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Execute Trading Policies on Optimal Portfolio When Stochastic Volatility and Inflation Effect Were Considered

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Abstract: Tempting to formulate the long-term investment strategy for investors who dynamically adjust her portfolio over her lifetime, we are interested to optimize the end-of-period terminal wealth using Bellman principles. We designed the portfolio to be replete with risky asset and risk-less asset/fixed-income asset in the continuous framework. The stochastic volatility model is depicted in risky asset dynamic known as Constant Elasticity of Variance (CEV) because the empirical bias of leverage effect in stock price evolution founded by Black Scholes can be directly examined. Meanwhile, the bond pricing analysis was no longer classified as risk-free asset because it was analyzed under the stochastic Inflation and interest rate of affine structures named Vasicek. Because we want to reflect their mean-reverting behavior as they're hovering around their long-term mean. Later, state space was constructed and portion of risky asset was elected to be control variables for supremum over value function. The concept of investment decision is intertemporal as today decision affected tomorrow's which finding its optimal rate would be trade-off for investor. For this, we framed the decision criteria with investor's utility function from class Decreasing Absolute Risk Aversion (DARA), the class that generally most investor mostly consistent with. The problem description above can be represented as stochastic optimal control problem and it was solved with dynamic programming argument with modified verification theorem to tackle the issue of Stochastic Differential Equation well-posedness violation. Through stages of change variables, we were able to find the closed form trading solution from corresponding Hamilton Jacobi Bellman (HJB) equation. Compare to standard Merton model, our trading strategies strength are determining interest rate, inflation rate and degree of leverage for improvement and hence have inline economic logic reasoning for our solutions.

Key words: CEV, dynamic programming, HJB, portfolio optimization, stochastic control, vasicek

INTRODUCTION

Meintyre was the frontierman of analyzing optimal asset allocation study for economic agent who wish to maximize her bequest wealth using static portfolio of two market asset model with the respect of return and variance as risk measurement for one period. He obtained singleton trading strategy for whole time in investment period as the distribution of asset return need to obey normal and elliptical families. Using Hamilton Jacobi Bellman arguments, dynamic optimization was later developed by Christine with uncertainty source from risky asset in discrete time framework. The seminal work of Neitz and Neitz (2000) framed Samuelson's work in continuous time framework with generalized Hyperbolic Absolute Risk Aversion (HARA) utility of terminal wealth and he acquired closed form trading strategy. Merton and Samuelson developed vary trading strategy as the corresponding from investor's wealth.

Following Merton's in this study we employed dynamic programming methodology which able to described good system characteristic with appropriate boundary condition employed as we want to maximize end-of-period wealth. In contrast, different method of solving optimal solution to the asset allocation problem also able to be tackled by utilize Maximum Principle which pioneered by Healy *et al.* (1992). This approach is favoured if we want to obtain random target based in the end of investment period, well known as Backward Stochastic Differential Equation (BSDE) (Brettel *et al.*, 1997) explored the BSDE theory for linear case in which had been debuted by Poret *et al.* (2009). The other

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different method of solving optimal in investment decision is enriched by exercising martingale technique. This methods was widely developed among researchers, i.e., Ohkubo and Kobayashi (2008), Plataniotis and Vinetsanopoulos (2000), Dowell (2008),Yang and Ro (2003), Kuo and Hsu (1996), Swain and Ballard (1991) and Birch (2012).

In the present years, Merton's model enhancement in continuous-time are in addressing stochastic economic parameters into the optimization model. Hood *et al.* (2006) who limited maximum risky asset belonging with stochastic borrowing constraint (Nathans *et al.*, 1986) put stochastic risk premium in the design of non-myopic portfolio (Sharpe *et al.*, 1999) research was about effect of endogenously stochastic risk factors in dynamics of risky asset (Walraven and Alferdinck, 1997) considered ergodic Markov interest in the model (Bimber *et al.*, 2007) exemplified the portfolio with stochastic interest rate, appreciation rate and volatility variables (20) worked on long term investment so that stochastic inflation rates had been taken into portfolio decision concern (Brettel *et al.*, 1997; Solem, 2012) for different aims had used Vasicek interest rate.

