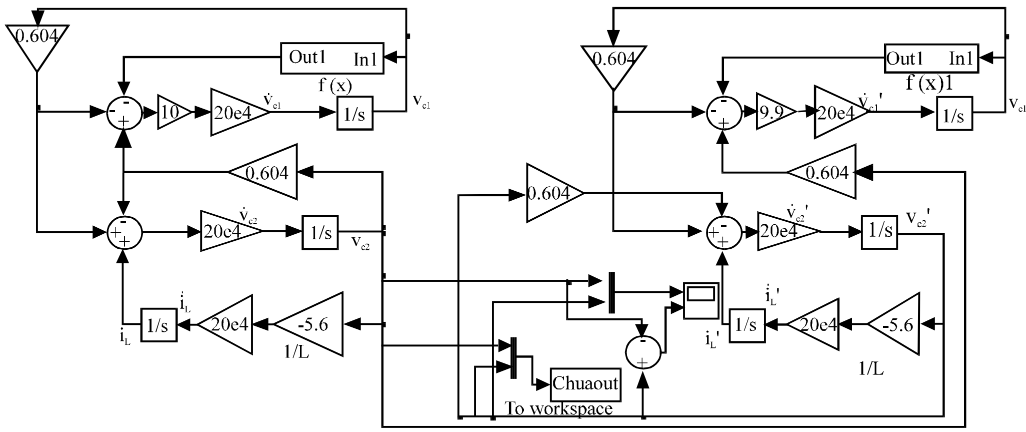

Fig. 1: Self synchronization in two Chua's circuits with different initial conditions and parameter values using v_{c1} as the drive signal

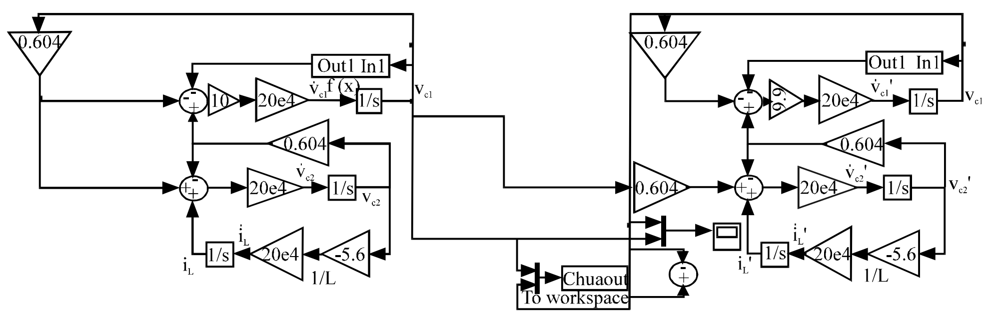

Fig. 2: Self synchronization in two Chua's circuits with different initial conditions and parameter values using v_{c2} as the drive signal

The second sub-system (v'_{c2}, i'_L) referred to as the second response sub-system is driven by v'_{c1} to give v'_{c2} as output:

$$\dot{v}'_{c2} = \frac{G}{C_2}(v'_{c1} - v'_{c2}) + \frac{1}{C_2} i'_L \, \dot{i}'_L = -\frac{1}{L} v'_{c2} \qquad (10)$$
$$\dot{i}'_L = -\frac{1}{L} v'_{c2}$$

The receiver is thus given by:

$$\dot{v}'_{c1} = \frac{G}{C_1}(v_{c2} - v'_{c1}) - \frac{1}{C_1} h(v'_{c1})$$
$$\dot{v}'_{c2} = \frac{G}{C_2}(v'_{c1} - v'_{c2}) + \frac{1}{C_2} i'_L \, \dot{i}'_L = -\frac{1}{L} v'_{c2} \qquad (11)$$
$$\dot{i}'_L = -\frac{1}{L} v'_{c2}$$

The transmitter and the receiver systems were again modeled with Simulink as shown in Fig. 2.

For the transmitter, the initial conditions were $v_{c1}(0) = 0.001$, $v_{c2}(0) = -0.05$, $i_L = -0.02$ and for the receiver, the initial conditions were $v'_{c1}(0) = 1.0$, $v'_{c2}(0) = -0.05$, $i'_L(0) = -0.02$. A slight parameter variation from 0.1 was introduced between the two copies.

