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## **Analytical Solution for a Multi-layer Thick Cylindrical Shell Subjected to Axial Inertia Applicable for Slump Estimations of Solid Propellant Rocket Motor Grains**

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**Abstract:** A simple procedure was established to obtain analytical solution for the general case of multi-layer thick cylindrical shell with each layer having different material properties under axial inertia loading. Design formulae in handbooks or monographs for a two-layer reinforced propellant grain, was shown to be a special case of the present general analytical solution. The solution of the problem was found to be useful for slump displacement evaluation of the propellant grain in a rocket motor under vertical storage condition. The slump displacement at the inner bore of the propellant grain was found to increase rapidly within an hour and later on increases slowly with time. When the elastic modulus varied monotonically in the ascending order from the inner radius to outer radius, the slump displacement at the inner bore of the propellant grain was found to increase compared to the case where the modulus varied monotonically in the descending order. Finite element solutions of all the above problems were found to be in good agreement with the present analytical solutions.

**Key words:** Analytical solution, rocket motor, steel casing, insulation, propellant grain, slump estimation, cylindrical shell

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### **Introduction**

One of the important design conditions for a solid propellant rocket motor concerns its structural integrity under inertia loading. Problems associated with spin stabilization, storage, maneuvering, wind shear and acceleration fall into this category. The particular inertia problem to be discussed is a rather restricted and simple example of the more general one, which ordinarily occurs in practice. It is proposed to restrict our consideration to the problem of axial inertia alone, whether at rest as in a storage condition for a long time or in accelerated flights for short times.

The major portion of the solid propellant motor (Fig. 1) can be considered to be a multilayered thick cylinder consisting of three layers of different materials namely, solid propellant, insulation and metallic or composite casing. Fitzgerald and Hufferd (1971) as well as Anonymous (1973) were presented the design formulae for slump displacement across the web (along the radial direction) in an infinitely long cylindrical grain under vertical storage by assuming the grain to be reinforced by the rigid metallic casing. It may be noted that, though insulation is also a high polymeric material, its Young's modulus is usually lower than that of propellant material. Therefore, to get an accurate estimation of slump displacement and interface shear stresses at both the interfaces (i.e., to propellant as well as to

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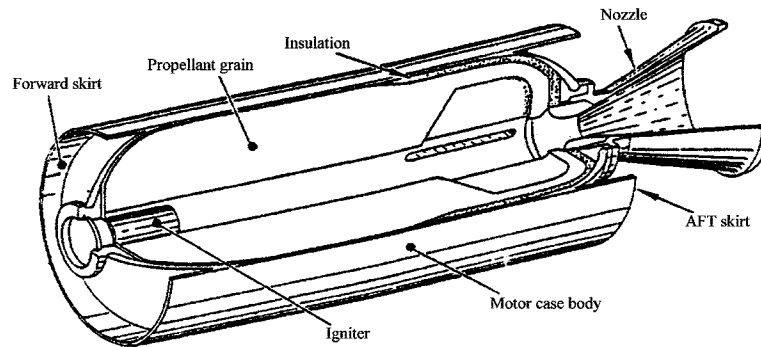


Fig. 1: A typical rocket motor

casing), it is necessary to consider the insulation as a separate layer in the analysis. Also, the conventional analysis of stresses and strains in a solid propellant rocket motor is based on the assumptions that the strains are small and a state of plane strain exists in the grain. This permits a simplified approach of the structural integrity evaluation of grains of higher length. In a region away from the end of the grain an approximate solution can be obtained if the length to radius ratio is large. However, for a short length propellant grain the effects of the end conditions should be considered, since the end effects do not damp out completely but influence the deformation as well as stress distribution throughout the grain in which case a three-dimensional analysis is required.

An elastic analysis would indicate that there is no essential difference between a 1-g storage loading for long time and n-g acceleration loading for short times, apart from the obvious magnitude factor due to the increased effective weight. However, because the grain material is viscoelastic, a time dependency arises due to the sensitivity of the material. Hence, it is essential to carry out the viscoelastic analysis for slump estimation of propellant grains in a rocket motor.

