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Research Article
Effect of Water Parameters on Mortality Rate of River Catfish (*Pangasius hypopthalmus*) Larvae Using Exponential Models with Cubic Interpolation

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Abstract

Background and Objective: Water quality problem would affect aquatic life described as the conditions of water, including chemical, physical, biological and aesthetic (appearance and smell) characteristics. This paper thus discusses on the effects of water parameters towards the mortality rate of the river catfish (*Pangasius Hypopthalmus*) larvae and henceforth, aims to identify factors affecting this catfish using exponential models. **Materials and Methods:** Data were collected from the Borneo Marine Research Institute (BMRI) in Universiti Malaysia Sabah which involves innovative research and dissemination of knowledge in aquaculture and marine sciences. This study was conducted for 15 days on 5 fish tanks with different salt concentrations and each had 200 fish larvae. Cubic spline interpolation was performed on the data missing values. Mathematical modelling procedures were applied involving the listing of all possible models, non-linear model transformation to linear models, selected models after multicollinearity and coefficient tests, best model selected based on the 8 selection criteria (8SC) and the goodness-of-fit (GOF) of the best model. **Results:** Comparisons of the best models between the tanks was made and it was found that best model was M19.5.2: $P = e^{0.42X_3 - 0.592X_{23}}$ from Tank 3 (4 ppt) where, X_3 represents the change in pH value of the water and X_{23} represents the interaction factor between pH value and effect of temperature. **Conclusion:** Results had conclusively indicated that the survival and bio-diversity of this river catfish were influenced by their surroundings and can be exponentially predicted.

Key words: Marine biological sciences, mortality rate, water parameters, cubic interpolation, exponential models

INTRODUCTION

Among problems affecting aquatic life is the water quality problem described as the conditions of water, including chemical, physical, biological and aesthetic (appearance and smell) characteristics usually with respect to its suitability for a particular purpose such as drinking or swimming. World aquaculture production continues to grow even though at a slowing rate. According to the latest available statistics collected globally by FAO[1], world aquaculture production attained another all-time high of 90.4 million t (live weight equivalent) in 2012 (US$144.4 billion), including 66.6 million t of food fish (US$137.7 billion) and 23.8 million t of aquatic algae (mostly seaweeds, US$6.4 billion). The term "food fish" includes finfishes, crustaceans, molluscs, amphibians, freshwater turtles and other aquatic animals (such as sea cucumbers, sea urchins, sea squirts and edible jellyfish) produced for the intended use as food for human consumption. Hence, this study is about the effect of water quality parameters on the mortality rate of river catfish (*Pangasius hypopthalmus*) larvae identified using exponential regression models. River catfish (*Pangasius hypophthalmus*) is the native species of Mekong basin as well as the Chao Phraya River and Maeklong basin, chiefly in Thailand, Myanmar, Laos and Vietnam. The river catfish (*Pangasius hypopthalmus*) larvae are often called Sutchi catfish, iridescent shark-catfish and striped catfish as they have a variety of common English names. Unusual high survival rates beneath faint light surroundings were found in sutchi catfish *Pangasianodon hypophthalmus* larval feeding behaviour and rearing environments[2]. Results showed that the feeding behaviour of sutchi catfish larvae most probably relied on chemo-sensing rather than optical sensing.

Exponential model is a non-linear polynomial. Exponential model has been effectively used in population growth or radioactive decay. Lee *et al.*[3] had studied a systematic review based on a negative exponential model, where the decline in excess risk of chronic obstructive pulmonary disease following quitting smoking was estimated, while Lee *et al.*[4] also did a systematic review to estimate the decline in excess risk of cerebrovascular disease following quitting smoking, based on the negative exponential model. Besides, Huang[5] conducted a study to see the effect of temperature on microbial growth rate using exponential models. Dos Santos *et al.*[6] applied exponential growth model of Nile tilapia (*Oreochromis niloticus*) strains when considering heteroscedastic variance. Khodabin *et al.*[7] studied generalized stochastic exponential population growth model with interpolation solution. Moreover, according to Baptista *et al.*[8] fatigue crack growth and load ratio effects in AL 2524 T3 alloy can be explained using an enhanced exponential model which is commonly used in aircraft engineering applications.