Most scholars utilized standard Geometric Brownian Motion (GBM) for describing the dynamics of stock price evolution. Meanwhile many empirical bias found by Black and Scholes that stock price evolution exhibited leverage effect. As the stock price has decreased, the volatility will increase and vice versa. This finding will argue that behavior described by GBM will be improved by Constant Elasticity of Variance (CEV). Cox and Ross. Firstly introduced to CEV process for modeling the option pricing. But in last decades many researcher have delivered the investment-consumption optimization employing CEV process as stock price movement, i.e., in pension plan terminal wealth optimization, combined CEV process in stock with deterministic bond for multi-assets. Extended the work of with consumption case used optimal dynamic mean-variance with CEV with borrowing constraint.

However, above mentioned CEV's paper used deterministic economic parameter (interest rate) with no consideration in random inflation rate in which researchers found very contrast to reality. Thus we want to use the idea of using stochastic volatility, the paper that has one pattern with us is which is present the partially observed price level in terms of inflation effect on portfolio. But what distinguish our work is that the consideration of the leverage effect on stock price on the CEV Model.

We exercised with log-utility function as the branch of utility function Decreasing Absolute Risk Aversion (DARA) risk preference shown through experiments that DARA type of utility function was mostly consistent on the demand for risky asset. We designed our portfolio with risky asset and risk-less asset and the setting is working under stochastic environment. Using Martingale technique asset pricing to derive solution of optimal policy with Vasicek process on inflation rate and interest rate and applying model of standardized Brownian motion of stock price evolution. Extended the work of them, we employed CEV process for stock price evolution for leverage effect examination. Explicit solution to the optimal trading strategy through the derivation of HJB equation which was distinguished from martingale technique was obtained. Result of this paper showed that the optimal policy did not perform as feedback form of wealth process. Hence, this finding was aligning to the Merton's as well as. However, this model can depict well that the trading strategies for stock and bond were depending on the degree of leverage asset characteristic as well as inflation rate and interest rate.

MATERIALS AND METHODS

Analysis of portfolio problem: Considering the market which was driven by uncertainty environment as it can be described by Probability Space (Ω, F, P) with natural filtration $F = \{F_t\}$ $0<t<T$. As Ω is given set, F is a σ-algebra of Ω and P is probability measure on measurable space (Ω, F) that mapped F into (0,1). The filtration was conducted by Wiener Process as representation of market information flow availability. Assuming that the price process of n available traded asset is affected by rate of interest, rate of inflation and some uncertainty. All variables mentioned here were to be F-measurable. The wealth process $X_t^{x,\theta}$ is the self-financing portfolio with the sum of relative assets weight equal to one.

The problem setup: Suppose an economic agent have the purpose of investing her initial wealth so that regularly she could have her bequest in the end of investment end period. She would like to apply the diversification principle over her portfolio by investing some portion in risky asset and risk-less asset. The risky assets behave randomly in capital market and to show the stochastic volatility of CEV we represent its price dynamic as in Eq. 1:

$$dS(t) = S(t)\mu_s(t)dt+S^{\gamma}(t)\sigma_s dW(t) \quad (1)$$

with drift coefficient $\mu_s(t) = \lambda_s+r(t)$. The $\mu_s(t)$ an σ_s are respectively mean and standard deviation associated with stock evolution $W_s(t)$ is the standard Brownian motion associated with stock evolution and λ_s is the risk premium over interest rate r(t). In this system we would like to introduce the stochastic inflation rate π which is compiled to Vasicek process in Eq. 2:

$$d\pi(t) = \alpha(\bar{\pi}\text{-}\pi(t))dt+\sigma_\pi dW_\pi(t) \quad (2)$$

The closed-form solution $\pi(t)$ is given by:

$$\pi(t) = \bar{\pi}+e^{-\alpha t}(\bar{\pi}\text{-}\pi(t))+\sigma_\pi e^{-\alpha t}\int_0^t e^{\alpha u}dW_\pi(t)$$

The inflation dynamics is the process of price level $d\psi_t = \psi_t(\pi dt)$. In this world too, the instantaneous interest rate, r(t) was underwent stochastic process of Vasicek, its dynamic given by Eq. 3:

$$dr(t) = k(\bar{r}\text{-}r(t))dt+\sigma_r dW_r(t) \quad (3)$$

and this affine term structure solution is:

$$r(t) = \bar{r}+e^{-kt}(r_0\text{-}\bar{r})+\omega_r e^{-kt}\int_0^t e^{\alpha u}dW_r(t) \quad (4)$$

The bond pricing will be done in the manner of Eq. 4:

$$B_0(t,T_B) = e^{E}[\int_t^T -r(u)du - \pi(u)du] + \frac{1}{2}Var_t[\int_t^T -r(u)du - \pi(u)du]$$