Most of the engineering viscoelastic materials, such as polymers, follow the Boltzman's superposition principle (Flugge, 1967) and so their constitutive or stress-strain relationship can be expressed in terms of either linear differential operators or linear integral operators. The viscoelastic constitutive relations in the linear differential operator form can be associated with the response of a model consisting of a series of springs and dashpots, i.e., in the form of the simple Kelvin and the Maxwell units. However, in view of certain limitations of these models, a realistic representation of the behavior of a viscoelastic material can be achieved only by a sufficiently large number of these elements. Accuracy over long time ranges requires a combination of a very large number of these elements. Such a combination would, however, give rise to constitutive relations with higher order differential operators containing a large number of unknown constants, which renders the analytical treatment quite cumbersome.

The main problem that occurs in the viscoelastic stress analysis is that the stresses, strains and displacements are all functions of time. If this time variable could be removed by a transform operation, the resulting problem would be an equivalent problem in the theory of elasticity (called the associated elastic problem) in terms of the transform parameter, with the load and boundary conditions in the form of transforms of the original time-dependent functions. The inverse transformation of the solution of the associated elastic problem into the real time variable would give the solution to the original viscoelastic problem. The disadvantage of the method of elastic-viscoelastic analogy is the difficulty

involved in the inversion of the associated elastic solution into the real time variable. To overcome this difficulty, Schapery (1961) proposed a direct method of inversion which is simple to apply, and is given by:

$$f(t) = [s \bar{f}(s)]_{s=1/2t} \quad (1)$$

Where,  $\bar{f}(s)$  is the Laplace transform of a function  $f(t)$  and  $s$  is the transform parameter. To apply the method it is necessary to multiply the transformed function by  $s$  and obtain the results at  $s = 1/2t$ . The above method of inversion generally represents satisfactorily the overall behavior of the time-dependent quantities. If the expressions for transformed quantities are obtained numerically, the subsequent inversion is accomplished by multiplying them by  $s$  and obtaining their values at  $s = 1/2t$ . But, if the values of these expressions result from some numerical solution, then the inversion is done by substituting  $s = 1/2t$  for discrete values of  $t$  in the operational quantities involving the Laplace parameter, like  $E(s) = s\bar{E}(s)$ ,  $\nu(s) = s\bar{\nu}(s)$ , etc. Thus, for each value of the transform parameters, there are corresponding values of  $E(s)$  and  $\nu(s)$  which are the associated Young's modulus and Poisson's ratio to be used in the associated elastic solution. The finite element solution obtained for these values at  $s = 1/2t$  gives directly the viscoelastic solution at time  $(t)$ . Adey and Brebbia (1973) have applied a similar procedure for the analysis of nuclear reactor pressure vessels.

The general assumption of homogeneity of solid propellant materials is subjected to certain restrictions. Such an assumption is valid only if a good quality control can be achieved and the mechanical properties are confined within narrow limits. In reality, these properties may have large variations within the same grain. Also, during the casting of large size grains due to various reasons related to the propellant mixing and slurry flowing characteristics, regions of multiple voids/porosity are noticed at certain radial locations almost throughout the circumference of the grain. Presence of these multiple voids/porosity can change the compressibility properties of the propellant at those locations. Therefore, it is essential to examine the influence of moduli (Tensile/shear and bulk moduli) variation on structural integrity of rocket motors considering grain to be made of various circumferential layers of different properties.

The purpose of this study was to develop a methodology to obtain an analytical solution for the general case of multi-layer thick cylindrical shell with each layer having different material properties, for axial inertia loading. The method of analysis is validated through comparison of existing analytical expressions for a two-layer case, and finite element solutions for a typical three-layer cylindrical shell portion of the solid propellant rocket motor.

### **Analytical Method of Analysis**

The analysis for a thick cylindrical multi-layer grain in a metallic casing of a rocket motor is based on the equations for stresses and deformation in a hollow thick cylinder (Fig. 2) with uniform thickness and subjected to the axial inertia. The radial coordinate is  $r$ , and the axial coordinates is  $z$  in the direction of the acceleration  $ng$ . For these conditions of axial symmetry of the geometry and body force loading  $png$  in the  $z$  direction, the three equations of equilibrium reduce simply to

$$\frac{1}{r} \frac{\partial(r\tau_{rz})}{\partial r} + png = 0 \quad (2)$$

Where,  $\tau_{rz}$  is the shear stress.

