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One Transducer Method for Quantifying Vibration Power Flow in Orthotropic Plates for Both General and Far-field Conditions

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Abstract: Frequency response function technique is used to quantify a vibration field in orthotropic plates considering both general-field and far-field conditions. If vibration field is stationary, one transducer technique using Frequency Response Function (FRF) is an alternative to conventional structural intensity measurements. It is free from finite difference and phase mismatch errors that occur during data acquisition. In this study, one-transducer technique using FRF is employed to formulate a structural intensity field in the frequency domain without the usage of finite difference approximation. As a result, one transducer may be employed sequentially to quantify the vibration fields.

Key words: Structural intensity, flexural wave, one-transducer FRF technique, orthotropy

Introduction

It is a good practice to subside vibration at the sources. It may also be possible to control its propagation along structures before radiating, in the form of noise, into the surroundings. Vibration Power Flow (VPF) or Structural Intensity (SI) is a good tool to use to solve Noise and Vibration Harness (NVH) problems in industries. In the case of SI, measurements can be employed on various points on the structures so as to produce an intensity map that gives a path of vibration propagation. Proper damping treatment can be introduced only to the area of vibration propagation. Therefore, SI gives important information for controlling noise and vibration in the structures. This is why the method is widely used in industrial applications.

SI technique has a unique aspect. It does not depend on boundary conditions of the structures. This enables SI to investigate the edge effects of vibration power transmission of pipes, plates and beams. Most of the earlier methods using SI are useful for simple structures typically beams and thin plates in flexure (Noiseux, 1970; Pavic, 1976; Verheij, 1980; Linjama and Lathi, 1992; Bauman, 1994; Arruda and Campos, 1996). In far field measurement two transducers are necessary (Noiseux, 1970) but in the general field condition, eight transducers are useful (Pavic, 1976). Some of these formulations are in time domain (Pavic, 1976) and others are in frequency domain (Verheij, 1980; Linjama and Lathi, 1992). Other than this contact method, non-contact such as acoustic holography (Williams *et al.*, 1985a; Maynard *et al.*, 1985; Williams *et al.*, 1985b; Romano and Williams, 1993) and optical measurements (Linjama, 1992; Pascal *et al.*, 1993; McDevitt *et al.*, 1993; Berthelot *et al.*, 1993) are also available. Numerical analysis using a finite element approach is a good alternative (Gavric and Pavic, 1993; Hambric and Taylor, 1994). Recently, SI is employed in more complex structures such as in orthotropic plates in far-field conditions (Mandal *et al.*, 2002) and in general field conditions (Mandal *et al.*, 2003) using flexural waves considering either the two-transducer or eight-transducer

method, respectively. It requires simultaneous acquisition of all field signals at a same time and ensemble averaging should be performed. Proper attention should be provided in instrumentation so as to make errors in measurements minimum.

From the literature search, it is identified that there is no literature relating to SI uses FRF method for orthotropic plates. As these types of structures (corrugated plates, beam stiffened plates, beam grid structures) are very useful in industries, any SI method applicable to orthotropic plates free from possible error sources (finite difference and random) would be an important solution in the area of noise and vibration control.

The objective this study is to modify the SI equations both in far-field (Mandal *et al.*, 2002) and general field (Mandal *et al.*, 2003) conditions using one transducer technique. As a result, a transducer can be used sequentially on every measurement point of structures when the vibration field is stationary. Consequently, it provides a SI free from phase mismatch error due to phase mismatch of transducers by virtue of small transducer spacing and finite difference error due to large transducer spacing. This formulation is thus an extension of previous methods (Mandal *et al.*, 2002; Mandal *et al.*, 2003).