The bond is somewhat classified risky too due to affected by stochastic interest rate as well asinflation rate but won't be default. The analysis gave its dynamics in Eq. 5. The other analysis of bond pricing may refer to for default-ableassets:

$$\frac{dB_0(t,T_B)}{B_0(t,T_B)} = [r(t)+\pi(t)+b(t,T)]dt - [P(t,T)\sigma_r dW_r(t)] - [Q(t,T)\sigma_r dW_r(t)] \quad (5)$$

with constant b (t, T) is defined by Eq. 6:

$$b(t,T) = -\left[\frac{\sigma_r^2}{2k^2}\right]-\left[\frac{\sigma_\pi^2}{2\alpha^2}\right]+\sigma_\pi^2\left[-\frac{e^{-K(T-t)}}{K^2}+\frac{e^{-2K(T-t)}}{2K^2}\right]+ \sigma_\pi^2\left[-\frac{e^{-\alpha(T-t)}}{\alpha^2}+\frac{e^{-2\alpha(T-t)}}{2\alpha^2}\right]+ \bar{r}\left[-2+2e^{-K(T-t)}\right]+\bar{\pi}\left[-2+2e^{-\alpha(T-t)}\right] \quad (6)$$

and:

$$P(t,T_B) = \frac{1}{k}\left(1-e^{-k(T_B-t)}\right) \quad (7)$$

and:

$$Q(t,T_B) = \frac{1}{k}\left(1-e^{-\alpha(T_B-t)}\right) \quad (8)$$

Now that the trading strategies were embodied as portion of risky asset, $\theta_1(t)$ and portion of fixed-income (risky bond), $\theta_2(t)$ as we underline constraint $\theta_1(t)+\theta_2(t) = 1$. Our model allow short selling and condition $\theta_{1,2}>0$ are not necessary.

Wealth dynamics and target function: The agent's wealth dynamics over investment period is outlined in Eq. 9 for simplification some variables will be written in short form without losing their generality:

$$\frac{dX(t)}{X(t)} = \begin{bmatrix}\theta_1(t).S.\mu_s \\ +\theta_2(t).B_0(r+\pi+b)\end{bmatrix}.dt + \begin{bmatrix}\theta_1(t)S_t^Y\sigma_s \\ -\theta_1(t)B_0P\sigma_r \\ -\theta_2(t)B_0Q\sigma_\pi\end{bmatrix}.\begin{bmatrix}dW_s \\ dW_r \\ dW_\pi\end{bmatrix} \quad (9)$$

The related target/cost function in which comprised by terminal cost was given by Eq. 10:

$$V(s;y) = E\left\{U\left(\frac{X_T}{\psi_T}\right)\right\} \quad (10)$$

while value function is supremum over the terminal wealth with inflation effect which was described in following Eq. 11:

$$V(s, y) = \sup_{u\in A_{ad}} J(s, y, u) \quad (11)$$

We would like to set wealth to be positive, $X(t)\geq 0$, for every $t\geq 0$, for the purpose of solvency conditions. We must solve the Hamilton Jacobi Bellman equation given Eq. 12:

$$V_t + \sup_{u\in A_{Ad}} G(t, z, u, V_z, V_{zz}) = 0 \quad (12)$$

with G (t, z, u, V_z, V_{zz}) = L^uV. Noting that our Stochastic Differential Equation system problem which is not satisfied the condition of lipschitz growth due to such two unbounded processes met, i.e., product of X(t) r(t), etc. Henceforth, we would like to adopt technique introduced by altogether, with their proof of modified verification technique for the sake of Stochastic Differential Equation solution's uniqueness and existence of well-posed conditions. Proof is to consult Korn and Kraft. The Dykin's operator is introduced by Eq. 13:

$$G(t, z, u, V_z, V_{zz}) = \frac{1}{2}Tr\left[g*(t, Z_t, u_t)^T V_{zz}\right] + V_z^T f(t, Z_t, u_t) \quad (13)$$

with $g^* = g.g^T$ and g is the diffusion matrix, f is the drift matrix of state space Z = (dXdSdr dπ). Later we have our Hamilton Jacobi Bellman (HJB) equation as following Eq. 14 where $\tilde{P} = P^2\sigma_2^r + Q^2\sigma_\mu^r$ and $\tilde{Q}(r, \sigma) = r + \sigma + b(t, T)$ is:

