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## **Carrier Interferometry Based VOFDM with Sub-optimal Modulation Scheme and Low PAPR**

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### **ABSTRACT**

Sub-optimal modulation scheme was proposed to reduce the Bit Error Rate (BER) in Vector Orthogonal Frequency Division Multiplexing (VOFDM). Suboptimal modulation scheme is connected with the minimum Euclidean distance of all possible received vector symbols. VOFDM using the suboptimal modulation has shown an improvement on eliminating channel spectral nulls. However, it has a high Peak to Average Power Ratio (PAPR) of the transmitted signal which may increase the complexity of the system. This paper introduces the carrier interferometry technique in suboptimal modulation-VOFDM to reduce the PAPR and improve the BER performance. The simulation results of the proposed system show about 10dB PAPR reduction compared to the conventional VOFDM system.

**Key words:** Vector OFDM, peak to average power ratio, sub-optimal modulation scheme, carrier interferometry

### **INTRODUCTION**

Orthogonal Frequency Division Multiplexing (OFDM) has become an attractive technology for high data rate transmission in wireless and wired communication systems. It may combat the intersymbol interference (ISI) by using the Cyclic Prefix (CP). Also when the ISI channel length is large, the Cyclic Prefix (CP) length is large too which results in a high data rate overhead.

In (Xia, 2001), precoded-OFDM was proposed to eliminate channel spectral nulls basically by adding (P-V) zeros between each two V consecutive information symbols. However, the insertion of zeros increases the data rate of the system; hence the Peak to Average Power Ratio (PAPR) increases too. Vector-OFDM was proposed to reduce the data rate overhead of the CP, where there is no zeros inserted between two V information symbols but did not show a robustness to channel spectral nulls.

Huang proposed an optimal and suboptimal modulation scheme to improve the performance of VOFDM over channels with spectral nulls (Huang *et al.*, 2010). The modulation scheme was designed based on the minimum Euclidean distance among received vector symbols. The performance of VOFDM system using suboptimal modulation was better than the performance of conventional VOFDM system. However, there was no improvement on reducing the PAPR which is higher than the conventional VOFDM. In the application case, the Peak to Average Power Ratio (PAPR) should be well addressed, since the PAPR performance directly determines the linear requirement on power amplifier. Research works have been proposed to reduce PAPR values in OFDM systems like amplitude clipping, tone reservation, selective mapping, partial transmit

sequence, interleaving and coding techniques (Han and Lee, 2005). The rotated precoder was studied in Wu and Nassar (2005) to reduce the PAPR values of the conventional Precoded-OFDM system by using a precoder which is not a constant matrix. The simulation results showed that its PAPR performance was comparable to the conventional OFDM system and it has the same BER performance as the conventional precoded-OFDM system. The carrier interferometry OFDM (CI/OFDM) was proposed by Natarajan *et al.* (2001) to improve Bit Error Rate (BER) performance and reduce the PAPR values through spreading each information symbol across all subcarriers with orthogonal CI phase codes. The CI technique produces the high BER performances due to the frequency diversity exploited in each symbol.

In Rugumira *et al.* (2011), we proposed the hybridization of CI spreading codes and VOFDM using BPSK modulation where the PAPR of VOFDM was dramatically reduced. In this paper, we study and evaluate the performance of the carrier interferometry based VOFDM (CI-VOFDM) system using suboptimal modulation scheme which is an extension of VOFDM with suboptimal modulation. The proposed OFDM spreads each information symbol in a vector sequence across all subcarriers using orthogonal CI spreading codes. We also propose the appropriate weights based on minimum mean square error combining (MMSEC). The CI-VOFDM using suboptimal modulation has a reduced PAPR of about 10 dB compared to the conventional VOFDM.

The remaining of the paper is arranged as follows: Section II presents the structure of suboptimal modulation for CI-VOFDM transmitter. A study of PAPR benefit and the performance evaluation are provided in section III and we concluded in section V.

