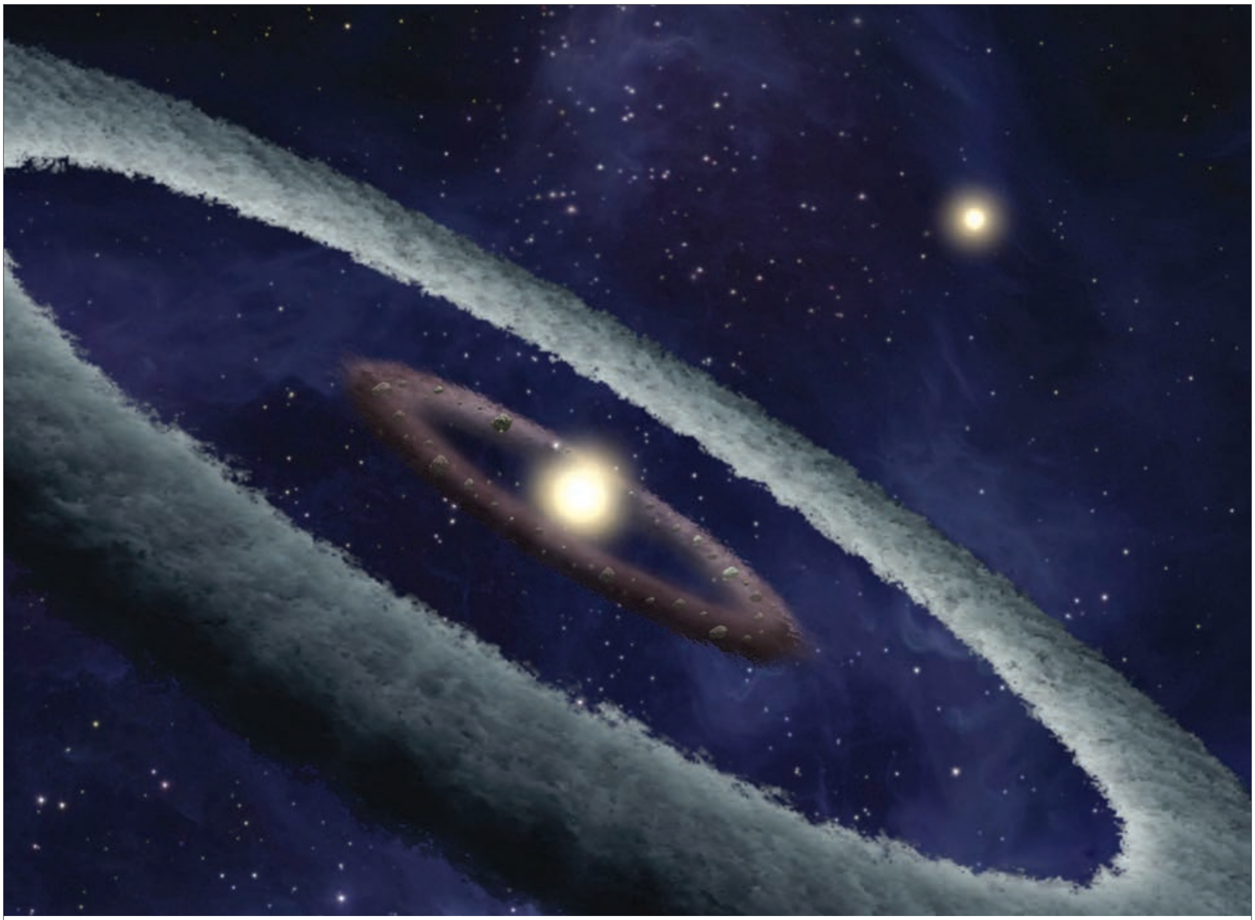

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Dynamical Analysis of Prestressed Euler-Bernouli Beam

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Abstract: The analysis of Pre-stressed Euler-Bernoulli beam traversed by uniform partially distributed moving mass is carried out. Equations of motion are solved systematically which results to the use of finite difference algorithm. Results from the numerical solution are shown graphically. It was observed that amplitude deflection of the moving mass Pre-stressed Euler-Bernoulli Beam was greater than those of the moving force. Also the moving force is not an upper bound for the accurate solution for the moving mass problem

Key words: Pre-stressed Euler-bernoulli beam, finite difference algorithm, moving mass, moving force, distributed moving mass, simply supported

INTRODUCTION

The study of dynamical systems of moving loads have attracted the attention of several researchers, notably among them are Esmailzedeh and Ghorashi (1994 and 1995), Akin and Mofid, (1989) and Gbadeyan and Oni (1995).