$$\begin{aligned}&\theta_1(t)^2\left[\frac{1}{2}x^2V_{xx}(S^{2\gamma}\sigma_s^2 + B_0^2\tilde{P})\right] + \\ &\theta_1(t)\{-x^2B_0^2\tilde{P}.V_{xx} + xS^{2\gamma}\sigma_s^2V_{xs} + \\ &xB_0(P\sigma_r^2.V_{xr} + Q\sigma_\pi^2.V_{x\pi}) + \\ &x.V_x\left((S\mu_s - B_0\tilde{Q}(r,\pi)\right)\} + \\ &[\frac{1}{2}x^2B_0^2\tilde{P}.V_{xx} + \frac{1}{2}S^{2\gamma}\sigma_s^2.V_{ss} + \\ &\frac{1}{2}\sigma_r^2.V_{rr} + \frac{1}{2}\sigma_\pi^2.V_{\pi\pi} - \\ &xB_0(P\sigma_r^2.V_{xr} + Q\sigma_\pi^2.V_{x\pi}) + \\ &V_t + xB_0\tilde{Q}(r,\pi).V_s + S\mu_{Ss}.V_s + \\ &k(\bar{r} - r).V_r + \alpha(\bar{\pi} - \pi).V_\pi] = 0\end{aligned} \quad (14)$$

In which the subscribe in the V represented the partial derivative with the respect to. By static Euler derivation we obtained the candidate for optimal control in following Eq. 15:

$$\begin{aligned}\theta_1(t)^* = &\frac{xB_0^2\tilde{P}.V_{xx} - S^{2\gamma}s_s^2.V_{xs}}{xV_{xx}(S^{2?}s_S^2 + B_0^2\tilde{P})} \\ &-\frac{B_0(Ps_r^2V_{xr} + Qs_\pi^2.V_{xp})}{xV_{xx}(S^{2\gamma}s_s^2 + B_0^2\tilde{P}} \\ &-\frac{V_x(S\mu_s + B_0\tilde{Q}(r,p))}{xV_{xx}(S^{2\gamma}s_s^2 + B_0^2\tilde{P})}\end{aligned} \quad (15)$$

Write our value function in the form of Eq. 16 (t, x, r, π, s) = a(t, r, π, s). ln(x)+b(t, r, π, s). Direct substitution will result to our system:

$$\begin{aligned}&\ln(x)\ \{a_t + S\mu_s.a_s + k(\bar{r}\text{-}r).a_r + a(\bar{p}\text{-}p).a_p + \\ &\frac{1}{2}S^2s_s^2.a_{ss} + + \frac{1}{2}s_r^2.a_{rr} + \frac{1}{2}s_\pi^2.a_{pp}\} + \\ &b_t + S\mu_s.b_s + k(\bar{r}\text{-}r).b_r + a(\bar{p}\text{-}p).b_p + \\ &\frac{1}{2}S^{2\gamma}s_s^2.b_{ss} + + \frac{1}{2}s_r^2.b_{rr} + \frac{1}{2}s_\pi^2.b_{pp} + \\ &f(t, r, p, s) = 0\end{aligned} \quad (16)$$

Where:

$$f(t,r,\pi,s) = (B_0\tilde{Q}(r,\pi) - \frac{1}{2}B_0^2\tilde{P}) + \frac{[-B_0^2\tilde{P} - S\mu_s + B_0\tilde{Q}(r,\pi)]^2}{2.S^{2\gamma}s_s^2 + B_0^2\tilde{P}}$$

We split our system into PDE I and PDE II. Sub-system PDE I represented on following (Eq. 17) would be solved as following Second Order Homogenous case:

$$\begin{aligned}&\{a_t + S\mu_s.a_s + k(\bar{r}\text{-}r).a_r + a(\bar{p}\text{-}p).a_p + \\ &\frac{1}{2}S^{2\gamma}\sigma_s^2.a_{ss} + \frac{1}{2}s_r^2.a_{rr} + \frac{1}{2}s_\pi^2.a_{\pi\pi} = 0\end{aligned} \quad (17)$$

Meanwhile, the PDE II in Eq. 18 would be raised as second order quasi-linear case:

$$\begin{aligned}&B_t\left(B1_t + B1B2_t.r\right), e^{B2(t).r};\ B_r = B1B2e^{B2(t).r}; \\ &B_{rr} = B1B2;\ B_p = B_{1p}e^{B2(t).r} \text{ and } B_{pp} = B1_{pp}.e\end{aligned} \quad (18)$$

