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Interim Crown Ratio Models for a Mixed *Tectona grandis* and *Gmelina arborea* Stand in the University of Ibadan, Nigeria*

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Abstract: This study aimed at developing interim crown ratio prediction models for mixed *Tectona grandis* and *Gmelina arborea* stand in the University of Ibadan, Nigeria. Based on the data set from the mixed *Gmelina arborea* and *Tectona grandis* stand in the University of Ibadan, several versions of four non-linear, individual tree functions (i.e., logistic, exponential, Weibull and Richards) for interim crown ratio predictions were tested. The total data set was used to fit and evaluate the functions. The functions were evaluated in terms of measures of fit (i.e., R^2, standard error of estimate, mean prediction residual, residual coefficient of variation) and prediction ability within the range of dataset. The significance of the estimated parameters was verified. The plots of residuals versus estimated crown ratio were also observed. The logistic and Richards functions were found to give better fit and were therefore selected as interim crown ratio models for the stand. The selected functions used stem form and tree size as independent variables.

Key words: Crown ratio, mixed hardwood plantation, modeling, tree growth parameters

INTRODUCTION

Crown Ratio (CR), which is the ratio of live crown length to tree total height is used to predict growth and yield of trees and forests (Bella, 1971; Sprinz and Burkhart, 1987; Daniels *et al*., 1979; Wykoff *et al*., 1982; Monserud and Sterba, 1996). Crown ratio is a very useful parameter in forest health assessment. It is a good indicator of tree vigour (Assmaun, 1970; Hasenauer and Monserud, 1996). It is also a good indicator of competition and survival potential (Oliver and Larson, 1996). It is used as an indicator of wood quality (Kershaw *et al*., 1990), wind firmness (Navratil, 1997) and stand density (Clutter *et al*., 1983). It is also a feature of interest in the multiple goal forest management including wildlife habitat, recreation, shade potentials and visual quality (McGaughey, 1997). It is however note worthy that CR measurement can be time consuming resulting in measures of few sub sampled trees in plots. In addition to this, the base of live crown may be difficult to locate in very dense stands and for very large trees. These observations were also made by Temesgen *et al*. (2005).

Empirical models have been used by some studies to predict CR (Belcher *et al*., 1982; Wykoff *et al*., 1982; Dyer and Burkhart, 1987; Maguire and Hann, 1990; Hynynen, 1995; Hasenauer and Monserud, 1996; Soares and Tome, 2001; Temesgen *et al*., 2005). Most of these models were formulated for either even aged single species stands or multi-species stands comprising trees of different ages. There are, however relatively little research on planted mixed stands with two species especially. Growth measures including crown ratio are obviously unique for such stands. Research has shown that crown size is a prime indicator of a tree's potential response to thinning (Smith, 1986; Chapman, 1953). The timing of silvicultural operations under mixed planted stands could be enhanced by obtaining a suitable crown model. Furthermore, CR is used as an input variable to estimate growth

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and mortality of individual trees and also is used to display changes in appearances of stands over time for habitat suitability and visual changes. The objective of this research was to develop interim crown ratio prediction models for mixed *Tectona grandis* and *Gmelina arborea* stand in the University of Ibadan, Nigeria using tree size and stem form as possible predictor variables. These variables are commonly used for more homogenous stands such as even-aged single species. For this stand, a competition measure, ratio of the tree basal area to tree basal area of the largest was included in the models represent competition.

MATERIALS AND METHODS

Study Area

The University of Ibadan is about 5 km to the North of the city of Ibadan at Longitude 3° 54′ east and Latitude 7° 26′ North and at a mean altitude of 200 m above sea level. The annual rainfall is approximately 1200 mm most of which falls within the period of April and October giving a predominantly dry season from November to March. The mixed *Tectona grandis* and *Gmelina arborea* stand was established in 1985 by the Department of Forest Resources Management. The stand is located at the North-Western part of the University.