By integrating Eq. 2 with the boundary condition,

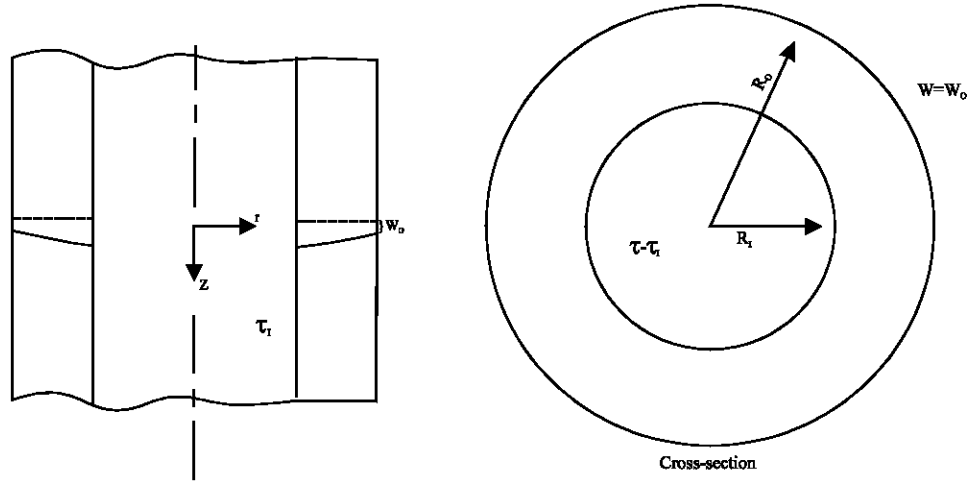


Fig. 2: A hollow thick cylinder under axial inertia

$$\tau_{rz} = \tau_i, \quad \text{at } r = R_i \quad (3)$$

We get

$$\tau_r = \frac{R_i}{r} \tau_i + \frac{1}{2} \rho g \left\{ \frac{R_i^2}{r} - r \right\} \quad (4)$$

Here,  $R_i$  is the inner radius of a hollow cylinder and  $\tau_i$  is the shear stress. The stress-strain-deformation relation for the present problem is

$$\tau_r = \frac{E}{2(1+\nu)} \gamma_r = \frac{E}{2(1+\nu)} \frac{\partial w}{\partial r} \quad (5)$$

Here  $E$  and  $\nu$  are the Young's modulus and the Poisson's ratio, respectively  $\gamma_r$  is the shear strain and  $w$  is the axial displacement, respectively.

Using Eq. 5 in 4 and integrating with the boundary condition,

$$w = w_o, \quad \text{at } r = R_o \quad (6)$$

we get

$$w = w_o + \frac{2(1+\nu)}{E} \left[ R_i \tau_i \log\left(\frac{r}{R_o}\right) + \frac{1}{2} \rho g \left\{ R_i^2 \log\left(\frac{r}{R_o}\right) + \frac{R_o^2 - r^2}{2} \right\} \right] \quad (7)$$

The multi-layer grain in the present study is an  $m$ -layer cylindrical vessel subjected to axial inertia. Each layer is having different material properties. When the boundary condition,  $\tau_i = 0$  is applied to the bore of the inner cylinder ( $r = R_i$ ), a shear stress distribution is induced in the composite structure, and at the interfaces ( $r = R_2, R_3, \dots, R_m$ ). Here  $E_i$ ,  $\nu_i$  and  $\rho_i$  are the Young's modulus, Poisson's ratio and the density of the  $i$ th cylinder. The  $i$ th cylinder is subjected to an internal shear stress (at  $r = R_i$ ) and an external shear stress  $\tau_{i+1}$ . From Eq. 4, we obtain a relation:

$$\tau_{i+1} = \frac{R_i}{R_{i+1}} \tau_i + \frac{1}{2} \rho_i \text{ng} \left\{ \frac{R_i^2}{R_{i+1}} - R_{i+1} \right\}, \quad \text{for } i=1,2,\dots,m \quad (8)$$

i.e., 
$$\tau_2 = \frac{1}{2} \rho_1 \text{ng} \left\{ \frac{R_1^2}{R_2} - R_2 \right\} \quad (\text{from boundary condition, } \tau_1 = 0)$$

$$\tau_3 = \frac{R_2}{R_3} \tau_2 + \frac{1}{2} \rho_2 \text{ng} \left\{ \frac{R_2^2}{R_3} - R_3 \right\}, \quad \text{etc.}$$