Now we are interested to resolve PDE I that we can define Eq. 19 as:

$$a(t, r, \pi, s) = c(t, r, \pi, y) \quad (19)$$

with transformation in power form y = s -2γ +2. We have the partial derivative as follows:

$$a_t = c_t;\ a_r = a_{rr} = c_{rr};\ a_\pi = c_\pi;\ a_{\pi\pi} = c_{\pi\pi}$$

and:

$$a_s = c_y(-2\gamma + 2).S^{-2\gamma+1}$$

and:

$$\begin{aligned}a_{ss} = &c_{yy}(-2\gamma + 2)^2.S^{-4\gamma+2} + \\ &c_y(-2\gamma + 2)\ (-2\lambda + 1).S^{-2y}\end{aligned}$$

Hence, we got our system to become in Eq. 20:

$$\begin{aligned}&y\left[\mu_s(-2\gamma + 2).c_y + \frac{1}{2}s_s^2(-2\gamma + 2)^2.c_{yy}\right].c_{yy} + \\ &\frac{1}{2}s_s^2(-2\gamma + 2)(-2\gamma + 1).c_y + \\ &c_t + k(\bar{r} - r).c_r + \alpha(\bar{\pi} - \pi).c_\pi + \\ &\frac{1}{2}s_r^2.c_{rr} + \frac{1}{2}s_\pi^2.c_{\pi\pi} = 0\end{aligned} \quad (20)$$

We defined the system into the linearisation process that (Eq. 21) describes on following:

$$c(t, r, \pi, y) = A(t, r, \pi) + B(t, r, \pi).y \quad (21)$$

Therefore, we will obtain our PDE as following Eq. 22):

$$\begin{aligned}&\{A_{t+\kappa(\bar{r}-r).A_r} + \alpha(\bar{\pi}-\pi).A_\pi + \\ &\frac{1}{2}\sigma_r^2.A_{rr}. + \frac{1}{2}\sigma_\pi^2.A_{\pi\pi} + \\ &\frac{1}{2}\sigma_s^2(-2y+2)(-2y+1).B\} \\ &y[B_t + \kappa(\bar{r}-r).B_r + \alpha(\bar{\pi}-\pi).B_\pi + \\ &\frac{1}{2}\sigma_r^2.B_{rr} + \frac{1}{2}\sigma_\pi^2.B_{\pi\pi} + \mu_s(-2y+2).B] = 0\end{aligned} \tag{22}$$

This PDE in Eq. 22 can be splited into upper and lower row. We are interested to solve the lower one in following Eq. 23 as we are narrowing down our system into:

$$\begin{aligned}&B_t + k(\bar{r}-r).B_r + a(\bar{p}-p).B_p + \\ &\frac{1}{2}\sigma_r^2.B_{rr} + \frac{1}{2}\sigma_p^2.B_{pp} + \mu_s(-2\gamma+2).B = 0\end{aligned} \tag{23}$$

and we define our ansats in Eq. 24 assuming that the PDE is separable:

$$B(t,\pi,r) = B1(t,\pi).e^{B2(t).r} \tag{24}$$

We again have the partial derivatives as follows:

$$\begin{aligned}&B_t = (B1_t + B1B2_t r).e^{B2(t).r};\ B_r = B1B2.e^{B2(t).r} \\ &Brr = B1B2^2.e^{B2(t).r};\ B_\pi = B1\pi.e^{B2(t).r} \text{ and} \\ &B_{\pi\pi} = B1_{\pi\pi}.e^{B2(t).r}\end{aligned}$$

Remind the boundary value we had; a(T, r, π, s) 1, c(T, r, π, y) = 1, A(T, r, π) = 1 and B(T, r, π) = 0. Thus, we had B1(T, π) = 0 and B2(T) = 0. Direct substitution gives us following system in Eq. 25:

$$\begin{aligned}&r(B1B2_t - \kappa B1B2) + B1_t + \\ &\alpha(\bar{\pi}-\pi).B1_\pi + + \frac{1}{2}\sigma_\pi^2.B1_{\pi\pi} + \\ &\kappa\bar{r}B1B2 + \mu_s(-2\gamma+2).B1 + \\ &\frac{1}{2}\sigma_r^2.B1B2^2 = 0\end{aligned} \tag{25}$$