The time series and synchronization error for the system were displayed on the scope and the XY graph of the model.

Self-synchronization in lorenz system: The Lorenz System used as the transmitter is given as (Kamil and Fakolujo, 2012):

$$\dot{u} = \sigma(v - u)$$
$$\dot{v} = ru - v - 20\,uw \qquad (12)$$
$$\dot{w} = 5\,uv - bw$$

There are also two possible ways of decomposing the Lorenz System for self-synchronization. In the first approach using u as the drive signal, the first response sub-system (v', w') is given by:

$$\dot{v}' = ru - v' - 20\,uw' \qquad (13)$$
$$\dot{w}' = 5\,uv' - bw'$$

The second response sub-system (u') is driven with input signal v' to give the output u':

$$\dot{u}' = \sigma(v' - u') \tag{14}$$

The complete response system is thus given by:

$$\begin{aligned} \dot{u}' &= \sigma\,(v' - u') \\ \dot{v}' &= ru - v' - 20\,uw' \\ \dot{w}' &= 5uv' - bw' \end{aligned} \tag{15}$$

Since the two sub-systems are stable, $u \approx u'$ as $t \to \infty$. Thus, synchronization is achieved. The Simulink Model for self-synchronizing Lorenz systems is shown in Fig. 3 with different initial conditions. The time series and the synchronization error of the synchronizing systems are displayed on the XY graph and the scope in the model. In the second approach using v as the drive signal, the first sub-system (u') is given by:

$$\dot{u}' = \sigma\,(v - u') \tag{16}$$

The second response sub-system (v', w') is given by:

$$\begin{aligned} \dot{v}' &= ru' - v' - 20\,u'w' \\ \dot{w}' &= 5\,u'v' - bw' \end{aligned} \tag{17}$$

The complete response system is therefore given by:

$$\begin{aligned} \dot{u}' &= \sigma\,(v - u') \\ \dot{v}' &= ru' - v' - 20u'w' \\ \dot{w}' &= 5u'v' - bw' \end{aligned} \tag{18}$$

Since the two sub-systems are stable $v \approx v'$ as $t \to \infty$. Thus synchronization is achieved. The Simulink Model for self-synchronizing Lorenz Systems is shown in Fig. 4 with different initial conditions. The time series and the synchronization error of the synchronizing systems are displayed on the XY graph and the scope in the model.

Self-synchronization in Rossler System: The Rossler system used as the drive system is given as:

$$\begin{aligned} \dot{x} &= -y - z \\ \dot{y} &= x + ay \\ \dot{z} &= b + z\,(x - c) \end{aligned} \tag{19}$$

Decomposition of Rossler System for self-synchronization can only be done by using y as the drive signal. The first response sub-system, the sub-system is given by:

$$\dot{y}' = x' + ay \tag{20}$$

The second response sub-system, the sub-system is given by:

$$\begin{aligned} \dot{x}' &= -y' - z' \\ \dot{z}' &= b + z'(x' - c) \end{aligned} \tag{21}$$

The complete response system is given by:

$$\begin{aligned} \dot{x}' &= -y' - z' \\ \dot{y}' &= x' + ay \\ \dot{z}' &= b + z'\,(x' - c) \end{aligned} \tag{22}$$

Simulink Model for self-synchronizing Rossler Systems is shown in Fig. 5 with different initial conditions and parameter variation. The time series and the synchronization error of the synchronizing systems were displayed on the XY graph and the scope in the model.