The Data

The data used for this study were collected in October 2005. Five temporary sample plots of size 20×20 m were randomly laid in the mixed stand. The following tree parameters were measured in each sample plot: diameter at breast (dbh) over bark of all trees (cm), diameter over bark at the base, middle and top of all the trees (cm), total height (m), merchantable height (m) and crown length (m). The following individual tree variables were also computed from the data collected; stem basal area (m^2), total height-dbh ratio, crown ratio, the ratio of diameter at the top to diameter at the base (a measure of stem form) and the ratio of stem basal area to the basal area of the largest tree (a measure of competition). The mean, maximum, minimum and coefficient of variation of the main dendrometric variables and other derived variables are presented in Table 1.

Model Description

Crown ratio values usually range from 0.0 (i.e., no crown, dead, top broken or defoliated) to 1.0 (i.e., crown extending over the entire tree bole). Since logistic model is restricted to 0-1 range, some studies have based crown ratio equations on the logistic function (Ritchie and Hann, 1987; Hasenauer and Monserud, 1996; Temesgen *et al*., 2005). Other studies used exponential, Richards and Weibull cumulative distribution functions (Deusen and Biging, 1985; Dyer and Bhurkhart, 1987; Soares and Tome, 2001). In this study, these four functions were investigated. The original forms of the investigated models and the restrictions imposed on them are presented in Table 2. The variables that are commonly used for crown ratio modeling are tree age, tree size (diameter, height, height/diameter ratio), stand density (number of planted trees or live trees/ha, basal area), maximum tree dimension (diameter), mean tree dimension (diameter, dominant diameter), site productivity (dominant height, site index) and stand-level competition. For this study, tree age, site productivity and stand-level competition were not included, since the stand under study comprise of single age and location. However, a measure of stem form (ratio of diameter at the top to diameter at the base) was included.

The linear function X (Table 2) was expressed as a combination of tree size (diameter, merchantable height, total height and total height/Dbh ratio) and stem form (diameter at the top/diameter at the base ratio). The number of variables was restricted by the presence of only one

Table 1: Characteristics of the tree variables used for model fitting (N = 78)

		Species		
Variables	Statistics	*Gmelina arborea* (n = 65)	*Tectona grandis* (n = 11)	Pooled (N = 78)
Dbh	Mean	0.2855	0.2611	0.2801
	Maximum	0.6050	0.3060	0.6050
	Minimum	0.1660	0.2200	0.1560
	CV	0.2700	0.1000	0.2600
THT	Mean	19.30	19.60	19.30
	Maximum	29.30	24.80	29.30
	Minimum	11.00	14.00	11.00
	CV	0.19	0.17	0.18
MHT	Mean	13.50	12.50	13.30
	Maximum	19.50	16.50	19.50
	Minimum	7.30	8.50	7.30
	CV	0.23	0.20	0.23
CR	Mean	0.30	0.36	0.31
	Maximum	0.58	0.48	0.58
	Minimum	0.06	0.26	0.06
	CV	0.35	0.24	0.30
CL	Mean	5.80	7.10	6.00
	Maximum	11.50	10.50	11.50
	Minimum	1.00	4.00	1.00
	CV	0.31	0.31	0.34
HDR	Mean	70.71	75.08	71.94
	Maximum	123.60	91.51	123.60
	Minimum	36.70	60.61	36.70
	CV	0.25	0.14	0.24
D_t/D_b	Mean	0.33	0.38	0.34
	Maximum	0.52	0.53	0.53
	Minimum	0.10	0.27	0.10
	CV	0.29	0.19	0.28

Dbh = Diameter at breast height (m), THT = Total height (m), MHT = Merchantable height, CL = Crown length (m), Dt/Db = Ratio of diameter at the top to diameter at the base, HDR = Ratio of total height to Dbh, CV = Coefficient of variation

Table 2: Functions tested and restrictions imposed on the prediction of tree crown ratio

Function	Restrictions
Logistic	
$Y = \dfrac{a_0}{a_1 + a_2 e^{(m-cX)}}$	$a_1 = 1$, $a_2 = -1$, $m = 1$
Richards	
$Y = \dfrac{a_0}{a_1 + a_2 e^{(m-cX)^{1/k}}}$	$a_1 = 1$, $a_2 = -1$, $m = 1$, $k = 2$*
Weibull	
$Y = a_0(a_1 + a_2 e^{-kX^m})$	$a_0 = 1$, $a_2 = 1$, $m = 1/5$*
Exponential	
$Y = a_0(a_1 + a_2 e^{-kX})$	$a_0 = 1$, $a_2 = 1$