Our PDE will be meaningfully iffr (B1B2 t-k B1B2) = 0 and we solve for B2(t) in Eq. 26:

$$B2(t) = B2(T).e^{\kappa(t-T)} \tag{26}$$

Applying boundary condition on B2(T) = 0 give us B2(t) = 0 as we solve the remaining from last PDE in Eq. 27:

$$\begin{aligned}&B1_t + \alpha(\tilde{\pi}-\pi).B1_p + \frac{1}{2}s_\pi^2.B1_{\pi\pi} + \\ &\mu_s(-2\gamma+2).B1 = 0\end{aligned} \tag{27}$$

and we had B1(t, π) = B3(t).e B4(t).π with the same pattern of partial derivatives from Eq. 28. Remind that our boundary condition B1(T, π) = 0 hence, B3(T) = 0 and B4(T) = 0. The system is to become (Eq. 28):

$$\begin{aligned}&\pi[B3B4_t - aB3B4] + B3_t + \\ &\mu_s(-2\gamma+2).B3 + \alpha\bar{\pi}B3B4 + \\ &\frac{1}{2}s_\pi^2.B3B4^2 = 0\end{aligned} \tag{28}$$

As usual we solve π (B3B4 t-α B3B4) = 0 and get (Eq. 29):

$$B4(t) = B4(T).e^{\alpha(t-T)} \tag{29}$$

as the boundary value been substituted we find that B4(t) = 0. The remind parts of last PDE has already reduced to Ordinary Differential Equation (ODE) in (Eq. 30):

$$B3_t + \mu_s(-2\gamma+2).B3 = 0 \tag{30}$$

The solution is B3(t) = B3(T).e ((2γ-2)μ s(t-T)) and by substituting our boundary we obtain B3(t) = 0 as B1(t, π) = B(t, r, π) = 0. For our big system we hence get second order homogenous PDE on Eq. 31 which is referring to Eq. 22:

$$\begin{aligned}&A_t + k(\bar{r}-r).A_r + \alpha(\bar{\pi}-\pi).A_\pi + \\ &\frac{1}{2}\sigma_r^2.A_{rr} + \frac{1}{2}\sigma_\pi^2.A_{\pi\pi} = 0\end{aligned} \tag{31}$$

Written in the form Eq. 32:

$$A(t,r,\pi) = A1(t,\pi).e^{A2(t).r} \tag{32}$$

We will have following PDE in Eq. 33:

$$\begin{aligned}&r[A1A2_t - kA1A2] + A1_t + \\ &\alpha(\bar{\pi}-\pi).A1_\pi + \frac{1}{2}\sigma_\pi^2.A1_{\pi\pi} + \\ &k\bar{r}A1A2 + \frac{1}{2}\sigma_r^2 A1A2^2 = 0\end{aligned} \tag{33}$$

The boundary value A(T, r, π) = 1, A1(T, r, π) = 1 and A2(T, r, π) = 0, we resolve the r(A1A 2 t-k A1A2) = 0 which gives us Eq. 34: we got A2(t) = 0. Now, solving A1 t+ $a(\bar{\pi}-\pi)$ A1 π+1/2σπ^2.A1 ππ in the form A1(t, π) = A3(t). eA4(t).π, hence system in Eq. 34:

$$\begin{aligned}&\pi[A3A4_t - \alpha A3A4] + A3_t + \\ &\alpha\bar{\pi}A3A4 + \frac{1}{2}\sigma_\pi^2.A3A4^2 = 0\end{aligned} \tag{34}$$

We have meaningful PDE iff π (A3A4t-αA3A4) = 0 and A4(t) = A4(T).eα (t-T). Set A4 (T) = 0 we have A4(t) = 0 and we had ODE in Eq. 35:

$$A3_t = 0 \quad (35)$$

which resulted as A3(t) = constant. We know that A3(T) = 1 and hence must be A3(t) = 1 and so do A1(t, π) = 1 and A(t, r, π) = 1 . From previous analysis our linear system c(t, r, π, y) = A(t, r, π)+B(t, r, π).y and as above mentioned results we have c(t, r, π, y) = 1 and so a (t, r, π, y) = 1. And our trading solution of risky asset can be directly infer by following Eq. 36:

$$\theta_1^*(t) = \frac{B_0^2.\tilde{P}+S\mu_s-B_0\tilde{Q}}{S^{2\gamma}\sigma_s^2 + B_0^2.\tilde{P}} \quad (36)$$

Now we are ready to establish our theorems:

Theorem 1: The optimal investment of risky asset trading strategy, $\theta_1^*(t)$ in which the risky asset dynamics following Eq. 1 in the world that bond pricing analysis has been conducted under the environment of stochastic inflation rate in Eq. 2 and stochastic interest rate in Eq. 3 is acquired by:

$$\theta_1^*(t) = \frac{B_0^2.\tilde{P}+S\mu_s-B_0\tilde{Q}}{S^{2\gamma}s_s^2 + B_0^2.\tilde{P}}$$

with constants and variables mentioned above. Meanwhile the trading strategy for risky bond is given by:

$$\theta_2^*(t) = 1 - \theta_1^*(t)$$

The risky asset trading solution is depending on risky asset and risk-less asset characteristic. It is affected by interest rate and inflation rate in opposite direction. It is also shown contrast effect toward leverage effect and risky asset volatility, carrying in the stock dynamics. This is going by economic common logic.