## SYSTEM MODEL

**Vector-OFDM system:** Vector-OFDM was proposed for single antenna systems where the input data  $a(n)$  with length  $K$ , ( $K = VN$ ) is mapped and blocked into  $V \times N$  vector sequence, where  $V$  is the vector size and defined as:

$$\tilde{a}(n) = (\tilde{a}_0(n), \tilde{a}_1(n), \dots, \tilde{a}_{V-1}(n))^T \quad (1)$$

where,  $a_v(n) = (Vn+v)$ ,  $V = 0, 1, \dots, N-1$  and is of vector size  $V \times 1$ . Since the precoder is the identity matrix, its output is equal to its input. The vector sequence  $a(n)$  is blocked into  $N \times V$  vector sequences, given by:

$$\hat{a}(n) = [\tilde{a}^T(Nn), \tilde{a}^T(Nn+1), \dots, \tilde{a}^T(N(n+1)-1)], \quad n = 0, 1, \dots, N-1 \quad (2)$$

Then the individual components of the vector sequence is transformed by the  $N$ -point IFFT, its output is:

$$\tilde{s}_v = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{a}_n \exp(j2\pi nv/N), \quad v = 0, 1, \dots, N-1 \quad (3)$$

The vector sequence after adding the cyclic prefix is:

$$\hat{s} = (\tilde{s}_{N-1}^T, \tilde{s}_{N-1+H}^T, \dots, \tilde{s}_{N-1}^T, \tilde{s}_0^T, \tilde{s}_1^T, \dots, \tilde{s}_{N-1}^T) \quad (4)$$

where,  $\bar{\Gamma} = [L/V]$  is the length of the cyclic prefix. Then the vector sequence  $\hat{s}(n)$  is converted into scalar sequence  $s(n)$  and transmitted through a multipath channel.

At the receiver, the received scalar sequence  $r(n)$  after removing the cyclic prefix is blocked into vector sequence:

$$\tilde{r}(n) = [\bar{r}_0^T(n), \bar{r}_1^T(n), \dots, \bar{r}_{N-1}^T(n)]^T \quad (5)$$

Each  $\bar{r}_n$  is the vector with size  $V \times 1$ . The output of  $N$ -point FFT is a vector sequence with size  $V \times 1$ :

$$\hat{r}_v = \sum_{n=0}^{N-1} \bar{r}_n \exp(-j2\pi n v / N), \quad v = 0, 1, \dots, N-1 \quad (6)$$

**Suboptimal modulation scheme based-VOFDM:** The design of suboptimal modulation scheme was based on the Euclidean distance of the received vector symbols by considering the relationship between the input signal  $\tilde{a}_v(n)$  and the output signal  $\hat{r}_v(n)$ :

$$\hat{r}_v(n) = H_v \tilde{a}_v(n) + \tilde{\eta}_v(n), \quad v = 0, 1, \dots, N-1 \quad (7)$$

under the condition that, the cyclic prefix length  $\bar{\Gamma}$  should be greater than or equal to the order of the equivalent MIMO transfer function matrix  $H(z)$ . Where  $H(z)$  is the blocked version of  $H(z)$  and is given by:

$$H(z) = \begin{bmatrix} h_0(z) & z^{-1}h_{v-1}(z) & \dots & z^{-1}h_1(z) \\ h_1(z) & h_0(z) & \dots & z^{-1}h_2(z) \\ \vdots & \vdots & \ddots & \vdots \\ h_{v-2}(z) & h_{v-3}(z) & \dots & z^{-1}h_{v-1}(z) \\ h_{v-1}(z) & h_{v-2}(z) & \dots & h_0(z) \end{bmatrix} \quad (8)$$

where,  $h_v(z)$  is the  $v$ th polyphase component of  $H(z)$ , i.e.:

$$h_v(z) = \sum_l h(Vl + v)z^{-l}, \quad v = 0, 1, \dots, V-1$$

and  $H_v = H_v(z)|_{z = \exp(j2\pi v/N)}$ .  $\eta_v$  is the blocked version of the additive noise and its components have the same power spectral density.  $H(z)$  has a property that can be diagonalized since, is a factor of circulant matrix (Zhang *et al.*, 2005) and may be defined as:

$$H_v = U_v^H \bar{H}_v U_v \quad (9)$$

where,  $U_v$  is the unitary matrix whose entries  $[U_v]_{s,m}$  is as follows:

$$[U_v]_{s,m} = \frac{1}{\sqrt{V}} \times \exp(-j2\pi(1+(s-1)L)(m-1)/N), \quad (10)$$

$$s, m \in \{0, 1, \dots, V-1\}$$

and  $\bar{H}_v$  is the diagonal matrix;  $\bar{H}_v = \text{diag}\{H_v, H_{v+N}, \dots, H_{v+(V-1)N}\}$ .

We assume that the two different vector symbols after digital baseband modulation  $\tilde{a}_v^e$  and  $\tilde{a}_v^c$  are transmitted over the  $v$ th vector channel. We assume also that the noise  $\tilde{n}_v$  in Eq. 7 equals zero. The Euclidean distance of the received vector symbols between  $\hat{r}_v^e = U_v^H \bar{H}_v U_v \tilde{a}_v^e$  and  $\hat{r}_v^c = U_v^H \bar{H}_v U_v \tilde{a}_v^c$  is illustrated as:

$$\begin{aligned} d_{\tilde{a}_v^e, \tilde{a}_v^c}^2 &= \left\| U_v^H \bar{H}_v U_v (\tilde{a}_v^e - \tilde{a}_v^c) \right\|_F^2 \\ &= (\tilde{a}_v^e - \tilde{a}_v^c)^H U_v^H \bar{H}_v^H \bar{H}_v U_v (\tilde{a}_v^e - \tilde{a}_v^c) \\ &= \frac{1}{V} \left( |H_v|^2 |A_v|^2 + |H_{v+N}|^2 |A_{v+N}|^2 + \dots \right. \\ &\quad \left. + |H_{v+(V-1)N}|^2 |A_{v+(V-1)N}|^2 \right) \end{aligned} \quad (11)$$

where,  $|A_v|^2 + |A_{v+N}|^2 + \dots + |A_{v+(V-1)N}|^2$  are the  $N$ -point FFT of  $(a_0^v, a_1^v, \dots, a_{V-1}^v)^*$ .

It was explained by Goeckel and Ananthaswamy (2002) that BER performance depends on the Euclidean distance of the received vector symbols. Therefore, in order to improve the BER performance of VOFDM system the suboptimal modulation scheme was designed to make the Euclidean distance as large as possible.

In suboptimal modulation, the vector size of the blocked symbol must be equal to the length of the channel impulse response. Therefore, Eq. 11 become:

$$(\tilde{a}_v^e - \tilde{a}_v^c)^H U_v^H \bar{H}_v^H \bar{H}_v U_v (\tilde{a}_v^e - \tilde{a}_v^c) = \frac{d_v^2}{V} \left( |H_v|^4 + |H_{v+N}|^4 + \dots + |H_{v+(V-1)N}|^4 \right) \quad (12)$$

According to the constellation figure of the suboptimal modulation given by Huang *et al.* (2010), this scheme is a generalization of MQAM modulation scheme.

**CI-VOFDM system with suboptimal modulation scheme:** Suboptimal modulation scheme based CI-VOFDM is an extension of the VOFDM system with suboptimal modulation scheme where the CI spreading code introduces orthogonal properties among all transmitted information symbols in the vector sequences. Each vector sequence  $\tilde{a}_v(n)$  after serial to parallel conversion is modulated onto all of  $N$  subcarriers. The information symbols are separated from each other by establishing the complex spreading codes:

$$\delta_{k,n} = \exp(j(2\pi/N) \cdot k \cdot n) \quad (13)$$

Let  $S_k^v(n)$ ,  $k = 0, 1, \dots, N-1$  and  $v = 0, 1, \dots, V-1$  be the spread signal of vector sequence  $a(n)$  which means:

$$s_k^v(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \hat{a}_k^v(n) \delta_{k,n}, \quad n = 0, 1, \dots, N-1 \quad (14)$$

The constant value of  $1/\sqrt{N}$  is multiplied in Eq. 14 to the phase offset to normalize the vector sequence, since CI codes may enlarge the power of data symbol.