Note: Y is tree crown; X is linear function of tree size and stem form; a_0 is asymptote; a_1, a_2, c, k and m are function parameters; *The restrictions k = 2 (under Richards function) and m = 1/5 (under Weibull function) are explained under results section of this study

from each group. Due to the peculiar situation of the stand, the crown ratio models developed were based on individual trees. All the variables were tested with individual tree crown ratio as dependent variable. For all the models, the following statistics were calculated:

(a) Estimated Standard Error of Estimates (SEE)

$$SEE = \sqrt{\frac{\sum_{i=1}^{n} \hat{e}_i^2}{n-k}} \quad (1)$$

(b) Coefficient of determination (R^2)

$$R^2 = 1 - \frac{SSE}{SST} \quad (2)$$

where, $\hat{e}$ is the difference between the observed (y_i) and the estimated crown ratio values $(\hat{y})$; SEE is the error sum of squares, SST is the total sum of squares, n is the number of trees in the model-fitting dataset and k is the number of coefficients in the fitted equation. Furthermore, residual plots for each fitted model were used to check for lack of fit. Evaluation of the functions was also achieved through the observation of the nature of contribution of the parameter estimates and the computation of mean residual, standard deviation of the residual and the Coefficient of Variation (CV) of the residual. Different versions of the logistic, Weibull, Richards and exponential functions were fitted first, separately to the two species and also to the whole data set. The parameter estimation of these non-linear functions was based on the least squares method associated with Quasi-Newton minimization technique of non-linear estimation option of STATISTICA version 5.1 (1997). Both the significance and the stability of the parameters estimated were checked based on the asymptotic t-statistic and standard errors of the parameters. When the parameter estimated was not significantly different from zero the variable and the parameter were discarded.

Model evaluation was based on the computation of the following statistics for the comparison of the selected functions:

(i) Mean Prediction Residual (MPR), which is the average residual given as

$$MPR = \frac{\sum_{i=1}^{n} (\text{Observed} - \text{Predicted})}{n} \quad (3)$$

Residual Standard Deviation (RSD), which is a measure of the variation in residuals and hence a measure of prediction precision.

Residual Coefficient of Variation (RCV) was also computed to address the weakness of RSD. It is note worthy that generally, standard deviation or its square (i.e., variance) can not be very useful in comparing two or more series where either the units of measurement are different or the mean values are different. A standard deviation of 5, for example, with associated mean value of 30 has an altogether different meaning compared to the same standard deviation associated with a mean of 90. Apparently, the variability in the second case is much less than the first case. Coefficient of variation therefore takes care of this problem. RCV is computed as:

$$RCV = \frac{RSD}{MPR} \quad (4)$$

Prediction sum of squares (PRESS) statistic is defined as

$$PRESS = \sum_{i=1}^{n} (\text{Observed} - \text{Predicted})^2 \quad (5)$$

The modeling efficiency, also called adjusted R^2 (ME) was also computed.

This statistic provides a simple index of performance on a relative scale, where 1 indicates a perfect fit, 0 reveals that the model is no better than a simple average and negative values indicate a poor model indeed (Vanclay and Skovsgaard, 1997). The modeling efficiency is computed as:

$$ME = 1 - \left(\frac{SSE}{n-p}\right)\left(\frac{n-1}{SST}\right) \tag{6}$$

Where:
SSE = Sum of squares error
SST = Sum of squares total
n = No. of observation
p = No. of parameters in the equation

RESULTS AND DISCUSSION

Fitting the models to individual species data set separately consistently gave a good fit for *Gmelina arborea*, while *Tectona grandis* data set consistently gave a poor fit. One possible reason for the poor fit of the *Tectona grandis* data set could be as a result of the sample size. The two species were coded as 0 for *Tectona grandis* and 1 for *Gmelina arborea* and incorporated as a dummy variable. This effort did not improve the relationships. The functions consistently indicated that species was not significant in the estimation of individual tree crown ratio of the mixed stand. Hence, the two species' data set were merged together for modeling.