RESULTS AND DISCUSSION

Benchmark; Merton Model: In this subsection we were presenting the Merton Model for logarithmic utility function. Without undergoing stochastic interest rate, the risky bond will be the asset which gives sure return, dB/B = r.dt. Meanwhile, the stock evolution follows the standard geometric Brownian motion in Eq. 1. We have 2 assets on this standardized model. Reader may refer to for more general utility class function:

$$\frac{dS}{S} = \mu dt + \sigma dW(t) \quad (37)$$

The wealth process will be defined in Eq. 38 which θ1M is the risky asset Merton's trading strategy that later will be the control function of our target function in Eq. 38:

$$\begin{aligned} dX &= X\left[\theta_{1M}\frac{dS}{S} + (1-\theta_{1M})\frac{dB}{B}\right] \\ &= X[r + \theta_{1M}(\mu - r)]dt + X[\theta_{1M}\sigma]dW(t) \end{aligned} \quad (38)$$

Our objective is to maximize the portfolio value function given by Eq. 10 by taking supremum over the expectation of utility of terminal wealth. Our chosen benchmark utility function is logarithmic which U(.) = log (.). The value function should obey the Hamilton Jacobi Bellman of V_t+fV_x+1/2g.V_{xx}.g = 0 and we obtain following Eq. 39:

$$\begin{aligned} &V_t + (xr)V_x + \\ &\theta_{1M}.x(\mu - r).V_x \\ &\theta_{1M}^2.\frac{1}{2}(x\sigma)^2.V_{xx} = 0 \end{aligned} \quad (39)$$

Using the same procedure we obtain the risky asset trading solution for Merton's problem for logarithmic utility in Eq. 40 which λ is the Sharpe reward to volatility ratio, λ = (μ-r)/σ:

$$\theta_{1M}^* = \frac{\lambda}{\sigma} = \frac{(\mu - r)}{\sigma^2} \quad (40)$$

and the trading solution for risk free asset given by:

$$\theta_{2M}^* = 1 - \theta_{1M}^*$$

Comparative analysis to our risky asset trading solution above. The Merton Model stated that risky asset trading solution have positive correlation to Sharpe ratio and conversely with volatility. We have our stochastic interest rate, stochastic inflation rate and the leverage degree of stock movement to be accounted in our trading solution. The bigger uncertainty from interest rate and inflation rate will reduce the portion of stock. Meanwhile, the bigger return of risky asset will boost the decision for stock belonging, directly inverse with volatility and degree of leverage.

Simulation: We would like to do simulation in our Indonesia emerging market JKSE with monthly basis for 103 periods from June 2005 December 2013 but effectively taken into simulation from January 2007 October 2012 for convenience purpose. The selected stock is PT. Telkom (TLKM) as the government enterprise of Indonesia main telecommunication company enlisted as the 45 most liquid ones. The Blue chip stock TLKM had best fit to CEV. The inflation and interest rate which were generated by Monte Carlo simulation were accessed with Vasicek process. All

