Trends in

Applied Sciences Research

ISSN 1819-3579

Academic
Journals Inc.

www.academicjournals.com

Trends in Applied Sciences Research 1 (2): 184-193, 2006
ISSN 1819-3579

Flow of an Electrically Conducting Second-Grade Fluid under an Oscillating Rigid Moving Plate

[1]Swamy, N.S., [2]H.R. Nataraja, [3]K.S. Sai, [4]S.B. Tiwari and [4]B. Nageswara Rao
[1]Department of Computer Science and Engineering, Vivekananda Institute of Technology, Gudimavu, Bangalore-560 074, India
[2]Compressor Group, Gas Turbine Research Establishment, C.V. Raman Nagar, Bangalore-560 093, India
[3]Department of Mathematics, S.V.H. College of Engineering, Machalipatnam-521 002, India
[4]Structural Analysis and Testing Group, Vikram Sarabhai Space Centre, Trivandrum- 695 022, India

Abstract: This study considers the Rivlin-Ericksen constitutive equation for the Cauchy stress in the equation of motion to examine the flow of an incompressible second-grade fluid with an oscillating rigid moving plate. Simple and reliable numerical procedures are used to obtain the parameters in the analytical expressions for the velocity field and the shearing stress on the moving plate. The Doppler effect is noticed from the increased frequency due to the motion of the plate. The thickness of the boundary layer reduces with an increase in the magnetic interaction parameter.

Key words: Second-grade fluid, rigid moving plate, frequency, velocity field, magnetic field, shear stress

Introduction

Rajagopal (1982), Rajagopal and Na (1983) and Hayat *et al*. (1998, 1999) have examined the problems on the oscillation of plates and disks in non-Newtonian fluids. In these problems, the plates and disks are subjected to various oscillations in their own planes. Asghar *et al*. (2002) have shown interest in investigating the problems in which the boundaries (plates and disks) are oscillating and moving in the fluid at the same time. Physically, these correspond to the problem of an oscillating piston moving into a gas with a constant velocity. Utilizing the perturbation expansion method of Foote *et al*. (1987) and Hinch (1992), they have presented analytical solutions for the flow of a second-grade fluid for a rigid moving plate oscillating in its own plane, which are valid only for small values of the non-Newtonian parameter. This paper presents simple and reliable numerical procedures for obtaining the parameters in the analytical expressions for the velocity field and the shearing stress on the moving plate applicable for all values of the non-Newtonian parameter.

Analysis

The constitutive equation for the Cauchy stress tensor (T) in the equation of motion is (Rivlin and Erickson, 1955):

Corresponding Author: Dr. B. Nageswara Rao, Structural Analysis and Testing Group, Vikram Sarabhai Space Centre, Trivandrum- 695 022, India

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \tag{1}$$

Here p is the pressure, I is the unit matrix, A_1 and A_2 are the first two Rivlin-Ericksen tensors defined by:

$$A_1 = \left(\text{grad}\,\vec{V}\right) + \left(\text{grad}\,\vec{V}\right)^T \tag{2}$$

$$A_2 = \frac{dA_1}{dt} + A_1 . \left(\text{grad}\,\vec{V}\right) + \left(\text{grad}\,\vec{V}\right)^T . A_1 \tag{3}$$

μ is the viscosity and α_1 and α_2 are material constants satisfying (Dunn and Fosdick, 1974; Fosdick and Rajagopal, 1979): $\alpha_1 \geq 0$, $\alpha_1 + \alpha_2 \geq 0$. In the present study, the velocity, $\vec{V}$ = [u(y,t), 0, 0], which satisfies exactly the continuity equation.

A rigid plate, initially located at y = 0 and making oscillations of the form $U\cos\beta t$ in its own plane, is moving into the fluid with a constant velocity, c_0 The fluid is occupied in the upper half plane (i.e., y > 0). The conducting fluid is permeated by an imposed uniform magnetic field, B_0in the positive y-direction normal to the plate. The applied magnetic field is perpendicular to the velocity field. The induced magnetic field is negligible compared with the applied field so that the magnetic Reynolds number is small. The electric field is assumed to be zero. With these assumptions, the Maxwell's equations are decoupled with the equation of motion. Due to the interaction of the velocity field with the applied magnetic field, the electromagnetic body force will be present in the equation of motion. The governing equations for the present moving boundary value problem in a second-grade fluid obeying (1) are:

$$\rho \frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial y^2} + \alpha_1 \frac{\partial^3 u}{\partial y^2 \partial t} - \sigma B_0^2 u \tag{4}$$

$$u = U\cos\beta t \qquad \text{at } y = c_0 t \tag{5}$$

$$u \to 0 \qquad \text{as } y \to \infty \tag{6}$$

Here μ is the velocity component along the flow direction; ρ is the density; β is the frequency of the oscillating plate; and σ is the electrical conductivity of the fluid. Specification of an asymptotic boundary condition (6) implies that all higher derivatives of the dependent variable approaches zero as the specified value of the independent variable approached.

Defining the moving coordinate (η,τ) by:

$\eta = \zeta - \tau$, $\zeta = \frac{\rho c_o y}{\mu}$ and $\tau = \frac{\rho c_o^2 t}{\mu}$; and introducing the dimensionless velocity component, $f(\eta,\tau) = \frac{u}{U}$

Eq. (4) and (6) can be expressed as:

$$\frac{\partial f}{\partial \tau\tau} - \frac{\partial f}{\partial \eta^2} = \frac{\partial^2 f}{\partial \eta^2} + \alpha\left(\frac{\partial^3 f}{\partial \eta^2 \partial \tau} - \frac{\partial^3 f}{\partial \eta^3}\right) - nf \tag{7}$$

$$f = \cos\omega\tau \qquad \text{at} \qquad \eta \to 0 \tag{8}$$

$$f \to 0 \qquad \text{as} \qquad \eta \to \infty \tag{9}$$

Here, $\alpha = \frac{\rho c_0^2 \alpha_1}{\mu^2}$, is the non-Newtonian parameter, $\omega = \frac{\mu\beta}{\rho c_o^2}$, is the frequency parameter and

$n = \frac{\sigma B_o^2 \mu}{\rho^2 c_o^2}$, is the magnetic interaction parameter.

Assuming
$$f(\eta,\tau)=\text{Re}[F(\eta)\,e^{i\omega\tau}] \tag{10}$$
One can obtain
$$\alpha F'''-(1+i\alpha\omega)F''-F'+(n+i\omega)F=0 \tag{11}$$

$$F=1 \quad \text{at} \quad \eta\to 0 \tag{12}$$
$$F\to 0 \quad \text{as} \quad \eta\to\infty \tag{13}$$

Here prime denotes differentiation with respect to η. The characteristic polynomial of Eq. (11) is of the form
$$\alpha\, m^3-(1+i\alpha\omega)m^2-m+(n+i\omega)=0 \tag{14}$$

To satisfy the asymptotic boundary condition (13), the real part of the complex root from the characteristic Eq. (14) should be negative. When $\alpha = 0$ the solution of the problem corresponds to the Newtonian case. If a + ib, is the required complex root of the characteristic Eq. (14), then the solution of Eq. (7) to (9) can be written in the from
$$f(\eta,\tau)=e^{a\eta}\cos(b\eta+\omega\tau) \tag{15}$$

It should be noted that the real part of the complex root, i.e., a<0. The value of a and b must satisfy the following two equations:

$$E_r=\alpha\left(a^3-3ab^2\right)+2\alpha\omega ab-a^2+b^2-a+n=0 \tag{16}$$

$$E_i=\alpha\left(3a^2b-b^3\right)-\alpha\omega\left(a^2-b^2\right)-2ab-b+\omega=0 \tag{17}$$

Equations (16) and (17) are obtained from Eq. (14) by substituting m = a + ib and equating the resulting real and imaginary parts to zero. When $\alpha = 0$, Eq. (14) gives the required complex root, $m = a_0 + ib_0$, in which the values of a_0 and b_0 in terms of n and ω are:

$$a_0=-\frac{1}{2}\left(1+\sqrt{\frac{r+1+4n}{2}}\right) \text{ and } b_0=-\frac{1}{2}\sqrt{\frac{r-1-4n}{2}} \text{ Here. } r=\sqrt{(1+4n)^2+16\omega^2}$$