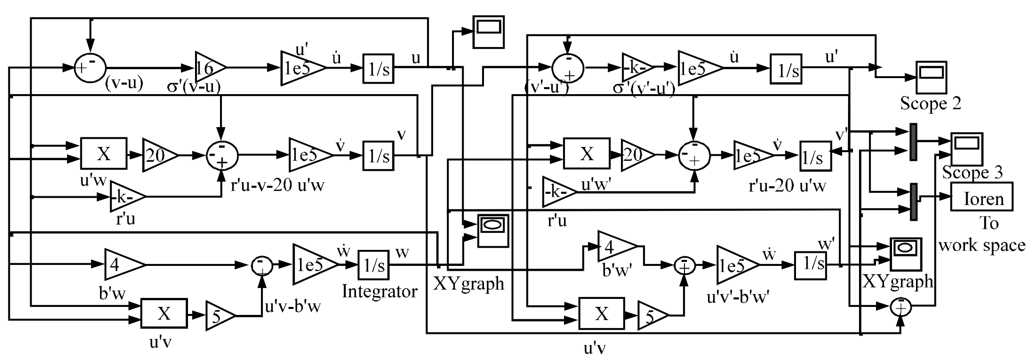

Fig. 3: Self synchronization in two Lorenz Systems with different initial conditions and parameter values using u as the drive signal

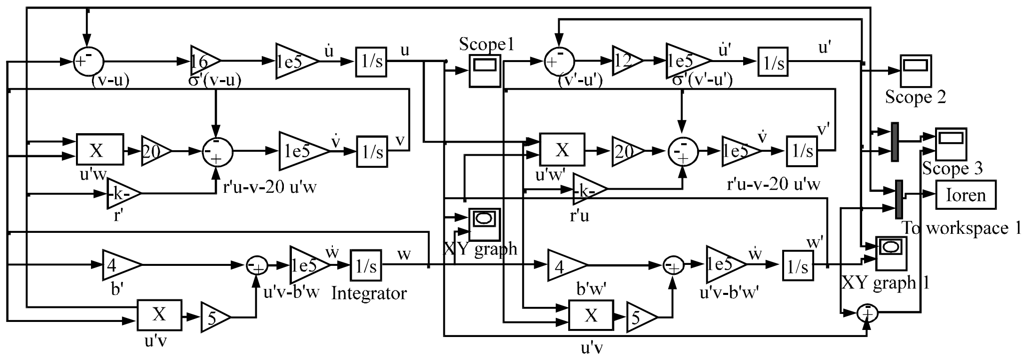

Fig. 4: Self synchronization in two Lorenz Systems with different initial conditions and parameter values using v as the drive signal

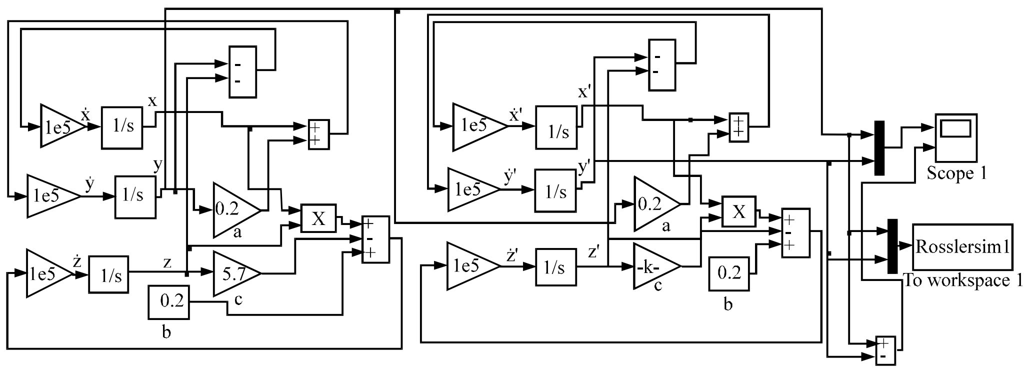

Fig. 5: Self synchronization in two Rossler Systems with different initial conditions and parameter values using y as the drive signal

RESULTS AND DISCUSSION

The simulation results of self-synchronization carried out in the three continuous chaotic systems namely Chua's circuit, Lorenz System and Rossler System modelled in Fig. 6-10.

From the results obtained, it was observed that a difference in initial conditions that would otherwise cause two systems to produce divergent time series had no effect when the two were synchronized using self-synchronization approach. The difference in the trajectories of two Chua's circuits with a difference of 0.999 in the initial conditions and 0.1 difference in a parameter value was reduced to zero after 0.075×10^{-5} simulation time and they became synchronized thereafter. The synchronization of Lorenz Systems was achieved within 0.2×10^{-4} simulation time in spite of the larger difference in initial conditions which was 400 and a parameter difference of 4.