Frequency Response Function (FRF) Method

The measurement methods (Mandal *et al.*, 2002, 2003) are applicable for thin orthotropic plates. The idea of thin plate results when the thickness of the plate (Fig. 1), h , is small compared to other dimensions. In thin plate flexural wave equation, the influence of rotary inertia and shear deformation is neglected. This approximation is valid when $h \ll \lambda$, the flexural wave length (Cremer and Heckl, 1988). In this following section, one-transducer FRF technique is employed to modify first the general field SI (Mandal *et al.*, 2003) and then for far-field SI (Mandal *et al.*, 2002). For general condition vibration power in x-direction of the plate (Fig. 1) for frequency domain using an eight-point transducer array (Fig. 2) may be obtained as:

$$P_{gx}(f) = \frac{1}{2\omega^3 d^3} [\text{Im}\{ (8D_x + 4D_x v_y + 4H)G_{36} - 2D_x G_{38} + 2D_x G_{61} + (H - D_x v_y)G_{32} + (H - D_x v_y)G_{34} - (H + D_x v_y)G_{35} - (H + D_x v_y)G_{37} + (H + D_x v_y)G_{62} + (H + D_x v_y)G_{64} - (H - D_x v_y)G_{65} - (H - D_x v_y)G_{67} + D_{xy}(G_{25} - G_{27} - G_{45} + G_{47}) \}]. \quad (1)$$

Where:

$P_x(f)$ = General field power in x-direction,

D_x = Flexural rigidity in x-direction,

G = Cross-spectra of acceleration signals in different transducer array (Fig. 2),

v_y = Poisson's ratio,

H = Effective torsional rigidity of the orthotropic plate (Troitsky, 1976),

ω = Circular frequency,

Im = Imaginary part of complex field signals and d = transducer spacing (Fig. 1 and 3).

The y-component intensity can be obtained by interchanging x and y co-ordinates of Eq. 1.

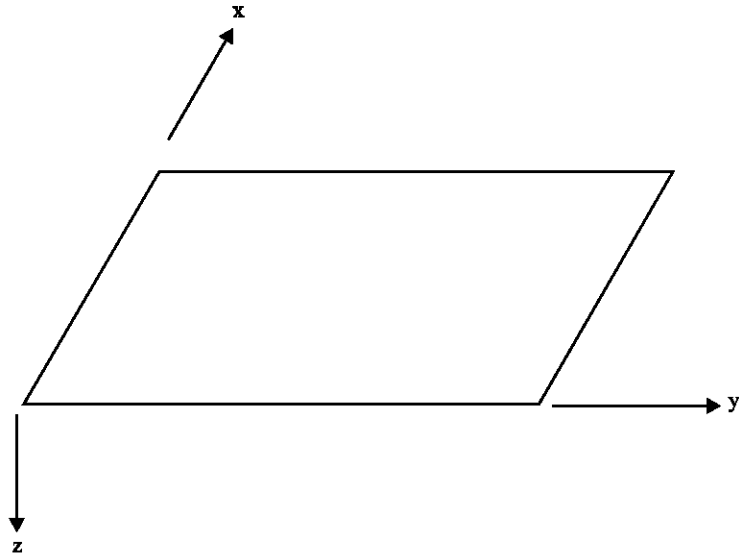


Fig. 1: Co-ordinate system of naturally orthotropic plate

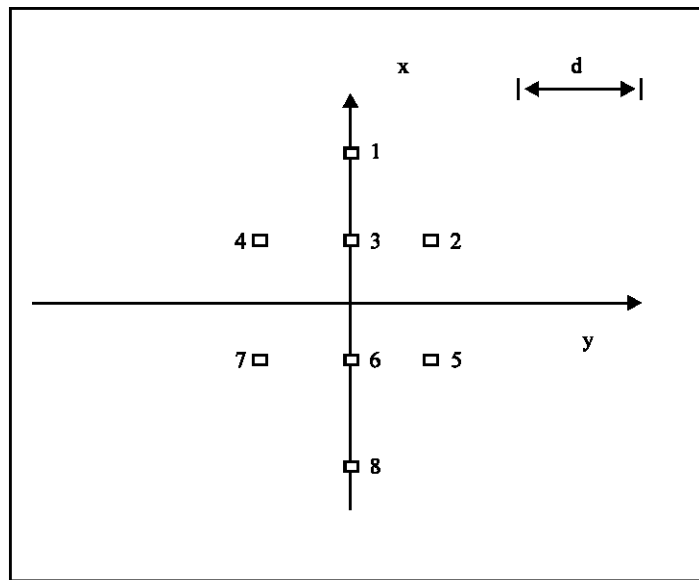


Fig. 2: 8-point measurement array required for the spatial derivative in the intensity equation