Model Fitting and Selection

The selected versions of the logistic, Weibull, Richards and exponential functions as well as their parameter estimates and fit statistics are presented in Table 3. All the functions have the ratio of top diameter to base diameter and merchantable height as the independent variables. These two variables were found to be important in defining the tree crown ratio of the mixed stand. The Height-Diameter ratio (H/D) usually considered as an important variable in modeling crown ratio (Hanus *et al.* 2000; Hasenauer and Monserud, 1996; Ritchie and Hann, 1987) was not significant in all the functions and was therefore not included. This finding is similar to the finding of Marshall *et al.* (2003) in their crown profile model. All other variables failed to explain tree crown ratio and were therefore not included in the models.

Convergence problems were detected in the fitting of Richards and Weibull functions when the index parameters were not restricted. This observation was also noted by Soares and Tome (2001). To estimate the index parameters associated, respectively with Richards and Weibull functions (i.e., k and m from Table 2), it was decided to test a set of fixed values of these parameters. The k and m corresponding to the smallest residual sum of squares and that allow for computation of asymptotic t-test statistic and standard error were selected. The final values of k = 2 and m = 1/5 were obtained for the Richards and Weibull, respectively.

Figure 1 shows the graphical relationship between the residuals (observed CR-predicted CR) and the crown ratio estimates obtained with logistics (L), Weibull (W), Richards (R) and the exponential (E) functions. For all the functions, a systematic variation was not observed, although a great dispersion associated with the smaller predicted values was evident (i.e., CR values between 0.2 and 0.3). This trend was similar to the findings of Soares and Tome (2001). In spite of the fact that the crown ratio values predicted by the four functions laid between 0 and 1, the exponential function predicted the lowest maximum values (0.48) which according to the characteristics of the total data set, seemed to underestimate the observed values. The Weibull function predicted the highest minimum values (0.22) which seemed to overestimate the observed values. However, none of the functions predicted negative crown ratio values. Therefore all the functions were proposed for the evaluation task.