Lorenz System may therefore not be suitable for application which requires the use of the same chaotic systems but with different parameters. The effect of parameter variation was almost unnoticeable in the two systems. The Rossler Systems synchronized within 0.5×10^{-3} simulation time.

This property demonstrated by the chaotic systems confirmed their applicability in communications as chaotic system at the transmitter end can easily synchronize with the one at the receiver which in practice will almost certainly have some variation in parameters.

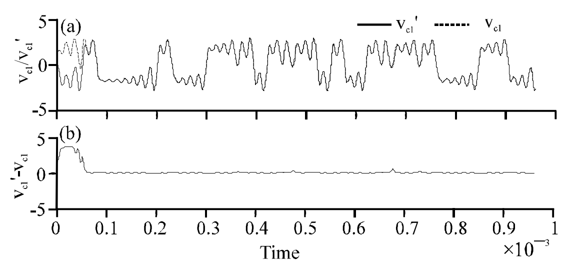

Fig. 6: Self synchronization of two Chua circuits using v_{c1} as drive signal with different initial conditions and parameter values; a) Time series of v_{c1} and v'_{c1}; b) Synchronization error

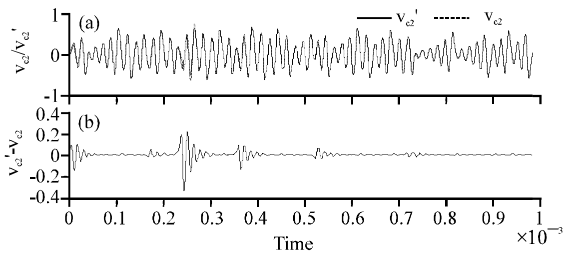

Fig. 7: Self synchronization of two Chua circuits using v_{C2} as drive signal with different initial conditions and parameter values; a) Time series of v_{c2} and v'_{c2}; b) Synchronization error

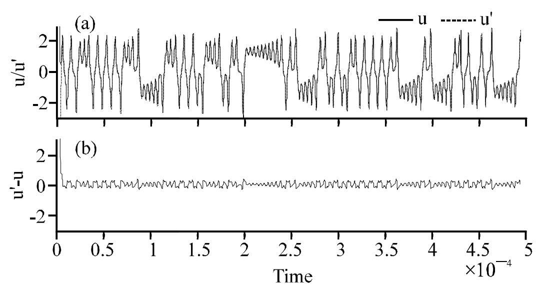

Fig. 8: Self synchronization of two Lorenz Systems using u as drive signal with different initial conditions and parameter values; a) Time series of u and u'; b) Synchronization error

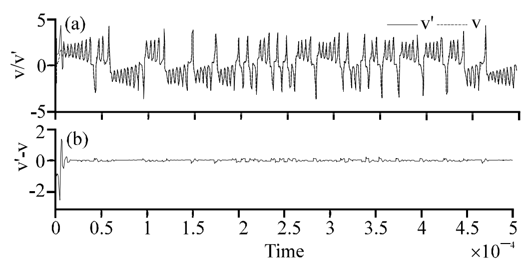

Fig. 9: Self synchronization of two Lorenz Systems using v as drive signal with different initial conditions and parameter values; a) Time series of v and v'; b) Synchronization error

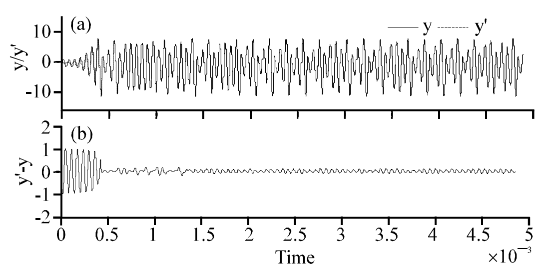

Fig. 10: Error feedback synchronization of two Rossler Systems using y as drive signal with different initial conditions and parameter values; a) Time series of y and y'; b) Synchronization error

CONCLUSION

The effectiveness of self-synchronization of Pecora and Caroll was confirmed on 3-dimensional chaotic systems, Chua circuit, Lorenz and Rossler System. Two copies of each chaotic system with different initial conditions and parameter values that would ordinarily produce divergent trajectories produced the same trajectories within a very short time interval.

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