Far-field intensity in x-direction, on the other hand, is very simple. It requires only one cross spectrum of acceleration signal as

$$I_x(f) = \frac{2\sqrt{D_x m^2}}{d \omega^2} \text{Im}\{G_{12}\} \quad (2)$$

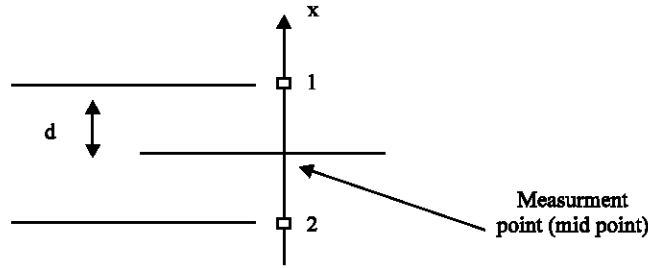


Fig. 3: Two-point transducer array for measurement of vibration power

Where, m'' is mass per unit area of the plate.

The assumptions and limitations are discussed briefly earlier in this section including thin plate and bending wavelength issues. In the case of far-field power (Eq. 2), the flexural rigidity in x-direction of the plate (D) is higher than that in y-direction. This assumption was carried out when the dimensionless parameters (Troitsky, 1976) were introduced to estimate far-field power. This assumption carries an important aspect in the case when the Eq. 2 may be applied for corrugated plates, where the rigidity in one direction is much greater than that in other orthogonal direction. It requires that the direction of corrugation would be parallel to x-direction.

FRF Method for General Field Conditions

It is necessary to get simultaneous acquisition of all cross-spectra of acceleration field signals of Eq. 1 for an intensity vector at a point on the structure to a particular direction. Consequently, an eight channel FFT analyser or more is required. If one-transducer FRF method be used, it is still able to obtain all cross-spectra by conventional two-channel FFT analyser. The first cross-spectrum of the intensity equation in Eq. 1 is G . Two accelerometers, one at point 3 and another one at point 6 (Fig. 2) are necessary. An estimate of this cross-spectrum G can be obtained with this FRF method (Bandat and Piersol, 1986) using one accelerometer as,

$$\tilde{G}_{36} = Z_{F3}^* Z_{F6} G_{FF} \quad (3)$$

Where, $\tilde{G}_{36}$ is the estimate of true G , Z and Z are the frequency response functions of a force (reference) signal to accelerations at points 3 and 6, Z is the auto spectrum of force signal and asterisk represents complex conjugate of respective FRF.

It is not possible to measure exciting forces in some practical situations. In such a situation, it is still possible to estimate the cross-spectra of the Eq. 1, using acceleration signal at any arbitrary point as reference. Using this idea, the same estimate of cross-spectrum in Eq. 3 can be obtained as:

$$\tilde{G}_{36} = Z_{a3}^* Z_{a6} G_{aa} \quad (4)$$

Here FRF is between reference signal of acceleration at any position and acceleration to the position 3 and 6 and G is auto-spectrum of acceleration of that point. If phase error is not at all a problem in the measurement, it is possible to make a reference acceleration signal at first measuring point 3, the first suffix point of the cross-spectrum, G , the Eq. 4 takes simpler form as:

$$\tilde{G}_{36} = Z_{36} G_{33} \quad (5)$$

Using the ideas presented in Eq. (3-5), the SI Eq. 1 can be formulated in three different forms (Eq. 6 to 8) incorporating FRF functions as