The analysis of moving loads finds application in several fields in Engineering, Applied Physics and Applied Mathematics. Detailed analysis of moving loads have been presented in Fryba (1971).

This study do take into consideration, beams that undergo compression when there is no external load acting on them. Such beams are called pre-stressed or initially-stressed beams and are of practical importance. For example they are commonly incorporated in the design of aeroplanes. Advances in technology have accelerated the utilization of such pre-stressed structural elements. In general an aircraft that is subjected to a wide range of temperature variations during flight which may cause considerable tensile or compressive pre-stressed in the beams when they are fixed in the airplane direction. It is, therefore of technological interest to investigate to what extent the dynamic response of the beam is affected by the moving loads.

The study presented in this study is an extension of our earlier paper Adetunde and Akinpehu (2007) which deals with Dynamical behaviour of Euler Bernoulli beam Traversed by Uniform partially distributed moving masses. In the present study we extend Adetunde and Akinpehu (2007) by trying to introduce the pre-stressed term in the governing differential equation to the beam of our consideration (pre-stressed Euler-Bernoulli beam) The analysis presented in the paper is for simply supported pre-stressed Euler-Bernoulli beam that should be easily applied to many situations and a variety of boundary conditions.

The main objectives of this study was to:

- Present a very simple technique for determining the response of simply supported pre-stressed Euler-Bernoulli beam traversed by uniform partially distributed moving masses.
- Determine the response of amplitude of the deflection of the simply supported pre-stressed Euler-Bernoulli beam.
- Determine the variation in the lateral displacement of simply supported pre-stressed Euler-Bernoulli beam.

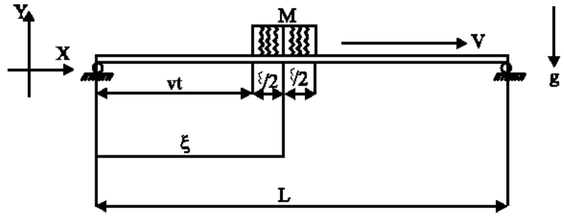

Fig. 1: The diagram of the pre-stressed Euler Bernoulli Beam with mass M

MATHEMATICAL MODEL

With reference to Fig. 1, a uniform simply supported Pre-stressed Euler Bernoulli Beam of length L is acted upon initially at time t = 0s, by mass M over fixed length ϵ of the beam with a specified constant velocity V. The load is in contact with the beam throughout the motion.

PROBLEM DEFINITION

A simply supported pre-stressed Euler-Bernoulli beam with a moving mass is shown in Fig. 1.

ASSUMPTIONS

I = Constant moment of inertia
m = Constant mass per unit length of the beam: no damping in the system.
g = Uniform gravitational field
M = Constant mass moving on the simply supported pre-stressed Euler-Bernoulli beam

Boundary/Initial Conditions

$$W = 0 \text{ at } x = 0, \frac{\partial W}{\partial x} = 0 \text{ at } x = L$$

$$m\frac{\partial^2 W}{\partial x^2} = 0 \text{ at } x = L, EI\frac{\partial^3 W}{\partial x^3} - N\frac{\partial^2 W}{\partial x^2} = 0 \text{ at } x = L$$

The corresponding initial conditions are

$$W(x,0) = \frac{\partial W}{\partial x} = 0$$

THE DIFFERENTIAL EQUATIONS

The partial differential equations governing motion of a simply supported pre-stressed Euler-Bernoulli beam under the moving mass, M, neglecting the damping, the rotary inertia and shearing force effect (based on the assumption of the constant inertia) can be written as

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 W(x,t)}{\partial x^2}\right] + m(x)\frac{\partial^2 W(x,t)}{\partial t^2} - N\frac{\partial^2 W(x,t)}{\partial x^2} = F(x,t) \quad (1)$$

Here E, is the Young modulus of the Elasticity, F(x,t) is the load inertia, t is the time, W is the deflection of the beam, x is the spatial co-ordinate and N is the pre-stressed constant.