When $\alpha \neq 0$, the value of *a* and *b* can be obtained by solving the differential equations:

$$\frac{da}{da}=-\frac{(B_2A_{12}+B_1A_{11})}{A_{11}^{\ 2}+A_{12}^{\ 2}} \tag{18}$$

$$\frac{da}{da}=-\frac{(B_1A_{12}+B_2A_{11})}{A_{11}^{\ 2}+A_{12}^{\ 2}} \tag{19}$$

With initial conditions:
$$a=a_0,\ b=b_0 \quad \text{at} \quad \alpha=0 \tag{20}$$

Here, $A_{11}=3\alpha\left(a^2-b^2\right)+2\alpha\omega b-2a-1$; $A_{12}=6\alpha ab-2\alpha\omega a-2b$; $B_1=a^3-3ab^2+2\omega ab$; $B_2=3a^2b-b^3-\omega\left(a^2-b^2\right)$.

Equation (18) and (19) are obtained by differentiating Eq. (16) and (17) with respect toα. Equation (18) to (20) are solved by a fourth-order Runge-Kutta integration scheme (Carnahan *et al*., 1969) with a fixed step size of 0.001. In the absence of magnetic field (i.e., n = 0), value of the parameters a and b at $\alpha = 1$ for the case of $\omega = 1$ are obtained as -0.79526 and -0.15962, whereas in the presence of magnetic field (i.e., n = 0.1), these values are -0.81402 and -0.13714, respectively.

Applying the Newton-Raphson iterative procedure (Carnahan *et al*., 1969) to the nonlinear Eq. (16) and (17), values of a and b can be obtained for the specified values of α, n and ω. The increments Δa and Δb in *a* and *b* for each iteration are obtained from

$$\Delta a = -\frac{\left(E_i A_{12} + E_r A_{11}\right)}{A_{11}^2 + A_{12}^2} \tag{21}$$

$$\Delta b = \frac{\left(E_r A_{12} - E_i A_{11}\right)}{A_{11}^2 + A_{12}^2} \tag{22}$$

In this iterative procedure, a_0 and b_0 are assumed as initial values and updating the values of *a* and *b* with the above increments for each iteration until the values $|\Delta a|$ and $|\Delta b|$ becomes negligibly small (say$<10^{-6}$).

Table 1 presents the values of the parameters *a* and *b* with the magnetic interaction parameter, n for the specified values of $\alpha = 1$ and $\omega = 1$. As n increases, the magnitude of parameter a increases which indicates reduction in the boundary layer thickness. These values are found to be the same as those obtained from the numerical solution of differential Eq. (18) and (19).

IBM scientific subroutine package (Anonymous, 1970) contains the subroutine POLRT, which can find the complex roots of a polynomial having real coefficients. An attempt is made to utilize this subroutine for finding the required complex root of Eq. (14). A six degree polynomial equation having real coefficients which gives the roots of Eq. (14), is obtained by squaring the Eq. (14) after re-arranging the real and imaginary terms, as:

$$\begin{aligned}&\alpha^2 m^6 - 2\alpha m^5 + (1 - 2\alpha^2\omega^2)m^4 + 2(1+\alpha n)m^3 \\ &+(1 - 2n - 2\alpha\omega^2)m^2 - 2nm + n^2 + \omega^2 = 0 \quad = R\sin(\omega\tau + \theta)\end{aligned} \tag{23}$$

For the specified values of α, *n* and ω, Eq. (23) gives six complex roots. Only three of them will satisfy Eq. (14). Among them, the required complex root is the one whose real part should be negative. It can be seen from the roots of Eq. (23) presented in Table 2 and 3 for $\alpha = \omega = 1$ and n = 0 and 0.1 that only three roots among six are satisfied Eq. (16) and (17). One of the complex roots, whose real part is negative, is found to be same as those obtained earlier by the Newton-Raphson iterative procedure as well as from the numerical integration of Eq. (18) and (19). All the above three numerical schemes were given the same values of a and b for the specified values of α, *n* and ω. Hence, any one of the numerical schemes can be adopted for obtaining the solution. Once *a* and *b* are known, the velocity, *u* can be obtained from Eq. (15).