$$P_{gx}(f) = \frac{1}{2\omega^3 d^3} \text{Im}[\{(8D_x + 4D_x v_y + 4H)Z_{F3}^* Z_{F6} - 2D_x Z_{F3}^* Z_{F8} + 2D_x Z_{F6}^* Z_{F1} + (H - D_x v_y)Z_{F3}^* Z_{F2} + (H - D_x v_y)Z_{F3}^* Z_{F4} - (H + D_x v_y)Z_{F3}^* Z_{F5} - (H + D_x v_y)Z_{F3}^* Z_{F7} + (H + D_x v_y)Z_{F6}^* Z_{F2} + (H + D_x v_y)Z_{F6}^* Z_{F4} - (H - D_x v_y)Z_{F6}^* Z_{F5} - (H - D_x v_y)Z_{F6}^* Z_{F7} + D_{xy}(Z_{F2}^* Z_{F5} - Z_{F2}^* Z_{F7} - Z_{F4}^* Z_{F5} + Z_{F4}^* Z_{F7})\}G_{FF}] \quad (6)$$

Above equation is the modified form of Eq. 1 using auto-spectrum of force signal and making force signal as reference. If acceleration is considered as reference, Eq. 6 may stand for another form as

$$P_{gx}(f) = \frac{1}{2\omega^3 d^3} \text{Im}[\{(8D_x + 4D_x v_y + 4H)Z_{a3}^* Z_{a6} - 2D_x Z_{a3}^* Z_{a8} + 2D_x Z_{a6}^* Z_{a1} + (H - D_x v_y)Z_{a3}^* Z_{a2} + (H - D_x v_y)Z_{a3}^* Z_{a4} - (H + D_x v_y)Z_{a3}^* Z_{a5} - (H + D_x v_y)Z_{a3}^* Z_{a7} + (H + D_x v_y)Z_{a6}^* Z_{a2} + (H + D_x v_y)Z_{a6}^* Z_{a4} - (H - D_x v_y)Z_{a6}^* Z_{a5} - (H - D_x v_y)Z_{a6}^* Z_{a7} + D_{xy}(Z_{a2}^* Z_{a5} - Z_{a2}^* Z_{a7} - Z_{a4}^* Z_{a5} + Z_{a4}^* Z_{a7})\}G_{aa}] \quad (7)$$

Using the idea of Eq. 5, making the acceleration reference signal to the first suffix point of cross-spectrum, the Eq. 7 can further be written as

$$P_{gx}(f) = \frac{1}{2\omega^3 d^3} \text{Im}[\{(8D_x + 4D_x v_y + 4H)Z_{36} - 2D_x Z_{38} + (H - D_x v_y)Z_{32} + (H - D_x v_y)Z_{34} - (H + D_x v_y)Z_{35} - (H + D_x v_y)Z_{37}\}G_{33} + \{2D_x Z_{61} + (H + D_x v_y)Z_{62} + (H + D_x v_y)Z_{64} - (H - D_x v_y)Z_{65} - (H - D_x v_y)Z_{67}\}G_{66} + D_{xy}\{(Z_{25} - Z_{27})G_{22} - (Z_{45} - Z_{47})G_{44}\}] \quad (8)$$

FRF Method for Far-field Conditions

Similarly in the case of far-field condition, a modified SI equation may be formulated using FRF of force and acceleration signals as reference. First using the force signal as reference, Eq. 2 can take a new form as

$$I_{fx}(f) = \frac{2\sqrt{D_x m''}}{d\omega^2} \text{Im}\{Z_{F1}^* Z_{F2} G_{FF}\} \quad (9)$$

Using acceleration as reference and making auto-spectrum of acceleration, the Eq. 9 may stand for

$$I_{fx}(f) = \frac{2\sqrt{D_x m''}}{d\omega^2} \text{Im}\{Z_{a1}^* Z_{a2} G_{aa}\} \quad (10)$$

If reference acceleration be at point 1 (Fig. 3), Eq. 10 will shape as

$$I_{fx}(f) = \frac{2\sqrt{D_x m''}}{d\omega^2} \text{Im}\{Z_{12} G_{11}\} \quad (11)$$