The load inertia takes the form described below

$$F(x,t)=\frac{1}{\varepsilon}\left[-Mg-M\nabla^2 W(x,t)\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right] \tag{2}$$

Here

$$H(x)=\begin{cases}0 & x<0\\ 1 & x>0\end{cases} \tag{3}$$

$$\nabla^2 W(x,t)=\frac{\partial^2 W(x,t)}{\partial t^2}+2V\frac{\partial^2 W(x,t)}{\partial x\partial t}+V^2\frac{\partial^2 W(x,t)}{\partial x^2}$$

Where,

V = Constant velocity of the load as defined earlier.
H = Heaviside unit function
ε = Fixed length of the beam
ξ = Length of the load (vt + ε/2)

For a particular distance along the length of the beam, as it has been defined above we employ Dirac Delta function.

$$\delta(x-x_o)=\frac{d}{dx}\left[H(x-x_o)\right] \text{ or } H(x-x_o)=\int_0^{x_o}\delta(x-x_o)dx$$

There three main properties of the Dirac Delta function employed are

$$\delta(t-a)=0 \quad t\neq a \tag{5a}$$

$$\int_{a-\delta}^{a+\delta}\delta(t-a)dt=1 \quad \varepsilon>0 \tag{5b}$$

$$\int_{-\infty}^{\infty}f(t)\delta(t-a)dt=f(a) \tag{5c}$$

In other words

$$\delta(x-x_o)dx=1$$

$$f(x_o)=\int_0^L\delta(x-x_o)dx=f(x_o)\int_0^L\delta(x-x_o) \quad o<x_o, x_o>L$$

OPERATIONAL SIMPLIFICATION OF THE GOVERNING EQUATION

By substituting Eq. 4 into Eq. 2 we have:

$$F(x,t)=\frac{1}{\varepsilon}\left[-Mg-M\left[\frac{\partial^2 W(x,t)}{\partial t^2}+2V\frac{\partial^2 W(x,t)}{\partial x\partial t}+V^2\frac{\partial^2 W(x,t)}{\partial x^2}\right]\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right] \tag{6}$$

Hence putting Eq. 6 into Eq. 1, we now have the governing equations describing the behaviour of a uniform Pre-stressed Euler-Bernoulli Beam written as

$$\begin{aligned}&\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 W(x,t)}{\partial x^2}\right]+m\frac{\partial^2 W(x,t)}{\partial t^2}-N\frac{\partial^2 W(x,t)}{\partial x^2}=\\ &\frac{1}{\varepsilon}\left[-Mg-M\left[\frac{\partial^2 W(x,t)}{\partial t^2}+2V\frac{\partial^2 W(x,t)}{\partial x\partial t}V^2\frac{\partial^2 W(x,t)}{\partial x^2}\right]\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]\end{aligned} \tag{7}$$

Assuming a solution, in the form of a variable separable for the traversed vibration of the beam.

$$W(x,t)=\sum_{i=1}^{\infty}\phi_i(x)Y_i(t) \tag{8}$$

Where, $\phi_i(x)$'s are the normalized deflection curve for the i^{th} mode of the vibrating uniform Pre-stressed Euler-Bernoulli Beam and $Y_i(t)$'s are the unknown functions of time to be determined.