The expression for the shearing stress, $\left(\text{i.e., } P_{xy} = \mu\frac{\partial u}{\partial y} + \alpha_1\frac{\partial^2 u}{\partial y \partial t}\right)$, on the moving plate is given by:

$$\begin{aligned}(P_{xy})_{y=c_o t} &= \left[\frac{\partial f}{\partial \eta} + \alpha\left(\frac{\partial^2 f}{\partial \eta \partial t} - \frac{\partial^2 f}{\partial \eta^2}\right)\right]_{at\ \eta=0} = R\sin(\omega\tau + \theta) \\ \theta &= \tan^{-1}\left[\frac{R_y}{R_x}\right]\end{aligned} \tag{24}$$

Where, $R = \sqrt{R_x^2 + R_y^2}$;

$R_x = b - \alpha\,\omega\,a - 2\alpha\,ab$; $R_y = \alpha - \alpha\,\omega\,b - \alpha\,(a^2 - b^2)$.

Table 1: The parameters a and b in the expression (15) for the velocity field obtained through the Newton-Raphson iterative procedure utilizing the increments Δa and Δb in Eq. (21) and (22) for $\alpha = 1$ and $\omega = 1$

n	a	b
0	-0.79526	-0.15962
0.1	-0.81402	-0.13714
0.2	-0.83378	-0.11619
0.3	-0.85423	-0.09683
0.4	-0.87514	-0.07903
0.5	-0.89626	-0.06274
0.6	-0.91742	-0.04785
0.7	-0.93846	-0.03425
0.8	-0.95928	-0.02182
0.9	-0.97982	-0.01044
1.0	-1.0	0.0

Table 2: Complex roots of Eq. (23) for m (= a + i b) for $\alpha = \omega = 1$ and n = 0

a	b	E_r Eq.(16)	E_i Eq. (17)
-0.79526	0.15962	-0.50775	0.78607
-0.79526	-0.15962	0.0	0.0
1.4022	-0.41349	-2.3192	-1.5905
1.4022	0.41349	0.0	0.0
0.39304	-0.74613	-1.1730	2.8045
0.39304	0.74613	0.0	0.0

Table 3: Complex roots of equation (23) for m (= a + i b) for $\alpha = \omega = 1$ and n = 0.1

a	b	E_r Eq. (16)	E_i Eq. (17)
1.3584	-0.41014	-2.2286	-1.3543
1.3584	0.41014	0.0	0.0
-0.81402	0.13714	-0.44654	0.71235
-0.81402	-0.13714	0.0	0.0
0.45558	-0.72700	-1.3248	2.6419
0.45558	0.72700	0.0	0.0

Results and Discussion

This paper examines the flow of an electrically conducting second-grade fluid with an oscillating rigid moving plate. Parameters a and b in Eq. (15) for the non-dimensional velocity component (f) are obtained by specifying the values for the non-Newtonian parameter (α), the frequency parameter (ω) and the magnetic interaction parameter (n). Figure 1 and 2 show the comparison of a and b for the specified values of α, n and ω. For very small α, the values of a and b from the perturbation method (Asghar *et al*., 2002) compare well with the present analysis results. For moderately large α, values of a and b obtained from the expression of Asghar *et al*. (2002), were not satisfied Eq. (16) and (17). Hence, the perturbation solution of Asghar *et al*. (2002) for the velocity field is applicable only for small values of α. Using the value of a and b from the present numerical analysis, in Eq. (15), velocity profiles are generated and presented in Fig. 3 and 4. The discrepancy in the results of Asghar *et al*. (2002) is mainly due to the limitation of the perturbation method. The shearing stress on the moving plate can be obtained using these parameters a and b in Eq. (24). Figure 5 and 6 show the variation of the magnitude (R) and the amplitude (θ) with the non-Newtonian parameter (α) for the specified values of $\omega = 1$ and n = 0 and 0.1.

The argument of the cosine in the non-dimensional velocity component in Eq. (15) can be expressed as:

$$b\eta + \omega t = b\zeta + (\omega - b)t = \frac{b\rho c_0}{\mu} y + \left(\beta - \frac{b\rho c_0^2}{\mu}\right)t \cdot$$

