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Analysis of the Heavy Nuclei Levels with the Newest Data

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Abstract: We have developed a special computing code for calculation of nuclear quadrupole moments versus deformation parameter δ . The calculated results for some heavy nuclei are compared with the 2001 experimental data which are the most new ones from Nilsson level diagrams we found new level energies for each nucleus by using new δ parameter, which would be useful for other studies that use Nilsson model and its diagrams. For some Isotopes, it has been seen that by increasing neutron number, deformation parameter also increase, which means more deformation from spherical shape.

Key words: Nuclear quadrupole moment, deformation parameter, Nilsson model, electric field gradient

INTRODUCTION

We know that nuclei in many cases have large quadrupole moments and they don't behave like a point charge, rather a spherical or elliptical shape with an axis of symmetry is considered for these nuclei. There are different methods for measuring nuclear electric quadrupole moments. Basically, the experimental techniques used to measure<Q>are based on the relationship of interaction energy E₂ of the quadrupole moment with the Electric Field Gradient (EFG) (Shalet and Feshbach, 1990),

$$E_2 = (W)_Q = \frac{e}{4} \langle Q \rangle_{exp} \frac{\partial^2 \phi}{\partial 2^2}$$
 (1)

where, EFG is assumed to have a cylindrical symmetry.

There are two fundamental ways to create large enough and symmetrical EFG to measure E₂. The first one is based on the electric charge provided by electrons around the nucleus, and in the other one, the atoms are embedded in crystalline fields, and then the surrounding fields may polarize electric charge density around the atoms to create a symmetrical EFG. The latter case is however, not easy because of chemical and physical difficulties in growing crystals with different atoms embedded in them. But for a few cases, the reliable values of EFG are available.

Experimental methods of measuring (W)_Q of the ground states are based on the following techniques: (1) the modification of nuclear magnetic resonance, NMR (Baumann *et al.*, 1998; Cederberg and Olson, 1998), (2) the nuclear quadrupole resonance, NQR (Pendrill *et al.*, 1992; Otsuka *et al.*, 1993) and (3) Mossbauer or perturbed angular correlation techniques (Nörtershäuser *et al.*, 1998; Brown and Wildenthal, 1987). By knowing the quadrupole moments, we can measure deformation parameters which can be used to define the shape of nuclei.

Nuclear Electric Quadrupole Moment

Some nuclei have permanent quadrupole moments that can be measured experimentally. It is expected that these nuclei have elliptical shape with a symmetrical axis. With this assumption, we define the intrinsic quadrupole moment Q' classically as:

$$Q' = \int (3z'^2 - r'^2)\rho' dz'$$
 (2)

To express the above equation quantum mechanically, we write operator q' op as:

$$q'_{op} = (3z'^2 - r'^2) = \frac{16\pi}{5}r'^2Y_{20}(\theta, \phi')$$
 (3)

and write the expectation value of Q' in terms of this operator:

$$< Q'> = \sum_{\alpha c c} p_i < \psi_{\alpha \Omega} |q'_{\alpha p}| \psi_{\alpha \Omega} >$$
 (4)

where, $p_i = 1$ or 0 for proton and neutron, respectively. For calculation of intrinsic quadrupole moments, proton eigenstates $\psi_{a,\Omega}$, of an elliptical nucleus are necessary. Here, we have used $\psi_{a,\Omega}$'s eigenstates that are introduced in Nilsson model for calculation of quadrupole moment versus deformation parameter δ which is a measure of departure from spherical shape.

The average shell model potential, for such non-spherical nuclei, was first written by (Nilsson, 1955), and has the shape of an oscillator potential with axial symmetry, and a strong spin-orbit coupling. The Hamiltonian of this model is

$$H = H_0 + H_5 + cl.S + Dl^2$$
 (5)

In this Hamiltonian, the energy levels of a single particle are given versus deformation parameter δ . Evidently new quantum's numbers N, l, Λ and Σ are involved (Nilsson, 1955). The quantum number N corresponds to the total numbers of oscillator quanta, while l, Λ and Σ are the quantum numbers, corresponding to operators l, l_z and S_z , respectively. The Nilsson's wavefunction $\psi\alpha$ which should be eigenfunction of new H, with eigenvalues $E\alpha$, can be written as $H\psi_\alpha = E_\alpha \ \psi_\alpha$ where ψ_α was written in terms of the expansion coefficients such that:

$$\psi_{\alpha\Omega} = \sum_{l\Lambda} a_{l\Lambda} | Nl\Lambda \Sigma \rangle \tag{6}$$

where, α represents any quantum numbers and $\Omega = \Lambda + \Sigma$. We have developed a special computing code for calculation of nuclear quadrupole moments versus deformation parameter δ .

RESULTS

We have compared deformation parameters of some nuclei in Table 1 to check the accuracy of a new programming code. Then we used it to compare the deformation parameters of different isotopes of Gd and Er nuclei in Table 2 and 3. As it can be seen from these Table 1-3, by increasing neutron number, deformation parameter also increase for more

Table 1: Calculation of deformation parameter of or old and new nuclear electric quadrupole moments

Nucleus	Old quadrupole	Calculated δ	New quadrupole (Stone, 2001)	Calculated δ
²³⁵ U	4.55	0.195	4.94	0.204
^{233}U	3.66	0.201	3.77	0.272
143Nd	0.48	0.176	0.61	0.225
155Gd	0.44	0.134	1.27	0.361
¹⁶⁷ Er	2.83	0.311	3.57	0.438
¹⁷⁷ Hf	3.79	0.413	3.93	0.521
189Os	0.63	0.078	0.86	0.181

Table 2: Calculation of deformation parameter δ for new quadrupole moments of Gd isotopes

õ	New quadrupole	Isotopes ($Z = 64$)
0.225	1.46	144 Gd
0.253	1.82	154 Gd
0.338	1.93	156 Gd
0.361	2.01	158 Gd

Table 3: Calculation of deformation parameter ô for new quadrupole moments of Er isotopes

ð	New quadrupole	Isotopes $(Z = 68)$
0.411	2.41	164 Er
0.423	2.71	165 Er
0.438	3.57	¹⁶⁷ Er
0.443	2.86	171 Er

heavier isotopes which means more deformation from spherical shape. From Nilsson level diagrams we found new level energies for each nucleus by using new δparameter which supports these deformations.

DISCUSSION

We have studied deformations of ²³⁵U, ²³³U, ¹⁴³Nd, ¹⁵⁵Gd, ¹⁶⁷Er, ¹⁷⁷Hf, ¹⁸⁹Os nuclei, where eigenfunctions of Nilsson model are used in all numerical calculations. Deformation parameters (δ) are chosen such that to deduce the best agreement between theoretical and experimental quadrupole moments. As we have illustrated in table1 for some nuclei, there are considerable differences between old (Raghavan, 1989) and new values (Stone, 2001) of experimental quadrupole moments. As it can be seen, the new quadrupole moments are higher, therefore, differences between calculated δ parameters is considerable. In Table 2 and 3, deformation parameters for some of the gadolinium (Gd) and erbium (Er) isotopes are shown which show more deformations as neutron numbers increase. Because we had no old values of experimental quadrupole moments for these isotopes, only the new ones are used. Also, we could not find any mentioned values for deformation parameters. Usually, these deformation parameters could be found from Nillson diagrams by some error, but here we gave an exact calculated value.

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