Thus, Eq. 8 becomes a recurrence relation for shear stress at interfaces. For the multi-layer case bonded solid propellant grain under vertical storage with un-supported base,  $\tau_m$  gives maximum interface shear stress value. Shear stress within the  $i^{\text{th}}$  cylinder can be obtained by substituting the interfacial shear stress,  $\tau_i$  in the equation:

$$\tau_r^{(i)} = \tau_i \frac{R_i}{r} + \frac{1}{2} \rho_i \text{ng} \left\{ \frac{R_i^2}{r} - r \right\} \quad (9)$$

Similarly, for the case of rigid casing material, the boundary condition for axial displacement at outer boundary can be written as:

$$w = w_{m+1} \text{ at } r = R_{m+1} \quad (10)$$

Replacing  $w_o = w_{i+1}$ ,  $\tau_i = \tau_i$ ,  $v = v_i$ ,  $E = E_i$ ,  $\rho = \rho_i$ ,  $R_i = R_i$  and  $R_o = R_{i+1}$  in Eq. 7, we get the axial displacement for  $i^{\text{th}}$  cylinder as:

$$w_r^{(i)} = w_{i+1} + \frac{2(1+v_i)}{E_i} \left[ R_i \tau_i \log\left(\frac{r}{R_{i+1}}\right) + \frac{1}{2} \rho_i \text{ng} \left\{ R_i^2 \log\left(\frac{r}{R_{i+1}}\right) + \frac{R_{i+1}^2 - r^2}{2} \right\} \right] \quad (11)$$

The axial displacement,  $w_i$  at inner radius ( $r = R_i$ ) of the  $i^{\text{th}}$  cylinder can be obtained from Eq. 11 as:

$$w_i = w_{i+1} + \frac{2(1+v_i)}{E_i} \left[ R_i \tau_i \log\left(\frac{R_i}{R_{i+1}}\right) + \frac{1}{2} \rho_i \text{ng} \left\{ R_i^2 \log\left(\frac{R_i}{R_{i+1}}\right) + \frac{R_{i+1}^2 - R_i^2}{2} \right\} \right] \quad (12)$$

which can be used for  $i = m, (m-1), \dots, 2, 1$

i.e., 
$$w_m = \frac{2(1+v_m)}{E_m} \left[ R_m \tau_m \log \frac{R_m}{R_{m+1}} + \frac{1}{2} \rho_m \text{ng} \left\{ R_m^2 \log \frac{R_m}{R_{m+1}} + \frac{R_{m+1}^2 - R_m^2}{2} \right\} \right] \quad (\text{from Eq. 10, } w_{m+1} = 0)$$

$$w_{m-1} = w_m + \frac{2(1+v_{m-1})}{E_{m-1}} \left[ R_{m-1} \tau_{m-1} \log \frac{R_{m-1}}{R_m} + \frac{1}{2} \rho_{m-1} \text{ng} \left\{ R_{m-1}^2 \log \frac{R_{m-1}}{R_m} + \frac{R_m^2 - R_{m-1}^2}{2} \right\} \right], \quad \text{etc.}$$

Thus Eq. 12 represents a recurrence relation for the axial displacement.  $w_1$  gives maximum slump value at the inner bore of the multi-layer grain. The axial displacement within the  $i^{\text{th}}$  cylinder can be obtained directly from Eq. 11 after determining displacements from Eq. 12 at interfaces.

Analytical expressions presented by Fitzgerald and Hufferd (1971) as well as Anonymous, (1973) for shear stress and axial displacement in a plane strain, circular port grain (with two layers i.e., propellant and rigid metal casing) under axial inertial load can be obtained easily from the present analytical solution assuming  $m = 1$ ,  $R_1 = a$ ,  $R_2 = b$ ,  $\tau_1 = 0$  and  $w_2 = 0$ . The properties of the propellant material are:  $E_1 = E$ ,  $V_1 = V$ , and  $\rho_1 = \rho$ . The interface shear stress ( $\tau_2$ ) at outer surface of the propellant (i.e., at  $r = R_2 = b$ ) can be obtained from Eq. 8 for  $i=1$  as:

$$\tau_2 = \frac{1}{2} \rho n g \left( \frac{a^2}{b^2} - b \right) \quad (13)$$

The maximum axial displacement ( $w_1$ ) at inner surface of the propellant (i.e., at  $r = R_1 = a$ ) can be obtained by using Eq. 13 in Eq. 11 for  $i=1$  as:

$$\begin{aligned} w_1 &= \frac{(1+\nu) \rho n g}{E} \left\{ a^2 \log\left(\frac{a}{b}\right) + \frac{b^2 - a^2}{2} \right\} \\ &= \frac{(1+\nu) \rho n g b^2}{2E} \left\{ 1 - \frac{a^2}{b^2} \log \frac{eb^2}{a^2} \right\} \end{aligned} \quad (14)$$

It is evident from Eq. 14 that rapid increase in slump displacement can be expected in the cylindrical grains with higher outer radius ( $b$ ) and lower  $a/b$  ratio (i.e., higher web thickness).