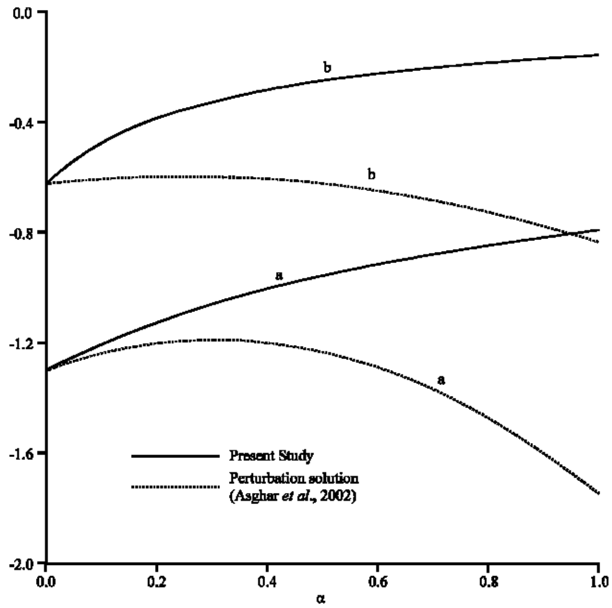

Fig. 1: Comparison of parameters *a* and *b* in Eq. (15) for the non-dimensional velocity component (*f*) in the absence of magnetic field (*n* = *0*)

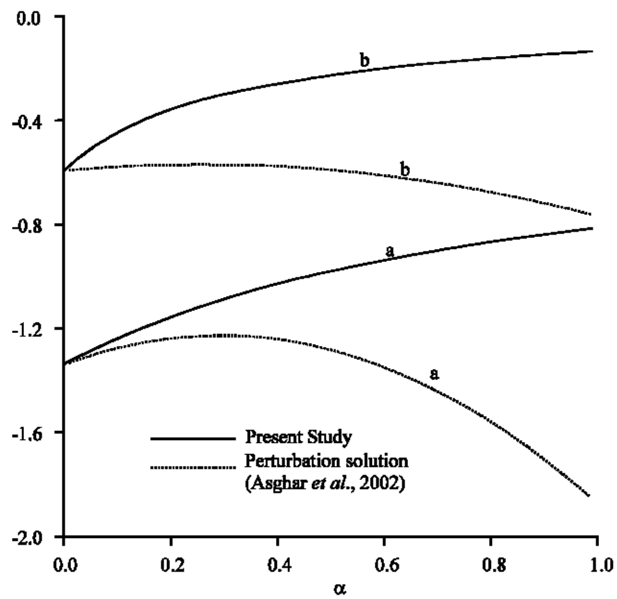

Fig. 2: Comparison of parameters *a* and *b* in Eq. (15) for the non-dimensional velocity component (*f*) in the presence of magnetic field (*n* = *0.1*)

The frequency of the wave received by the observer at *(y, t)* is: $\left(\beta - \frac{b\rho c_0^{\,2}}{\mu}\right)$