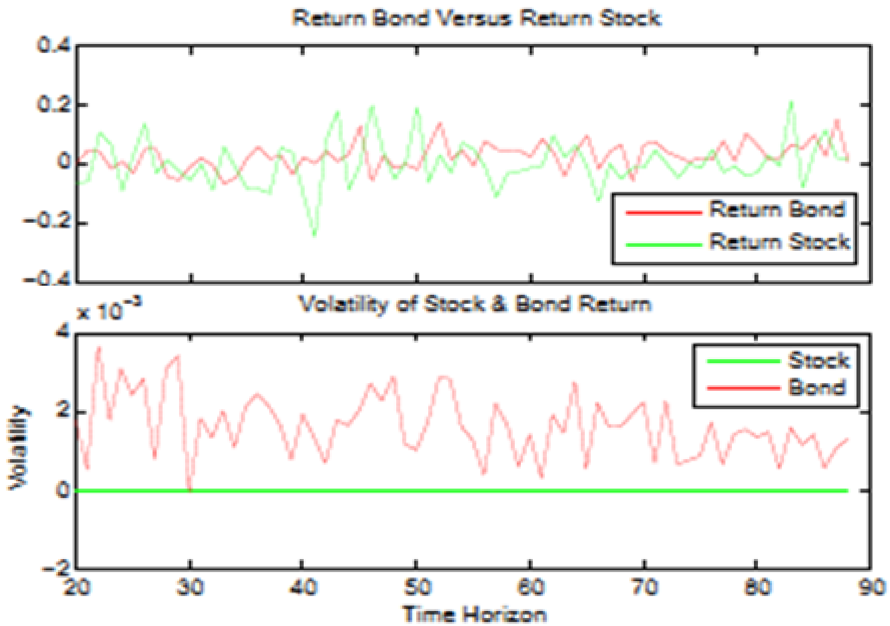

Fig. 1: Return and volatility for stock and bond

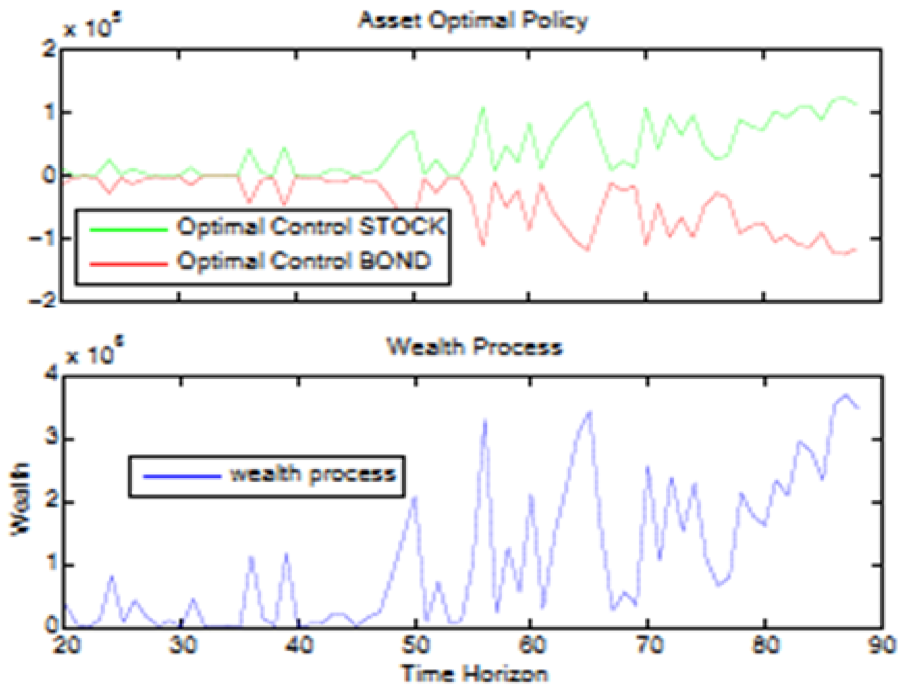

Fig. 2: Trading stragedy for optimal policy

of models above numerically convergent with Maximum Likelihood Estimation (MLE) using free toolbox provided by Sahalia and Kimmel. The return and volatility for each stock and bond is given on Fig. 1. We can observe directly that volatility for stock return is much more stable than bond return because our bond is valued under stochastic interest and inflation rates which we know that in our emerging market are not stable too. Hence, we cannot say that stock surely to be riskier than bond, at least in this context. Meanwhile the trading strategy for optimal policy shown in upper Fig. 2 as we strict the $\theta_1(t)+\theta_2(t) = 1$. The investor is risk averse in our defined utility function, so although in some period the bond gave higher return, she prefers to buy stock due to relatively stable volatility and she did not mind to short sell the bond. The wealth process during the investment period given by lower Fig. 2 as we can see in the end of period investor accumulated quadruple from her initial wealth (which is IDR 1.000.000) although for the first 20+ periods she suffers losses.

CONCLUSION

Researchers improve the Merton model with stochastic interest rate and stochastic inflation rates as well as the consideration of leverage effect in stock movement. The future works may consist series of improving these model that may mimics the real of market condition, i.e., the consideration of proportional transaction cost as well as the consider the risky assets may undergo default condition. It is something that needed to be serious accounted when we put our money in the emerging market.

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