MATERIALS AND METHODS

Data collection: Data of larvae from female fresh water catfish were collected from the fish hatchery which is located at Borneo Marine Research Institute in University Malaysia Sabah in Kota Kinabalu, Sabah as shown in Fig. 1.

The river catfish (*Pangasius hypopthalmus*) larvae were put into 5 fish tanks each with different concentration of salt. The concentrations of salt used in this study were 0 part per thousand, 2 parts per thousand, 4 parts per thousand, 6 parts per thousand and 8 parts per thousand. Total number of river catfish (*Pangasius hypopthalmus*) larvae before the study was 1000 and divided with putting 200 of them in each fish tank. The study was conducted for 15 days and the number of fish died everyday was taken during day time and night time periods. The river catfish (*Pangasius hypopthalmus*) larvae were fed using live *Artemia* at 8:00-8:30 am in the morning and 4:00-4:30 pm in the evening.

Exponential models: Exponential model is a non-linear polynomial model. When it comes to solving problems relating to changes in populations, pollution, temperature, bank savings, drugs in the bloodstream and radioactive

Kingdom	: Animalia
Phylum	: Chordate
Class	: Actinopterygii
Order	: Siluriformes
Family	: Pangasiidae
Genus	: *Pangasius*
Species	: *P. hypophthalmus*

Fig. 1: Brood stock of *Pangasius hypophthalmus*
(Source: FAO[9])

materials, the exponential models are frequently used. To classify an exponential function is to make sure that it has a base that is constant and an exponent that is a variable. The general functions of the exponential models are shown below: General exponential model equation:

$$P_i = a_i e^{b_i(X_i)}, b_i > 0 \qquad (1)$$

Where:
P = Dependent variable
X = Independent variable
a = Constant variable
b = Constant variable and
e = Base of the function, with $i = 1, \cdots, n$

It is important to bear in mind that the value of the constant variable, 'b' should be more than zero for the function to be valid. This is due to the fact that it is not possible to determine the value of the dependent variable, 'Y' as the value cannot be calculated.

Although the exponential function and the logarithmic function seem the same in the function but in fact, they are not. The inverses of the exponential function are regarded as the logarithmic function. The function for logarithmic function is shown below:

$$Y_i = LnP_i = a_i b_i^{c_i(X_i)} = \ln a_i + c_i \ln b_i(X_i) = \beta_0 + \beta_i X_i \qquad (2)$$

Where:
b = Base of the function
a = Constant variable
c = Constant variable
X = Independent variable
I = $1, \cdots, n$ with n is the number of variables

Data analysis: Preliminary, two initial steps had to be carried out on the data before modelling can be done. They were:

• **Solving missing values:** The non-linear models were transformed into linear models as it would be easier to run these models by using the SPSS software version 21. Based on the data obtained, it was recorded there were a few days where no larvae died at certain days during the experiment. After non-linear models are transformed to linear models, the independent value, P will be 'ln P'. Missing value occurs as the value of 'ln 0' is a

mathematical error. To solve this problem, a method known as interpolation is carried out to get the value of the missing values. Interpolation is carried out to obtain a value between two points from the data. Since, the data for 'ln 0' is unable to be figured out, interpolation method is applied using Microsoft Excel. The interpolation in Microsoft Excel is known as Cubic Spline Interpolation

• **Normality test:** Normality tests were applied to every data obtained in order to determine whether the data is normal or non-normal data. For this study, the model that is to be applied with the data is an exponential model, which is a non-linear model. In order to use non-linear model, the data has to be a non-normal data. For this study, the data is divided into five categories, based on five fish tanks, each with five types of salt concentration. Thus, for each tank, the sample size will be $n = 30$. For sample size which is less than 50, Shapiro-Wilk test is applied to determine the normality of the data. If the significant p-value is less than 0.05, the data is said to be non-normal

The model-building procedures for multiple regressions can be referred[10,11]. The phases of model-building approach can be described as follows:

• **Phase I-all possible models:** In the development of the Exponential models for this datasets, number of larvae died would be the Dependent Variable (DV) noted by P, whereas, salt concentration or salinity (X_1), oxygen concentration (X_2), pH value (X_3) and temperature (X_4) would be the Independent Variables (IV). Days of experiment (A) and feeding times (B) were included as independent dummy variables in the models. Dummy variables were not included during the calculation of the possible models but executed in the models before model-building procedures were carried out. All possible models, N can be calculated by using the formula:

$$N = \sum_{j=1}^{q} j \left(^q C_j \right)$$

where, N is the number of possible models generated and q is the number of variables and $j = 1, 2, \cdots, q$.