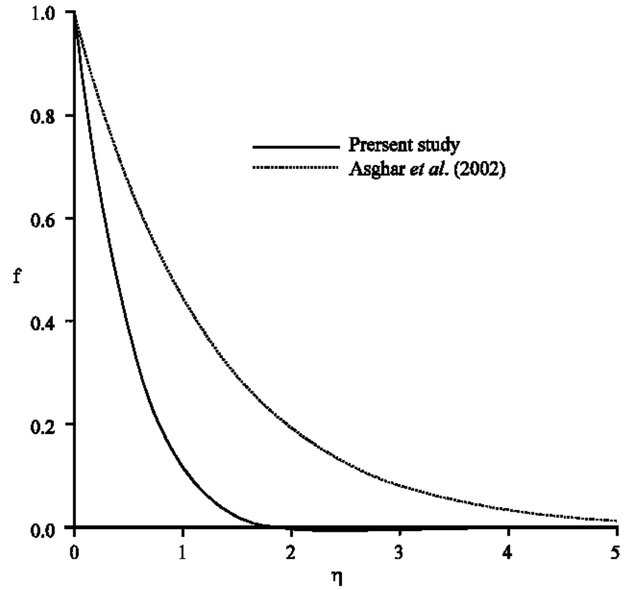

Fig. 3a: Comparison of velocity profiles for α = 1, ω = 1, τ = 0 and *n* = 0

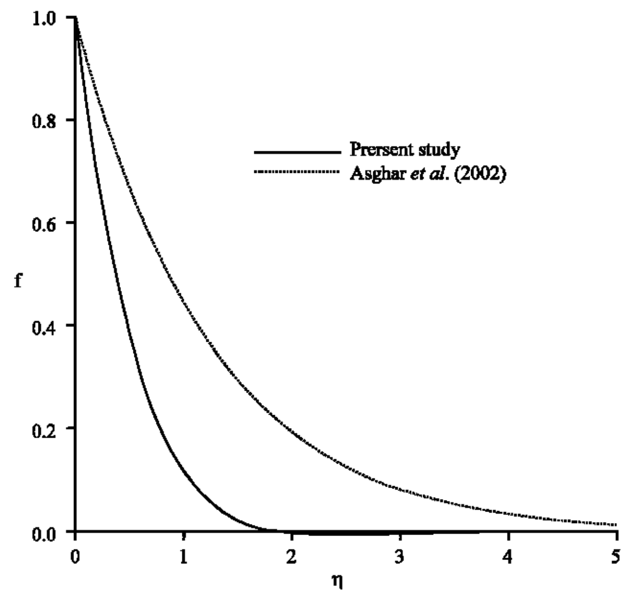

Fig. 3b: Comparison of velocity profiles for α = 1, ω = 1, τ = 0 and *n* = 0.1

The values of parameters *a* and *b* in Table 1 are negative. The frequency is increased, from the frequency of the oscillating plate *(β)* by an additive factor:

$$\left(\frac{-b?\,c_0^{\,2}}{\mu}\right)$$

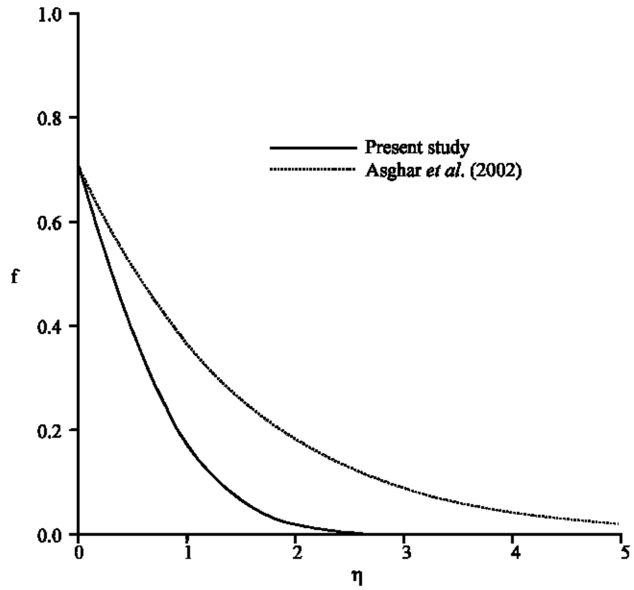

Fig. 4a: Comparison of velocity profiles for $\alpha = 1$, $\omega = 1$, $\tau = [\pi / 4]$ and $n = 0$

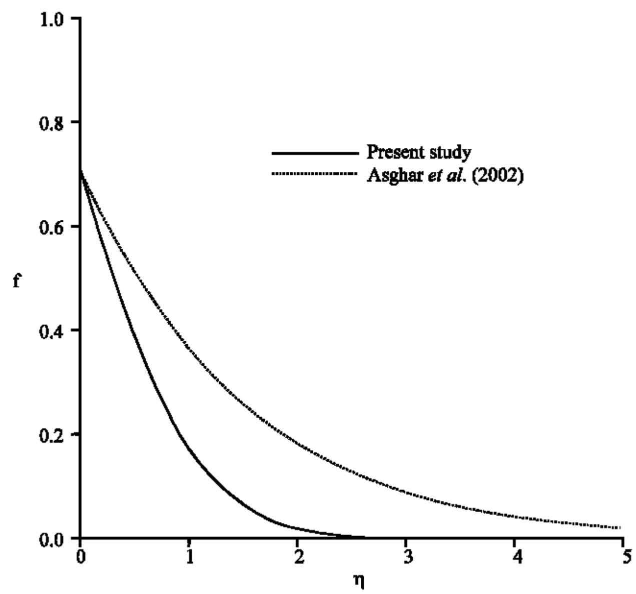

Fig. 4b: Comparison of velocity profiles for $\alpha = 1$, $\omega = 1$, $\tau = [\pi / 4]$ and $n = 0.1$

This increase in frequency is due to the motion of the plate towards the stationary observer (the Doppler-effect). The magnitude of the parameter *(a)* is found to decrease with an increase in the non-Newtonian parameter *(α)*, whereas the results of Asghar *et al*. (2002) show initially decrease and latter on increase with*α*. However, the magnitude of the parameter (*a*) increases with an increase in the magnetic interaction parameter *(n)*, which indicates reduction in the boundary layer thickness.