Now substituting Eq. 8 into Eq. 7 we have:

$$\begin{aligned}&\frac{\partial^2}{\partial x^2}\left[EI\sum_{i=1}^{\infty}\phi_i''(x)\right]Y_i(t)+m\sum_{i=1}^{\infty}\phi_i(x)\ddot{Y}_i(t)-N\sum_{i=1}^{\infty}Y_i(t)\phi_i''(x)=\\ &\frac{1}{\varepsilon}\left[-Mg-M\left[\sum_{i=1}^{\infty}\phi_i(x)\ddot{Y}_i(t)+2v\sum_{i=1}^{\infty}\phi_i'(x)\dot{Y}_i(t)+v^2\sum_{i=1}^{\infty}Y_i(t)\phi_i''(x)\right]\right]\\ &\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]\end{aligned} \tag{9}$$

Multiplying both sides of Eq. 9 by $\phi_n(x)$ and integrating along the length of the bean we have:

$$\begin{aligned}&\int_0^L\phi_n(x)\left\{\frac{\partial^2}{\partial x^2}\left[EI\sum_{i=1}^{\infty}\phi_i''(x)\right]Y_i(t)+m\phi_i(x)\ddot{Y}_i(t)-N\sum_{i=1}^{\infty}Y_i(t)\phi_i''(x)\right\}=\\ &\int_0^L\phi_n(x)\left\{\frac{1}{\varepsilon}\left[-Mg-M\left[\begin{matrix}\sum_{i=1}^{\infty}\phi_i(x)\ddot{Y}_i(t)+2V\sum_{i=1}^{\infty}\phi_i'(x)\dot{Y}_i(t)\\ +V^2\sum_{i=1}^{\infty}Y_i(t)\phi_i''(x)\end{matrix}\right]\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]\right\}\end{aligned}$$

$$\Rightarrow \int_0^L \phi_n(x)\left[\frac{\partial^2}{\partial x^2} EI\left(\sum_{i=1}^{\infty} \phi_i''(x) Y_i(t)\right)\right] dx + M\sum_{i=1}^{\infty} \ddot{Y}_i(t) \int_0^L \phi_n(x)\phi_i(x)dx - N\sum_{i=1}^{\infty} Y_i(t)$$

$$\int_0^L \phi_n(x)\phi_i''(x)dx = \frac{1}{\varepsilon}\left\{\begin{matrix} -Mg\int_0^L \phi_n(x)dx - m\sum_{i=1}^{\infty}\ddot{Y}_i(t)\int_0^L \phi_n(x)\phi_i(x)dx - 2VM \\ \sum_0^L \dot{Y}_i(t)\int_0^L \phi_n(x)\phi_i'(x)dx - V^2M\sum_0^L Y_i(t)\int_0^L \phi_n(x)\phi_i''(x)dx \end{matrix}\right\}* \quad (10)$$

$$\left[H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]$$

By considering the right hand side of Eq. 10, making the following denotations

$$A = -\frac{Mg}{\varepsilon}\int_0^L \phi_n(x)\left[H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]dx \quad (10a)$$

$$B = -\frac{M}{\varepsilon}\int_0^L \phi_n(x)\phi_i(x)\left[H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]dx \quad (10b)$$

$$C = \frac{-2VM}{\varepsilon}\int_0^L \phi_n(x)\phi_i'(x)\left[H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]dx \quad (10c)$$

$$D = -\frac{V^2M}{\varepsilon}\int_0^L \phi_n(x)\phi_i''(x)\left[H\left(x-\xi+\frac{\varepsilon}{2}\right) - H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]dx \quad (10d)$$

By solving Eq. 10a-d, we implore integration by parts and Heaviside unit function. We have Eq. 10a-d becoming:

$$A = \frac{Mg}{\varepsilon}\int_0^L \left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right) - \phi_n\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi \quad (10a)*$$

$$B = \frac{M}{\varepsilon}\int_0^L \left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i\left(\xi-\frac{\varepsilon}{2}\right) - \phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi_i\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi \quad (10b)*$$

$$C = \frac{2VM}{\varepsilon}\int_0^L \left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i'\left(\xi-\frac{\varepsilon}{2}\right) - \phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi_i'\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi \quad (10c)*$$

$$D = \frac{V^2M}{\varepsilon}\int_0^L \left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i''\left(\xi-\frac{\varepsilon}{2}\right) - \phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi''\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi \quad (10d)*$$