• **Phase II-selected models:** Multi-collinearity test can be carried out in two ways which are by testing the correlation based value or through Variance Inflation Factor (VIF). In this study, VIF value used will be 5. The
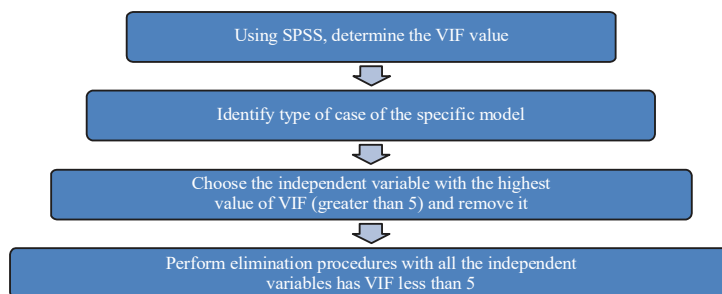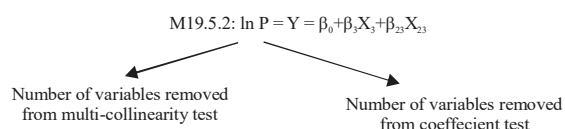
Fig. 2: VIF test procedures of multi-collinearity removals

Table 1: Eight selection criteria (8SC)

Variables	Formula	References	Variables	Formula	References
AIC	$\left(\dfrac{SSE}{n}\right)e^{\frac{2(k+1)}{n}}$	Akaike[14]	GCV	$\left(\dfrac{SSE}{n}\right)\left(1-\dfrac{k+1}{n}\right)^{-2}$	Golub *et al.*[18]
FPE	$\left(\dfrac{SSE}{n}\right)\dfrac{n+k+1}{n-(k+1)}$	Akaike[15]	SHIBATA	$\left(\dfrac{SSE}{n}\right)\dfrac{n+2(k+1)}{n}$	Shibata[19]
SCHWARZ	$\left(\dfrac{SSE}{n}\right)(n)^{\frac{k+1}{n}}$	Schwarz[16]	RICE	$\left(\dfrac{SSE}{n}\right)\left(1-\dfrac{2(k+1)}{n}\right)^{-1}$	Rice[20]
HQ	$\left(\dfrac{SSE}{n}\right)(\ln n)^{\frac{2(k+1)}{n}}$	Hannan and Quinn[17]	SGMASQ	$\left(\dfrac{SSE}{n}\right)\left(1-\dfrac{(k+1)}{n}\right)^{-2}$	Ramanathan[21]

overall procedure to remove the variables via multi-collinearity by using VIF is shown in Fig. 2. The types of cases and parameters counting can be refered[12]

Furthermore, the coefficient test was also applied to test the coefficients of the corresponding variables. The variable with a condition that has the highest p-value and greater than α = 0.05 would be removed where model labelling is given as: M19 as the 'parent' model with 5 multi-collinearity variables and 2 insignificant variables from coefficient test[13]:

$$M19.5.2: \ln P = Y = \beta_0 + \beta_3 X_3 + \beta_{23} X_{23}$$

Number of variables removed from multi-collinearity test

Number of variables removed from coeffecient test

- **Phase III-best model:** In order to achieve the BEST models, the Eight Selection Criteria (8SC) were also used in this study as shown in Table 1
- **Phase IV-goodness-of-fit:** Lastly, goodness-of-fit (GOF) test was also used to ensure how well the model fits into the problem or data based on the standardized of residuals. To determine the randomness of the residuals, a randomness test is done to determine them. If the value obtained is larger than 0.05, thus the null hypothesis is accepted and vice versa

RESULTS AND DISCUSSION

In solving missing values for data in Tank 2, cubic spline interpolation was applied and the results were shown circled as in Fig. 3.