## Results and Discussion

The analytical solution for multi-layer thick cylindrical shell subjected to axial inertia load is examined for a propellant grain with insulation and metallic casing (a three-layer cylindrical shell). The analytical solution of the problem is compared with finite element solution.

The basic input considered for the analysis is as follows:

|                            |            |
|----------------------------|------------|
| Grain inner radius ( $a$ ) | = 50 cm    |
| Grain outer radius ( $b$ ) | = 138.4 cm |
| Insulation thickness       | = 0.5 cm   |
| Casing thickness           | = 0.78 cm  |

The Young's modulus ( $E_c$ ), Poisson's ratio ( $\nu_c$ ) and density ( $\rho_c$ ) of the steel casing material are:

$$E_c = 1900000 \text{ kg cm}^{-2}, \nu_c = 0.3 \text{ and } \rho_c = 0.0178 \text{ kg cm}^{-3}$$

The Young's modulus ( $E_{ins}$ ), Poisson's ratio ( $\nu_{ins}$ ) and density ( $\rho_{ins}$ ) of the insulation material (assumed to be elastic) are:

$$E_{ins} = 20 \text{ kg cm}^{-2}, \nu_{ins} = 0.5; \text{ and } \rho_{ins} = 0.0012 \text{ kg cm}^{-3}$$

Bulk modulus ( $K$  assumed to be constant) and density ( $\rho_p$ ) of the propellant are :

$$K = 35300 \text{ kg cm}^{-2}, \rho_p = 0.00178 \text{ kg cm}^{-3} \text{ and Inertia load} = 1g$$

The relaxation modulus,  $E_{rel}$  for a HTPB based propellant material is expressed in the Prony series form as;

$$E_{rel}(t) = E_{\infty} + \sum_{k=1}^n A_k e^{-\frac{t}{\tau_k}} \quad (15)$$

where,  $E_{\infty}$  is the equilibrium modulus,  $\tau_k$  are the relaxation times and  $A_k$  are constants.

The first 16 values of  $\tau_k$  (in seconds) and  $A_k$  (in  $\text{kg cm}^{-2}$ ) are (Renganathan *et al.*, 2000):

$\tau_k = 10^{-k-9}$  for  $k = 1, 2 \dots 16$ . and the corresponding values of  $A_k$  are 1.17, 158.8, 387.3, 530.2, 225.6, 139.3, 52.2, 45.6, 13.9, 11.9, 4.46, 4.14, 0.26, 0.1, 0.445 and 0.655. The equilibrium modulus,  $E_{\infty} = 20 \text{ kg cm}^{-2}$ .

The Laplace transform for the relaxation modulus defined in Eq. 15, is:

$$\bar{E}_{ni}(s) = \frac{E_{\infty}}{s} + \sum_{k=1}^n \frac{A_k}{s + \frac{1}{2\tau_k}} \quad (16)$$

The operational modulus,  $E(s)$  is given by:

$$E(s) = s\bar{E}_{ni}(s) = E_{\infty} + \sum_{k=1}^n \frac{sA_k}{s + \frac{1}{2\tau_k}} \quad (17)$$

Using the bulk modulus ( $K$ ) and the operational modulus  $E(s)$ , we can write the operational Poisson's ratio in the form

$$\nu(s) = \frac{1}{2} \left[ 1 - \frac{E(s)}{3K} \right] \quad (18)$$

At any time  $t$ , substituting the value of  $s = 1/2t$  in Eq. 17 and 18, the value of operational modulus  $E(s)$  and the corresponding value of the Poisson's ratio  $\nu(s)$  can be obtained. The values of operational modulus  $E(s)$  and the Poisson's ratio,  $\nu(s)$  are used for  $E$  and  $\nu$  in the elastic analytical solutions to get time dependent viscoelastic solutions.