The final transmitted signal of  $N$  information symbols is:

$$s(n) = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} \sum_{k=0}^{N-1} \hat{a}_k^v(n) \exp(j(2\pi/N) \cdot k \cdot i) \times \delta_{k,n} \quad (15)$$

where,  $\tilde{a}(n)$  is the blocked vector sequence and  $\delta_{k,n}$  is the phase offset added to the  $n$ th subcarrier for symbol  $k$ . The CI code is the same as IDFT, therefore Eq. 15 may be written as:

$$s = \text{M.IFFT}[\text{N.IFFT}(\hat{a})] \quad (16)$$

The received scalar sequence  $r(n)$  after transmitting  $s$  over ISI channel and removing the CP is converted to the  $V \times 1$  vector sequence same as in Eq. 5. After detection, the output signal of the receiver is:

$$\hat{r}_v = \sum_{i=0}^{N-1} \sum_{k=0}^{N-1} C_i \tilde{r} \exp(j(2\pi/N) \cdot k \cdot i) \times \delta_{k,n} = \text{FFT}[C \cdot \text{FFT}(\tilde{r})] \quad (17)$$

where,  $C$  is the weights based on MMSEC and is defined as:

$$C = \sqrt{\frac{H^*}{H^* \cdot H + N_0/2}} \quad (18)$$

The length of the second IFFT/FFT is  $M = 4N \times \beta$  where  $\beta$  is the oversampling factor can be described as:

$$M = \left( \underbrace{m_0, m_1, \dots, m_{N-1}}_N, \underbrace{0, 0, \dots, 0}_{2N \times (2\beta - 1)}, \underbrace{0, 0, \dots, 0}_N \right)$$

## PAPR BENEFIT AND PERFORMANCE EVALUATION

CI-VOFDM system with suboptimal modulation scheme has PAPR benefits since when the energy of one information symbol in a vector sequence is at maximum value, the energies of other information symbols are at minimum value. Due to the uniformly spread of peaks in symbol energies over the symbol time in the vector sequence, it is not possible for all symbol energies to be combined at the same time. This results to a low peak power hence low PAPR value. The PAPR can be denoted as:

$$\text{PAPR} = \frac{\frac{1}{2} \max_{0 \leq n \leq N} |s(n)|^2}{E\{|s(n)|^2\}} \quad (19)$$

The PAPR performance comparison between VOFDM, VOFDM with suboptimal modulation scheme, CI/OFDM, CI-VOFDM and suboptimal modulation scheme based CI-VOFDM systems are illustrated in Fig. 1. The modulation schemes used are the BPSK for VOFDM, CI/OFDM, CI-VOFDM systems and suboptimal modulation scheme for VOFDM and CI-VOFDM with suboptimal modulation with oversampling factor ( $\beta$ ) = 2. We assumed the number of subcarriers to be 256 and the vector size  $V = 2$ . The results shown a spectacular reduction on PAPR value of the proposed VOFDM compared to the conventional VOFDM with suboptimal