For this study, q = 4 (excluded the 2 dummy variables), the number of all possible models would be:

$$N = 1(^4C_1) + 2(^4C_2) + 3((^4C_3) + 4(^4C_4) = 32$$

Since 5 fish tanks were experimented in this study, thus, a total of (32 models x 5 fish tanks) amounting to 160 possible model equations were obtained.

The list of non-linear models and linear model equations were partially shown below. After running the models using the SPSS version 22 software, there were a few selected models obtained from each fish tank of 0-8 ppt as shown in Table 2. The selected model was first determined using the multi-collinearity test via the VIF values obtained. Using SPSS, go to Analyse→Regression→Linear→Enter the required variables. The variable (s) with VIF>5.0, would be eliminated first. Subsequent elimination was carried out until the variables left were of VIF values less than 5.0. After the multi-collinearity test, the next step was to conduct the Coefficient test. Coefficient test[13] was done by eliminating sequentially one at a time the variable (s) that had p-values more than 0.05.
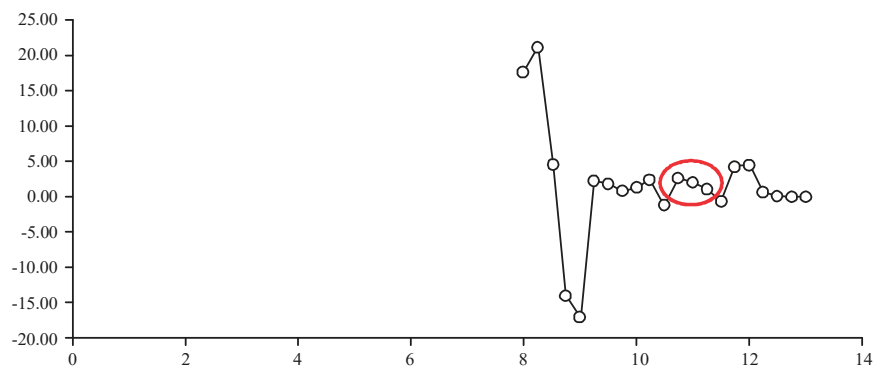
Fig. 3: Interpolated graph on missing values for tank 2