For an infinitely long multi-layer cylindrical shell under axial inertia load, the structure can be modeled as a strip by dividing into a number of quadrilateral elements along the radial direction and constraining radial displacements on top and bottom surfaces of the cylinder, in addition to the constraint of axial displacement at the outer surface of the cylindrical shell (Fig. 3).

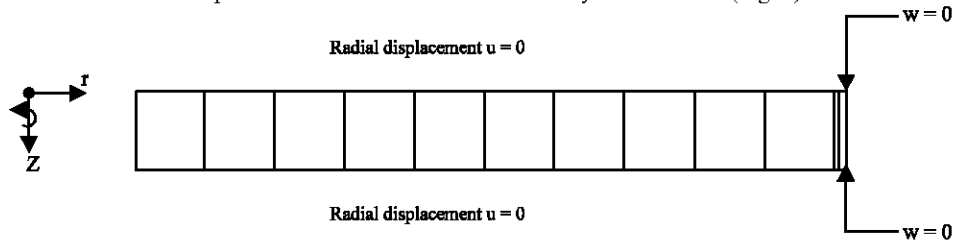


Fig. 3: Finite element idealization of an infinitely long case bonded propellant grain

Table 1: Comparison of maximum slump displacement at inner surface of the grain with time for 1-g vertical storage load

| Time, t<br>(sec)<br>$\log_{10} t$ | Operational modulus,<br>$E(s)$ of the propellant<br>(KSC) | Maximum slump at the inner surface (mm) |                         |
|-----------------------------------|-----------------------------------------------------------|-----------------------------------------|-------------------------|
|                                   |                                                           | Analytical solution                     | Finite element solution |
| -8                                | 1588                                                      | 0.179                                   | 0.176                   |
| -7                                | 1514                                                      | 0.183                                   | 0.180                   |
| -6                                | 1259                                                      | 0.204                                   | 0.202                   |
| -5                                | 832                                                       | 0.268                                   | 0.267                   |
| -4                                | 427                                                       | 0.446                                   | 0.445                   |
| -3                                | 237                                                       | 0.738                                   | 0.736                   |
| -2                                | 132                                                       | 1.262                                   | 1.259                   |
| -1                                | 83                                                        | 1.960                                   | 1.956                   |
| 0                                 | 50                                                        | 3.200                                   | 3.196                   |
| 1                                 | 37                                                        | 4.296                                   | 4.293                   |
| 2                                 | 28                                                        | 5.651                                   | 5.649                   |
| 3                                 | 24                                                        | 6.580                                   | 6.579                   |
| 4                                 | 21.4                                                      | 7.376                                   | 7.375                   |
| 5                                 | 21.1                                                      | 7.459                                   | 7.457                   |
| 6                                 | 20.9                                                      | 7.547                                   | 7.544                   |
| 7                                 | 20.4                                                      | 7.727                                   | 7.723                   |
| 8                                 | 20.0                                                      | 7.880                                   | 7.875                   |

Table 2: Comparison of slump displacement across the web of the propellant grain corresponding to the equilibrium modulus for 1-g vertical storage load

| Radial distance, r (cm)<br>(from inner bore to outer<br>surface of the propellant) | Slump displacement (mm) |                         |
|------------------------------------------------------------------------------------|-------------------------|-------------------------|
|                                                                                    | Analytical solution     | Finite element solution |
| 50.00                                                                              | 7.880                   | 7.875                   |
| 58.89                                                                              | 7.780                   | 7.775                   |
| 67.78                                                                              | 7.497                   | 7.493                   |
| 76.67                                                                              | 7.051                   | 7.047                   |
| 85.56                                                                              | 6.455                   | 6.451                   |
| 94.45                                                                              | 5.717                   | 5.713                   |
| 103.34                                                                             | 4.834                   | 4.841                   |
| 112.23                                                                             | 3.840                   | 3.837                   |
| 121.12                                                                             | 2.709                   | 2.708                   |
| 130.01                                                                             | 1.456                   | 1.455                   |
| 138.40                                                                             | 0.081                   | 0.081                   |