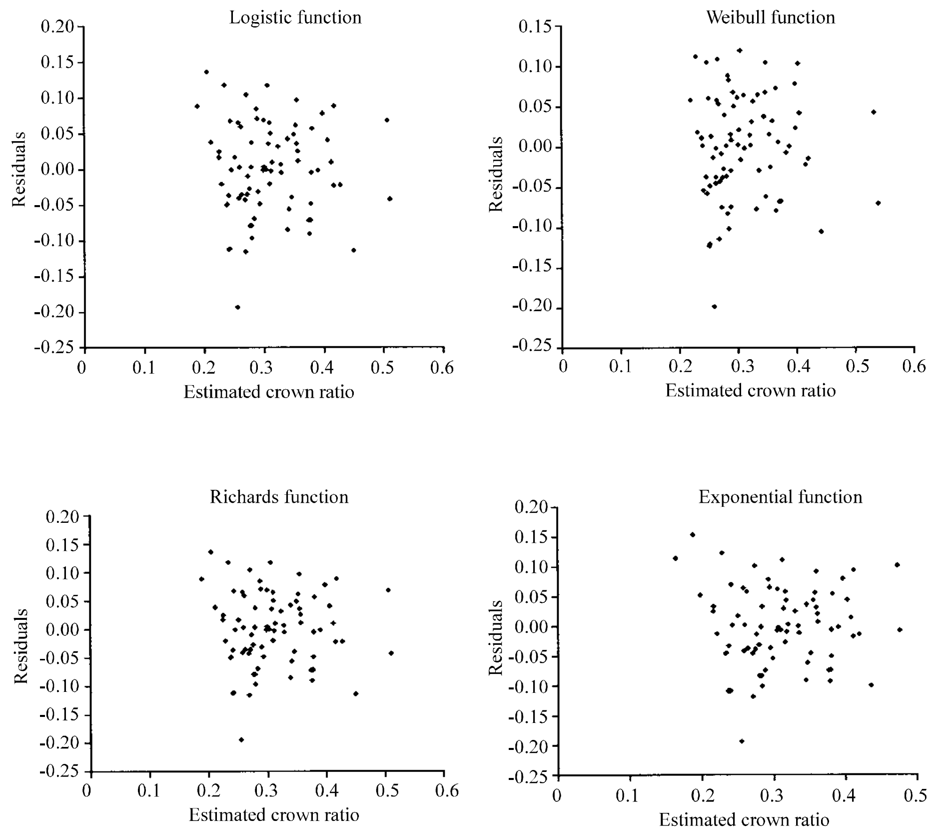

Fig. 1: Graphical relationship between residuals and crown ratio values estimated with logistic, Weibull, Richards and exponential functions

Table 3: Tree crown ratio models selected with parameter estimates and fit statistics, n = 78

Function	Parameter	Estimate	SE	t (df = 75)	p-level
Exponential					
$Cr = a_0 + e^{\left(-a_1\left(\frac{d_t}{d_b}\right)+a_2(MHT)\right)}$	a_0	- 0.571	0.054	10.538	0.000
	a_1	- 0.403	0.077	5.222	0.000
$R^2 = 0.50$, SE = 0.066	a_2	-0.020	0.004	4.966	0.000
Richards					
$Cr = \frac{a_0}{1-e^{\left(1-a_1\left(\frac{d_t}{d_b}\right)+a_2MHT\right)^{1/2}}}$	a_0	- 1.512	0.269	5.616	0.000
	a_1	0.801	0.151	5.287	0.000
$R^2 = 0.52$, SE = 0.065	a_2	0.039	0.008	4.986	0.000
Logistic					
$Cr = \frac{a_0}{1-e^{\left(1-a_1\left(\frac{d_t}{d_b}\right)+a_2MHT\right)}}$	a_0	-0.756	0.135	5.616	0.000
	a_1	0.801	0.151	5.287	0.000
$R^2 = 0.52$, SE = 0.065	a_2	0.039	0.008	4.986	0.000
Weibull					
$Cr = a_0 + e^{\left(-a_1\left(\frac{d_t}{d_b}\right)+a_2MHT\right)^{1/3}}$	a_0	0.183	0.040	4.562	0.000
	a_1	-2.950	0.588	5.015	0.000
$R^2 = 0.52$, SE = 0.065	a_2	-0.116	0.041	2.828	0.006

Cr = Tree crown ratio, d_t/d_b = Ratio of top diameter to base diameter, SE = Standard error of the estimates, t = asymptotic t statistic, R^2 = Coefficient of determination, p-level = probability level

Model Evaluation and Interpretation

Only very minor changes were observed in SE and R^2 of the selected functions. Since crown ratio is constrained to the interval of 0 and 1, SE differences were necessarily small (Temesgen *et al.*, 2005).