Table 2: List of selected models from each tank

Fish tanks	Best models
Tank 1 (0 ppt)	$M2.0.2 : Y = \beta_0 + \beta_2 X_2 + \mu_2$
	$M8.0.2 : Y = \beta_0 + \beta_2 X_2 + \beta_3 X_3 + \mu_8$
	$M17.4.2 : Y = \beta_0 + \beta_3 X_3 + (\beta_{1B} X_1 B) + (\beta_{3A} X_3 A) + \mu_{17}$
	$M18.4.3 : Y = \beta_0 + \beta_A A + (\beta_{1B} X_1 B) + \mu_{18}$
	$M24.8.3 : Y = \beta_0 + \beta_3 X_3 + A + (\beta_{1B} X_1 B) + \mu_{24}$
	$M25.8.4 : Y = \beta_0 + \beta_3 X_3 + \beta_{24} X_{24} + \mu_{25}$
	$M26.13.5 : Y = \beta_0 + \beta_{24} X_{24} + \beta_{34} X_{34} + \mu_{26}$
	$M31.17.5 : Y = \beta_0 + \beta_2 X_2 + \beta_{34} X_{34} + \mu_{31}$
Tank 2 (2 ppt)	$M2.0.1 : Y = \beta_0 + \beta_2 X_2 + \beta_A A + \mu_2$
	$M16.4.3 : Y = \beta_0 + \beta_2 X_2 + (\beta_{2A} X_2 A) + \mu_{16}$
	$M19.5.2 : Y = \beta_0 + \beta_{23} X_{23} + (\beta_{3A} X_3 A) + \mu_{19}$
	$M20.5.3 : Y = \beta_0 + \beta_{24} X_{24} + \mu_{20}$
	$M25.9.3 : Y = \beta_0 + \beta_{23} X_{23} + (\beta_{4A} X_4 A) + \mu_{25}$
	$M30.9.4 : Y = \beta_0 + \beta_{23} X_{23} + (\beta_{2A} X_2 A) + \mu_{30}$
Tank 3 (4 ppt)	$M1.0.2 : Y = \beta_0 + \beta_B B + \mu_1$
	$M2.0.2 : Y = \beta_0 + \beta_2 X_2 + \mu_2$
	$M3.0.2 : Y = \beta_0 + \beta_3 X_3 + \mu_3$
	$M17.5.1 : Y = \beta_0 + \beta_3 X_3 + \beta_B B + (\beta_{1A} X_1 A) + \mu_{17}$
	$M18.5.3 : Y = \beta_0 + \beta_A A + \mu_{18}$
	$M19.5.2 : Y = \beta_0 + \beta_3 X_3 + \beta_{23} X_{23} + \mu_{19}$
Tank 4 (6 ppt)	$M2.0.2 : Y = \beta_0 + \beta_2 X_2 + \mu_2$
	$M19.5.3 : Y = \beta_0 + \beta_{23} X_{23} + \mu_{19}$
	$M27.10.4 : Y = \beta_0 + \beta_{123} X_{123} + \mu_{27}$
Tank 5 (8 ppt)	$M2.0.2 : Y = \beta_0 + \beta_2 X_2 + \mu_2$
	$M3.0.2 : Y = \beta_0 + \beta_3 X_3 + \mu_3$
	$M6.0.2 : Y = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + \mu_6$
	$M8.0.2 : Y = \beta_0 + \beta_2 X_2 + \beta_3 X_3 + \mu_8$
	$M11.0.2 : Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \mu_{11}$
	$M19.5.2 : Y = \beta_0 + \beta_3 X_3 + \beta_{23} X_{23} + \mu_{19}$
	$M22.9.2 : Y = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + \beta_{23} X_{23} + \mu_{22}$
	$M23.8.5 : Y = \beta_0 + \beta_{24} X_{24} + \mu_{23}$
	$M28.10.3 : Y = \beta_0 + \beta_1 X_1 + \beta_{12} X_{12} + \mu_{28}$
	$M31.18.4 : Y = \beta_0 + \beta_3 X_3 + \beta_{123} X_{123} + \mu_{31}$

Table 3: List of best models for each fish tank

Fish tanks	Best models
Tank 1(0 ppt)	$M8.0.2 : P = ae^{\beta_0 + \beta_2 X_2 + \beta_3 X_3}$
Tank 2 (2 ppt)	$M30.9.4 : P = ae^{\beta_0 + \beta_{23} X_{23} + (\beta_{2A} X_2 A)}$
Tank 3 (4 ppt)	$M19.5.2 : P = ae^{\beta_0 + \beta_3 X_3 + \beta_{23} X_{23}}$
Tank 4 (6 ppt)	$M2.0.2 : P = ae^{\beta_0 + \beta_2 X_2}$
Tank 5 (8 ppt)	$M31.18.4 : P = ae^{\beta_0 + \beta_3 X_3 + \beta_{123} X_{123}}$

Best models obtained from each fish tank: The best models from each fish tank were tested using the 8 Selection Criteria (8SC). The selected models were then obtained the lowest value was selected as the overall best model as shown in Table 3.

Based on Table 3, each fish tank had its own best model. These best models obtained had the lowest value after using the 8 Selection Criteria (8SC). The randomness test can be done by doing the Runs Test in SPSS. The randomness test is to determine the randomness of the residuals. Tank 1, Tank 2, Tank 3 and Tank 5 were randomly distributed whereas Tank 4 was not randomly distributed.

In this study, each fish tank was able to attain its own best model. Among the best models from each fish tank, the overall best model was again determined using the Eight Selection Criteria (8SC). The model that shows the most minimum value will be chosen as the best model. The overall best model was M19.5.2 which was from Tank 3 (4 ppt) given as: $M19.5.2 : P = ae^{\beta_0 + \beta_3 X_3 + \beta_{23} X_{23} + \mu_{19}}$. The best model M19.5.2 was graphically exponential smoothing in micro-soft excel, comparing between the actual data and forecast as depicted in Fig. 4.