Table 3: Comparison of slump displacement along the radial direction (from the inner bore to outer surface of the propellant) for the variation of elastic modulus across the web in (Case-I) ascending order of 30 to 50 in steps of  $5 \text{ kg cm}^{-2}$  and (case-II) descending order of 50 to 30 in steps of  $-5 \text{ kg cm}^{-2}$ 

| Radial distance, r (cm) | Case-I              |                         | Case-II             |                         |
|-------------------------|---------------------|-------------------------|---------------------|-------------------------|
|                         | Analytical solution | Finite element solution | Analytical solution | Finite element solution |
| 50.00                   | 3.737               | 3.735                   | 4.475               | 4.472                   |
| 58.89                   | 3.670               | 3.668                   | 4.435               | 4.432                   |
| 67.78                   | 3.482               | 3.480                   | 4.322               | 4.319                   |
| 76.67                   | 3.227               | 3.225                   | 4.123               | 4.121                   |
| 85.56                   | 2.886               | 2.885                   | 3.858               | 3.856                   |
| 94.45                   | 2.517               | 2.516                   | 3.489               | 3.487                   |
| 103.34                  | 2.081               | 2.079                   | 3.053               | 3.051                   |
| 112.23                  | 1.635               | 1.634                   | 2.479               | 2.478                   |
| 121.12                  | 1.113               | 1.113                   | 1.833               | 1.832                   |
| 130.01                  | 0.631               | 0.630                   | 0.997               | 0.997                   |
| 138.40                  | 0.081               | 0.081                   | 0.081               | 0.081                   |

Results of analytical and finite element solutions in Table 1 show a good comparison of maximum slump displacement of inner surface with time. The slump displacement is found to increase rapidly within an hour, and later on increases slowly with time, and approaches a value corresponding to equilibrium modulus ( $E_\infty$ ). Table 2 gives comparison of slump displacement across the web of the propellant grain corresponding to the equilibrium modulus.

Using the finite element idealization of Fig. 3, a five-layer propellant grain with insulation and metallic casing (a seven-layer cylindrical shell) is analyzed considering: variation of elastic modulus in ascending order of 30 to 50  $\text{kg cm}^{-2}$  in steps of  $5 \text{ kg cm}^{-2}$  and descending order of 50 to 30  $\text{kg cm}^{-2}$  in steps of  $-5 \text{ kg cm}^{-2}$ , from inner radius to outer radius of the propellant. Since, the propellant material is nearly incompressible, the Poisson's ratio,  $\nu = 0.499$  is used in the analysis. In the finite element model, each layer of the five-layer propellant material is divided into two elements hence 10 elements are used for the propellant material (Fig. 3). Specifying the mechanical properties of propellant, insulation and metallic casing, the grain is analyzed for 1g axial inertia load. Results in Table 3 indicate a good comparison of analytical and finite element solutions of slump along the radial direction. The slump displacement at inner bore of the propellant grain increases when the Young's modulus of the material varies monotonically in the ascending order from the inner radius to outer radius, compared to the case where the modulus varies monotonically in the descending order.

## **Conclusions**

This study presents a simple methodology to obtain analytical solution for the general case of multi-layer thick cylindrical shell (with each layer having different material properties) for axial inertia loading. The existing analytical solution for a reinforced propellant grain is shown to be a special case of the present study. Finite element solutions, for grains having more numbers of layers with different material properties are in good agreement with the present analysis results.

## **References**

- Adey, R.A. and C.A. Brebbia, 1973. Efficient method of solution of viscoelastic problems. Proc. ASCE. J. Eng. Mech. Div., 99: 1119-1127.
- Anonymous, 1973. Solid propellant grain structural integrity analysis, National Aeronautics and Space Administration, Lewis Research Center (Design Criteria Office), Cleveland, Ohio, Report No.:NASA-SP-8073.
- Fitzgerald, J.E. and W.L. Hufferd, 1971. Handbook for the engineering structural Analysis of Solid Propellants, Prepared in cooperation with the Johns Hopkins University, Applied Physics Lab, Chemical Propulsion Information Agency, Silver Spring, MD, CPIA Publication No.214.
- Flugge, W., 1967. Viscoelasticity, Blasidell, New York.
- Renganathan, K., B. Nageswara Rao and M.K. Jana, 2000. An efficient axisymmetric hybrid-stress-displacement formulation for compressible/nearly incompressible material. Intl. J. Pressure Vessels and Piping.,77: 651-667.
- Schapery, R.A., 1961, Two simple approximate methods of Laplace transform inversion for viscoelastic stress analysis. Graduate Aeronautical Laboratories, California Institute of Technology SM, pp: 61-23.