Substituting Eq. 10a*-d* into the right hand side of Eq. 9 we have:

$$\int_0^L \phi_n(x)F(x,t)dx = \frac{Mg}{\varepsilon}\int_0^L\left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)-\phi\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi +$$
$$\frac{M}{\varepsilon}\sum_{i=1}^{\infty}\ddot{Y}_i(t)\int_0^L\left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i\left(\xi-\frac{\varepsilon}{2}\right)-\phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi_i\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi$$
$$+\frac{2MV}{\varepsilon}\sum_{i=1}^{\infty}\dot{Y}_i(t)\int_0^L\left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i'\left(\xi-\frac{\varepsilon}{2}\right)-\phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi_i'\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi +$$
$$\frac{V^2M}{\varepsilon}\sum_{i=1}^{\infty}Y_i(t)\int_0^L\left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i''\left(\xi-\frac{\varepsilon}{2}\right)-\phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi_i''\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi$$

Now considering the left hand side of Eq. 9, using orthogonality condition, we have:

$$w^2mY_i(t)\int_0^L\phi_n^2(x)dx + m\ddot{Y}_i(t) - N\sum_{i=1}^{\infty}Y_i(t)\int_0^L\phi_n(x)\phi_n''(x)dx \quad \text{for } i = n \tag{12}$$

Where $w^2 = \frac{d^2}{dx^2}\left[EI\sum_{i=1}^{\infty}\phi_i''(x)\right]$

Combining Eq. 12 and 11, that is LHS = RHS, we have:

$$w^2mY_i(t)\int_0^L\phi_n^2(x)dx + m\ddot{Y}_i(t) - N\sum_{i=1}^{\infty}Y_i(t)\int_0^L\phi_n(x)\phi_n''(x)dx =$$
$$\frac{Mg}{\varepsilon}\int_0^L\left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)-\phi_n\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi$$
$$+\frac{M}{\varepsilon}\sum_{i=1}^{\infty}\ddot{Y}_i(t)\int_0^L\left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i\left(\xi-\frac{\varepsilon}{2}\right)-\phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi_i\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi + \tag{13}$$
$$\frac{2MV}{\varepsilon}\sum_{i=1}^{\infty}\dot{Y}_i(t)\int_0^L\left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i'\left(\xi-\frac{\varepsilon}{2}\right)-\phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi_i'\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi +$$
$$\frac{MV^2}{\varepsilon}\sum_{i=1}^{\infty}Y_i(t)\int_0^L\left[\phi_n\left(\xi-\frac{\varepsilon}{2}\right)\phi_i''\left(\xi-\frac{\varepsilon}{2}\right)-\phi_n\left(\xi+\frac{\varepsilon}{2}\right)\phi_i''\left(\xi+\frac{\varepsilon}{2}\right)\right]d\xi$$

for i = n

SIMPLY SUPPORTED BEAM

For the illustration of the result of the foregoing analysis, we consider a simply supported configuration beam in which

$$\phi_n(x) = \sqrt{2/L}\,\sin(n\pi x/L) \quad n = 1, 2, 3, \ldots \tag{14}$$

We obtain the set of exact governing differential equation for the vibration of the beam by deriving exact governing equations by employing Eq. 14 and evaluating the exact values of the integral in Eq. 9, we finally obtain