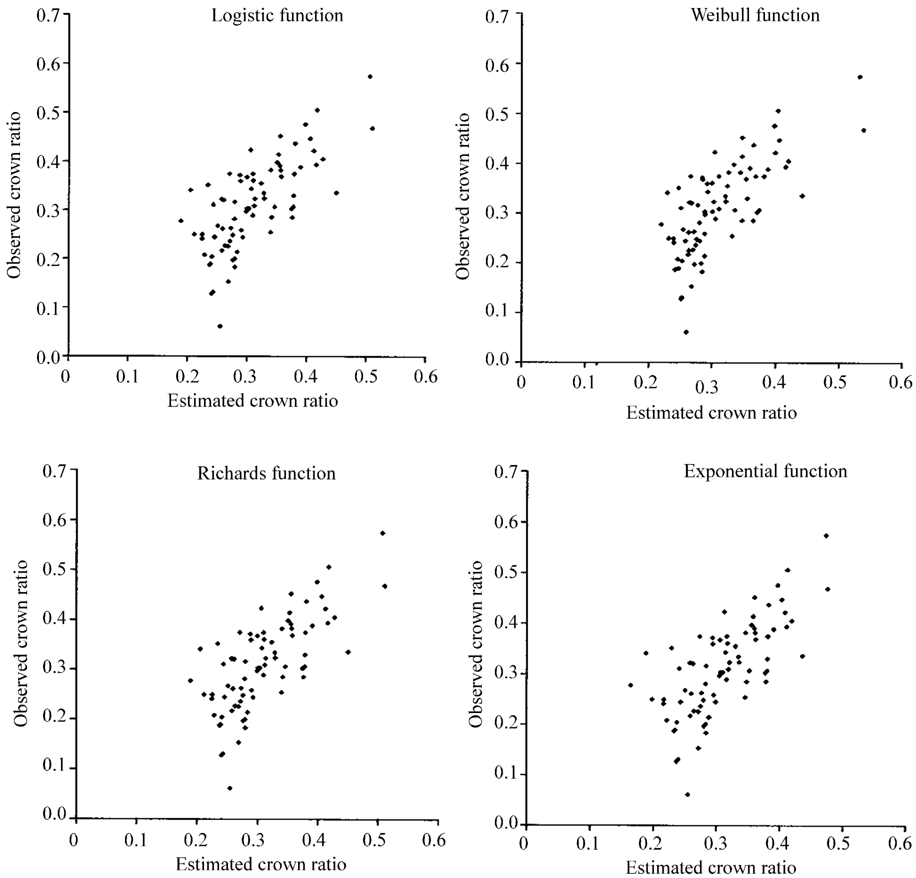

Fig. 2: Relationship between the observed crown ratio and the estimated crown ratio with the logistic, Weibull, Richards and exponential functions

Table 4: Evaluation of the logistic (L), Weibull (W), Richards (R) and exponential (E) functions

Function	MPR	RSD	RCV	PRESS	ME
L	0.00013365	0.06430671	481.16	0.318	0.51
W	0.00000000622374	0.064633243	107722207.17	0.322	0.50
R	0.00013334	0.06430671	482.276	0.318	0.51
E	-0.000000156402	0.0655	-419871.276	0.330	0.49

MPR, mean prediction residual; RSD, residual standard deviation; RCV, residual coefficient of variation; PRESS, PRESS statistic; ME, modelling efficiency

All the parameter estimates retained in the functions were found to be significantly different from zero. Table 4 presents the values of the evaluation statistics. The measures of precision and bias associated with the four functions as well as the corresponding modelling efficiency.

The mean prediction residual values associated with each of the functions were found to be negligible. The residual standard deviation values for the four functions were observed to be similar. However, residual coefficients of variation for the functions were quite different and revealing. Although, Weibull and exponential functions had the least mean prediction error, the logistic and Richards functions were more consistent (based on the RCV values), precise and equally had negligible bias values. The RCV values for W and E were consider very large and indicative of poor performance. A drop in predictability of W and E functions to 50 and 49%, respectively was also observed.

Furthermore, L and R had the least PRESS statistic and highest value of modelling efficiency. This finding confirms the suitability of Logistic and Richards functions as discovered by Temesgen *et al.* (2005) and Soares and Tome (2001), respectively.

The graphs of observed versus predicted crown ratio values for the four functions did not reveal a strong linear relationship (Fig. 2). However, the dispersion observed around the line (y = x) for crown ratio values between 0.25 and 0.50 was well balanced. This trend was similar to the findings of Soares and Tome (2001) under *Eucalyptus* plantations in Portuga.

CONCLUSION

Based on the evaluations of the functions examined in this study, the logistic and Richards functions are recommended as interim tree crown ratio prediction equations for mixed *Gmelina arborea* and *Tectona grandis* stand in the University of Ibadan, Nigeria:

Logistic Function

$$CR = \frac{-0.7561}{1 - e^{(1-0.8007 \, d_1/d_b + 0.0391MHT)}}$$

Richards Function

$$CR = \frac{-1.5122}{1 - e^{(1-8.007 \, d_1/d_b + 0.0391MHT)^{1/2}}}$$

The two functions are tree size and stem form dependent, reflecting the importance of stem form in describing the tree crown. It is note worthy that small sample data were used for the modelling exercise. As more data become available, the two functions can further be investigated through validation with independent data set final selection of tree crown prediction model for the mixed stand.

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