Discussion

This is a one-transducer method, not incorporating finite difference technique for vibration power flow estimation. The existing method (Mandal *et al.*, 2002, 2003) for SI in orthotropic plates considers finite difference technique to approximate spatial derivatives in shear force and moments relations. Both the methods used many transducers: a two-transducer array for far-field condition (Mandal *et al.*, 2002) and an eight-transducer array for general field conditions (Mandal *et al.*, 2003). Regarding this aspect, the present study is an extension of existing method. Considering possible error sources in measurements, the present method has more benefits compared to other methods (Mandal *et al.*, 2002, 2003).

Finite difference errors may readily occur if sensor spacing is not sufficiently small compared to the flexural wavelength. The ratio of spacing to wavelength (d/λ) should be equal or less than 0.2 (Linjama and Lathi, 1992; Kay and Swanson, 1996) so as to make finite difference errors minimum. As such it may never be possible to eliminate finite difference error completely. Because of application of one transducer in the present method, the finite difference error problem is not an issue in measurements.

Phase mismatch error, on the other hand, may be less if spacing is quite large compared to the wavelength. For phase mismatch error, it is necessary to make fully propagative waves and no reverberant fields in the plates. In fully propagative wave situation, however, the phase mismatch of the sensors may exceed the true phase difference between two sensors due to structural vibration. It is because the actual phase difference can be very small when they are located very closely (so as to keep finite difference errors to a minimum) relatively to a very large wavelength. Making fully propagating wave environment with no or little reverberation is a challenging task. This can be achieved by providing strong end damping using sand or other damping materials to suppress reflections of waves from the edges of the plates. As there is no scope of applying spacing between two sensors, phase error is also not a problem in this one-transducer technique.

Random errors arise due to fluctuation (in either direction) of measurement. A large number of averaging can reduce the error in the measurement. Systematic (bias) errors, by contrast, evolve due to improper transducer (not change). Systematic errors can be studied through inter-comparisons, calibrations and error propagation.

In contact method of measuring vibration, accelerometers are generally used. Mass of accelerometer is a critical issue in vibration measurements. An eight accelerometer array is necessary to measure vibration transmission in general field conditions (Mandal, *et al.*, 2003); two for far-field conditions (Mandal *et al.*, 2002). If the mass of accelerometer is not small, it may change a local vibration behavior of the plates. For one transducer method, this issue is not critical as only one miniature accelerometer is employed. The mass loading problem is more severe in other methods: eight-time for general conditions (Mandal *et al.*, 2003) and two-time for far-field conditions (Mandal *et al.*, 2002). Other than mass loading and local stiffness due to positioning of an accelerometer, cabling damping effects could be minimal in this case. An analysis due to mass loading and local stiffness is available in modal testing book (Ewins, 1986).

Through this study a thorough formulation of structural intensity is carried out using one transducer frequency response technique with no useful experimental examples. As a preliminary result, this note is put forward. However, it is possible, in future, to provide experimental investigations of vibration power flow fields which includes near fields in presence of evanescent waves. This is the most interesting aspect of intensity measurements with this method.

Considering all these factors, the FRF method may be a good alternative to existing finite difference approximation SI techniques. If the vibration field is stationary, one transducer can be used sequentially at different measurement locations to estimate required cross-spectra. Consequently, there is no possibility of inherent error due to finite difference and phase mismatch. However, it takes a little longer in measurement time.

Conclusions

The existing SI methods using finite difference technique are modified by one-transducer FRF technique. This provides an alternative way of measurement with no finite difference and phase mismatch errors. These models provide an extension of SI methods useful for orthotropic plates. Instrumentation of these models is very simple and requires only a stable vibration field. A popular two-transducer FFT analyzer can be easily used for data acquisition and there is no need to take all cross-spectra of acceleration signals at a same time.