$$\ddot{Y}_n(t)+\omega_n^2 Y_n(t)+N\left(\frac{n\pi}{L}\right)^2 Y_n(t)=\frac{M}{m}\left[-\frac{g}{n\pi\xi}\sqrt{8h}\,\mathrm{Sin}\frac{i\pi\xi}{2L}\mathrm{Sin}\frac{n\pi\xi}{2L}-\frac{2}{\in\pi}\sum_{i=1}^{\infty}\ddot{Y}_j(t)\right.$$
$$\left[\frac{1}{(i-n)}\mathrm{Cos}\frac{\pi\xi}{L}(i-n)\mathrm{Sin}\frac{\pi\in}{2L}(i-n)\right]+\frac{2}{\in\pi}\sum_{i=1}^{\infty}\ddot{Y}_i(t)\left[\frac{1}{(i+n)}\mathrm{Cos}\frac{\pi\xi}{L}(i+n)\mathrm{Sin}\frac{\pi\in}{2L}(i+n)\right]-$$
$$\frac{2V}{\in}\sum_{i=1}^{\infty}\dot{Y}_i(t)\left[\sqrt{\frac{2}{L}}\left(\frac{1}{i+n}\right)\mathrm{Sin}\frac{\pi\in}{2L}(i+n)\mathrm{Sin}\frac{\pi\xi}{L}(i+n)\right]-\frac{2V}{\in}\sum_{i=1}^{\infty}\dot{Y}_i(t)$$
$$\left[\sqrt{\frac{2}{L}}\left(\frac{1}{i-n}\right)\mathrm{Sin}\frac{\pi\in}{2L}(i-n)\mathrm{Sin}\frac{\pi\xi}{L}(i-n)\right] \qquad (14)$$
$$-\frac{V^2}{\in}\left(\frac{i\pi}{L}\right)\sum_{i=1}^{\infty}Y_i(t)\left[\sqrt{\frac{2}{L}}\left(\frac{1}{i-n}\right)\mathrm{Sin}\frac{\pi\in}{2L}(i-n)\mathrm{Sin}\frac{\pi\xi}{L}(i-n)\right]+$$
$$\left.\frac{V^2}{\in}\left(\frac{i\pi}{L}\right)\sum_{i=1}^{\infty}Y_i(t)\left[\sqrt{\frac{2}{L}}\left(\frac{1}{i+n}\right)\mathrm{Sin}\frac{\pi\in}{2L}(i+n)\mathrm{Sin}\frac{\pi\xi}{L}(i+n)\right]\right]$$

The Eq. 14 is the exact governing Equation of a simply supported Pre-stressed Euler-Bernoulli Beam.

For the case of i = n the expression involved should be replaced by $\pi\varepsilon/2L$

NUMERICAL CALCULATIONS AND DISCUSSION

We now use the finite difference method to solve the above Eq. (14) numerically which now leads to a system of equations and Visnal Basic programming language was employed. In order to illustrate the results, the numerical data (Esmailzadeth and Gorashi, 1995, 1994; Akin and Mofid, 1989; Adetunde, 2003; Akinpelu, 2003) were used for the purpose of comparisons. $E = 2.07 * 10^{11}$ Nm^{-2}, $I = 1.04 * 10^{-6}$ m^4, $V = 12$ km h^{-1}, m = 70 kg, g = 9.8 m s^{-1}, M = 10, 15 and 20 kg, t = 0.5, 1.0, 1.5 s, L = 10, $\in$ = 0.1 and 1.0 m, N = 0.5, 0.8, 1.0. The results are shown on the various graphs in Fig. 2-6. Figure 2 and 3 shows the displacement

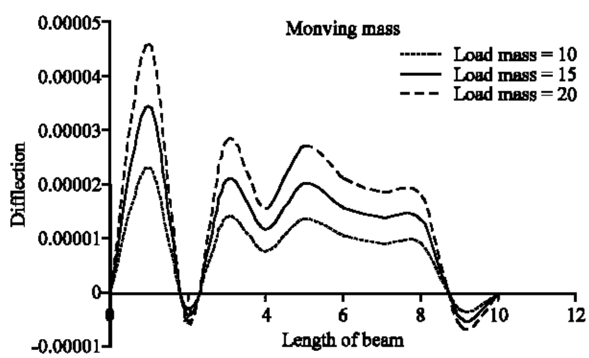

Fig. 2: Displacement response of moving mass for simply supported Euler-Bernoulli Beam for various values of M