The model produced an exponential curve which is almost linear. The value of constant 'a' in the equation is 1, with $\beta_3 = 0.420$ and the value for $\beta_{23} = -0.592$. The model has one single independent variable (X_3) and has a first order interaction (X_{23}). The variable, X_3, represents the pH value of the water in the fish tank throughout the experiment. The mortality rate of the river catfish larvae is affected by the pH value of water. If the water is too acidic or too alkaline, the river catfish larvae may not be able to survive in that kind of
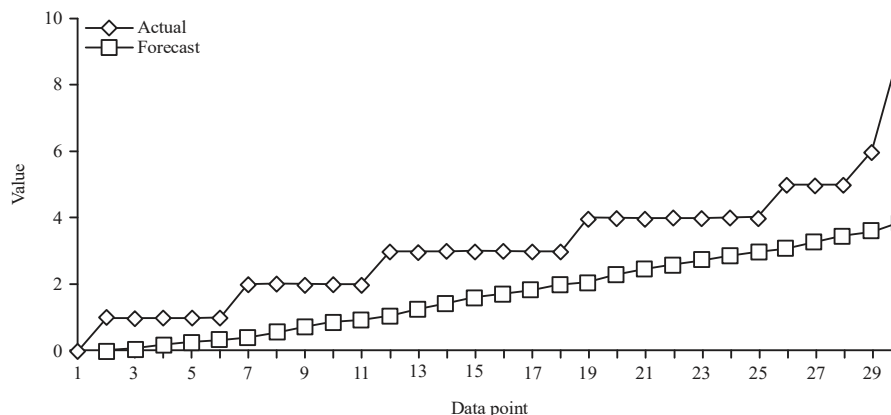
Fig. 4: Best model graph for tank 3 (4 ppt) from Microsoft excel

habitat. The reason is that they have small pores located on the surface of their skin. These pores are sensitive towards changes in their habitat or environment. The most suitable pH value for the river catfish larvae to live is near to neutral.

Huang[4] had stated that his study had regarded bacterial growth rate which was influenced by changes in temperature. The bacterial growth rate had a negative relationship with changes in temperature when an exponential model was applied. Similar effect was observed when applied to the mortality rate of the river catfish larvae. If the temperature is too high or too low, the environment will not be suitable for the larvae to live in. This thus shows that the mortality rate of the river catfish larvae is influenced by the isothermal conditions.

Meanwhile, a study done by Khodaibin et al.[7] had considered the exponential population growth where the dependent variable was not solely affected by only one variable but more. The model used was annually applied in Iran to predict the population of individuals. Similarly in this study, there were also many variables considered that might affect the mortality rate of the river catfish larvae and namely the variables were: changes in oxygen concentration, changes in pH values and changes in salt concentration. In order to increase the robustness and accuracy of this study, all possible variables that might be present in the environment were taken into account. Thus, with more variables, the probability of getting the model equation in this study will not be determined by only one variable is greater.

The mathematical model used viz. the exponential model is able to be applied in this study and the results discussed easily. The relationship between the water parameters and the mortality rate of the river catfish larvae showed an exponential growth graph. This means that the water parameters are directly proportional to the mortality rate of the river catfish larvae. As there are more variables of water parameters present in the environment, the higher the impact will be towards the larvae mortality rate.

The factors as in model M19.5.2 that are capable of affecting the mortality rate of the river catfish larvae are the pH value of the water in the fish tank (X_3) and interaction factor between the oxygen concentration and pH value (X_{23}) of the fish tank. These factors involving the water parameters that affect the mortality rate of the river catfish larvae showed a positively associated relationship.

CONCLUSION

Data obtained to mathematical model the mortality rate of the river catfish (Pangasius hypopthalmus) larvae were suitable to be carried out using an exponential model. The mortality rate was positively associated with the water parameters that existed throughout the experiment. These water parameters were essential but each had their own limitations within their habitat or environment. It is thus further recommended that reserved experimental data be provided as it can be used with ease to forecast targeted outcome and for future researchers who are interested in this type of research interest on biological sciences.

ACKNOWLEDGMENT

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