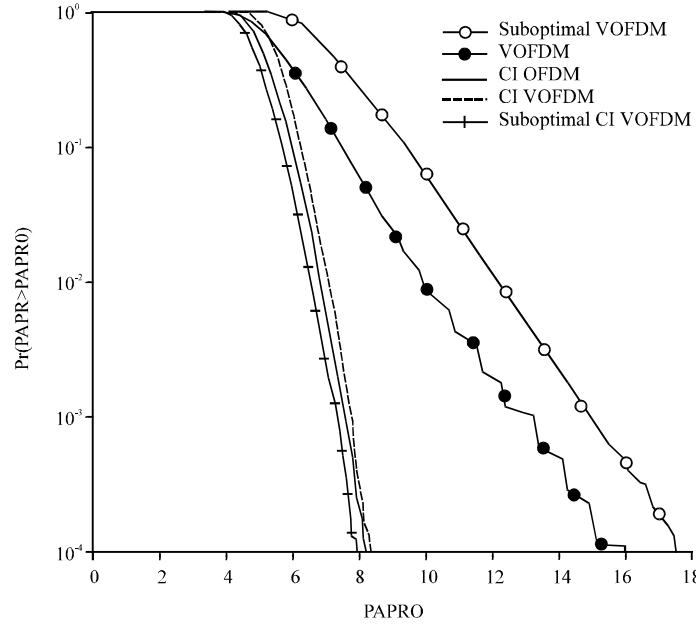


Fig. 1: PAPR performance for VOFDM, VOFDM with suboptimal modulation scheme, CI/OFDM, CI-VOFDM and suboptimal modulation scheme based CI-VOFDM systems

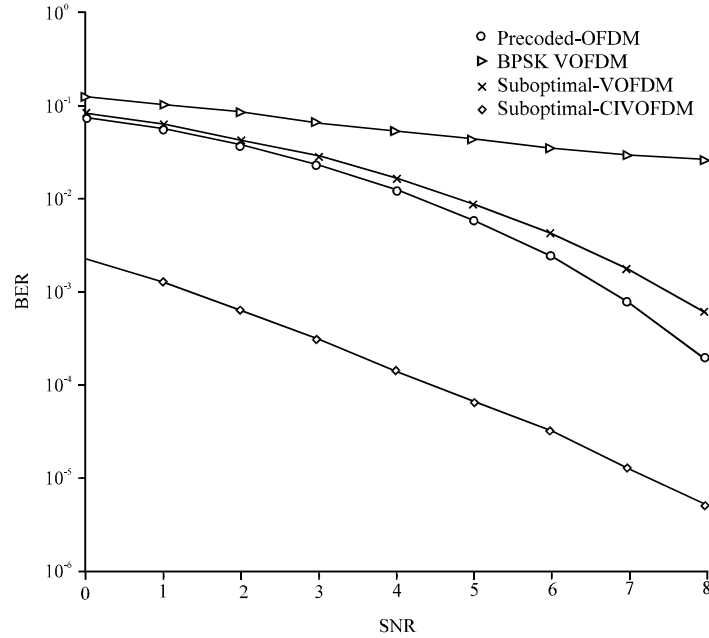


Fig. 2: BER performance on ISI channel with spectral nulls

modulation, it is even lower than CI/OFDM system. At the complementary cumulative distribution functions (CCDF) of  $10^{-4}$ , the P APR is reduced from 18-8 dB.

In Fig. 2, the BER performance of the proposed VOFDM, VOFDM with suboptimal modulation, BPSK VOFDM with  $V = 2$  and precoded-OFDM with  $V = 1$ ,  $P = 2$  over the ISI channel with channel

impulse responses  $h = [0.907, 0.915]$  and spectral nulls is demonstrated. The number of subcarriers  $N = 256$  and oversampling factor for CI-VOFDM is  $\beta = 2$ . The result shows an improvement of BER performance of the proposed VOFDM compared to BPSK VOFDM, VOFDM with suboptimal modulation and precoded-OFDM systems.

## CONCLUSIONS

The VOFDM with suboptimal modulation scheme show an improvement of BER performance on the ISI channel with spectral nulls. However, it has high PAPR compared to BPSK VOFDM. In this paper we introduced the CI technique in VOFDM with suboptimal modulation scheme for single transmit antenna system. The PAPR of the proposed VOFDM is reduced about 10dB at clip rate of  $10^{-4}$ . Also there is an improvement on BER performance of the proposed system compared to precoded-OFDM system.

Our future work will base on using higher modulation scheme (MQAM) in the proposed method.

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