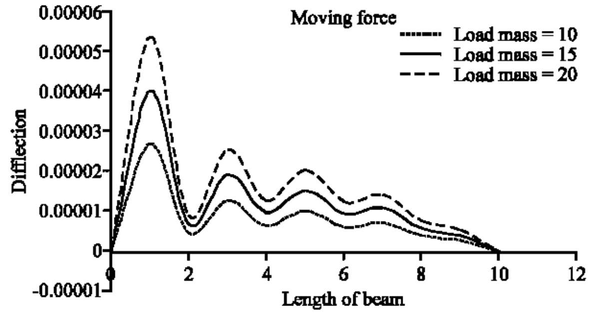

Fig. 3: Displacement response of moving force for simply supported Euler-Bernoulli Beam for various values of M

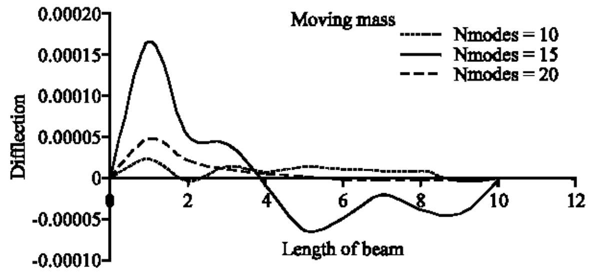

Fig. 4: Displacement response of moving mass for simply supported pre-stressed beam for various values of pre-stressed term N

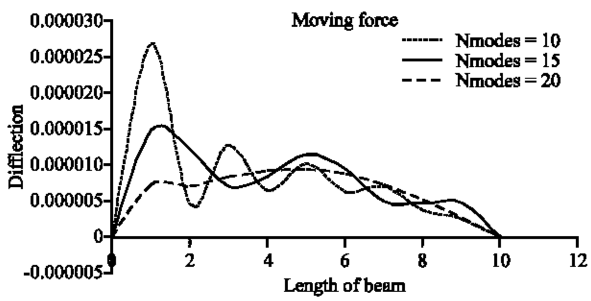

Fig. 5: Displacement response of moving force for simply supported pre-stressed beam for various values of pre-stressed term N

response of the simply supported pre-stressed Euler-Bernoulli Beam for both cases of moving force and moving mass for different values of M. Clearly, the results show that the response amplitude due to the moving mass is greater than that due to the moving force. Consequently, the moving force is not an upper bound for the accurate solution for the moving mass problem. The deflection profile for various values of Pre-stressed term N for both cases of a moving force and moving mass problems of the Pre-stressed uniform beam are displayed in Fig. 4 and 5. It was observed that as the Pre-stressed term increases, the transverse displacement of the beam decreases. Figure 6 shows the comparisons

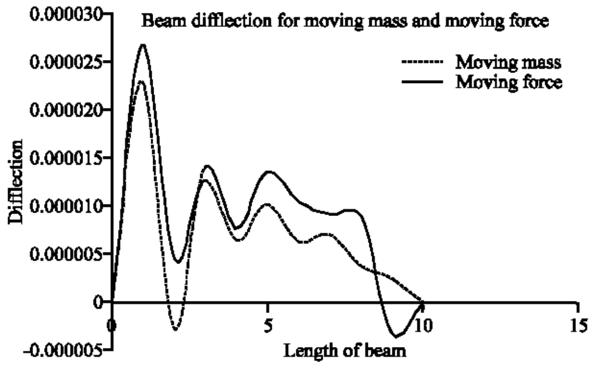

Fig. 6: Comparison of the deflection of moving force and moving mass cases for simply supported pre-stressed beam

between moving force and moving mass for fixed values of m, N and M. It is seen that the response amplitude of the moving mass is greater than the response amplitude of the moving force.

CONCLUSION

The problem of investigating the dynamical analysis of Pre-stressed Euler-Bernoulli Beam traversed by uniform partially distributed moving load is studied. Analytical numerical technique is used to solve the pertinent initial-boundary value problem. It was observed that:

Amplitude deflection of the moving mass Pre-stressed Euler Bernoulli Beam was greater than those of the moving force.

The moving force is not an upper bound for the accurate solution for the moving mass problem.

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