References

- Arruda, J.R.F. and J.P. Campos, 1996. Experimental determination of flexural power flow in beams using a modified prony method. *J. Sound and Vibration*, 197: 309-328
- Bandat, J.S. and A.G. Piersol, 1986. *Random Data: Analysis and Measurement Procedures*. New York: John Wiley, 2nd Edn.
- Bauman, P.D., 1994. Analytical and experimental evaluation of various techniques for the case of flexural waves in one-dimensional structures. *J. Sound and Vibration*, 174: 677-694.
- Berthelot, Y.H., M. Yang and J. Jarzynski, 1993. Recent progress on laser doppler measurement in structural acoustics. *Proceeding of the 4th International Congress on Intensity Technique*, Senlis, France pp: 199-206.
- Cremer, L. and M. Heckl, 1988. *Structure-Borne Sound: Structural Vibration and Sound Radiation at Audio Frequencies*. Springer-Verlag, Berlin.
- Ewins, D.J., 1980. *Modal Testing: Theory and Practice*, Letworth, Research Studies Press Ltd.
- Gavric, L. and G.A. Pavic, 1993. finite element method for computation of structural intensity by the normal mode approach. *J. Sound and Vibration*, 164: 29 -43.
- Hambric, S.A. and P.D. Taylor, 1994. Comparison of experimental and finite element structure-borne flexural power measurements for a straight beams. *J. Sound and Vibration*, 170: 595- 605.
- Kay, K.Q. and D.C. Swanson, 1996. Error in bending wave power measurements. *Noise Control Eng. J.*, 44: 185-192.
- Linjama, J. and T. Lahti, 1992. Estimation of bending wave intensity in beams using the frequency response technique. *J. Sound and Vibration*, 153: 21-36.
- Linjama, J., 1992. Structural intensity measurements using two laser vibrometers. *Inter-noise*, 92: 541-544.
- Mandal, N.K., R. Rahman Abd. and M.S. Leong, 2002. Prediction of Structure borne sound in orthotropic plates for far field conditions. *J. Vibration and Control*. 8: 3-12.
- Mandal, N.K., R. Rahman Abd. and M.S. Leong, 2003. Structure-borne power transmission in thin naturally orthotropic plates: General Case. *J. Vibration and Control*, 9: 1189-1199.
- Maynard, J.D., E.G. Williams and Y. Lee, 1985. Near field acoustic holography: I. Theory of generalised holography and development of NAH. *J. Acoustical Soc. Am.*, 78: 1395-1413.

- McDevitt, T.E., G.H. Koopmann and C.B. Burroughs, 1993. Two-channel laser vibrometer Techniques for vibration intensity measurements, Part 1: Flexural intensity. *J. Vibration and Acoustics*, 115: 436-440.
- Noiseux, D.U., 1970. Measurement of power flow in uniform beams and plates. *J. Acoustical Soc. Am.*, 47: 238-247.
- Pascal, J.C., T. Loyau and X. Carniel, 1993. Complete determination of structural intensity in plates using laser vibrometers. *J. Sound and Vibration*, 161: 527-531.
- Pavic, G., 1976. Measurement of structure borne wave intensity, part I: Formulation of the methods. *J. Sound and Vibration*, 49: 221-230.
- Romano, A.J. and E.G. Williams, 1993. On the use of acoustic holography for the determination of intensity of structures. *Proceeding of the 4th International Congress on Intensity Technique*, Senlis, France. 13-34.
- Troitsky, M.S., 1976. *Stiffened Plates: Bending, stability and vibration*. Amsterdam: Elsevier Scientific Publishing Company.
- Verheij, J.W., 1980. Cross-spectral density methods for measuring structure borne power flow on beams and pipes. *J. Sound and Vibration*, 70: 133-138.
- Williams, E.G. and H.G. Dardy, 1985a. Near field acoustic holography using an underwater-automated scanner. *J. Acoustical Society of Am.*, 78: 789-798.
- Williams, E.G., H.D. Dardy and R.G. Fink, 1985b. A technique for measurement of structure-borne intensity in plates. *J. Acoustical Soc. Am.*, 78: 2061-2068.