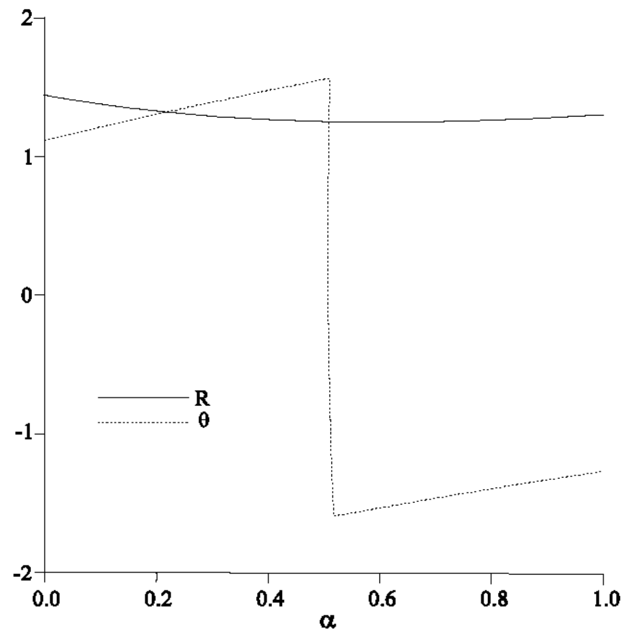

Fig. 5: The magnitude (R) and amplitude (θ) in the expression (24) for the shearing stress on the moving plate in the absence of magnetic field ($n = 0$ and $\omega = 1$)

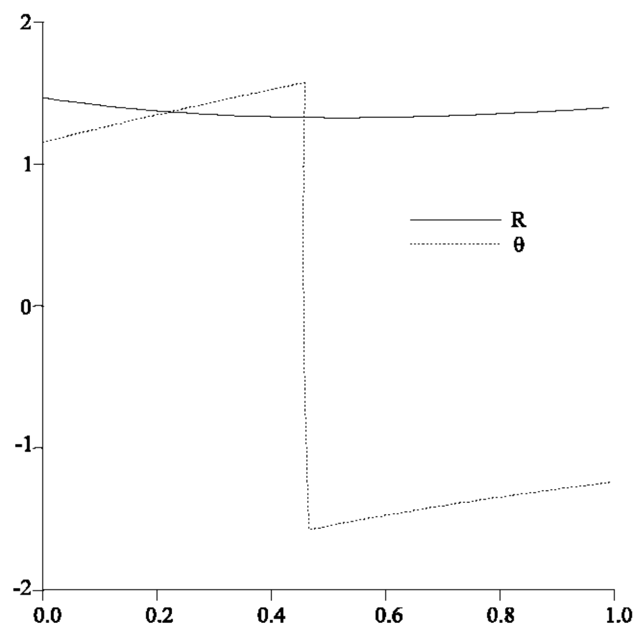

Fig. 6: The magnitude (R) and amplitude (θ) in the expression (24) for the shearing stress on the moving plate in the presence of magnetic field ($n = 0$ and $\omega = 1$)

Conclusions

The flow of an incompressible second-grade fluid with an oscillating rigid moving plate is examined in the presence of a uniform applied magnetic field. Simple and reliable numerical procedures are used to obtain the parameters for the derived velocity field expression. It is found that the thickness of the boundary layer increases with the non-Newtonian parameter and decreases with the magnetic interaction parameter.

References

Anonymous, 1970. System / 360 Scientific Subroutine Package. Version III. Programmer's Manual, Program Number 360 A -CM -03 X. IBM Application Program (GH20-0205-4), pp: 181-183.

Asghar, S., T. Hayat and A.M. Siddiqui, 2002. Moving boundary in a non- Newtonian fluid. Intl. J. Non-Linear Mech., 37: 75-80.

Carnahan, B., H.A. Luther, J.O. Wilkes, 1969. Applied Numerical Methods. John Wiley and Sons, Inc., New York.

Dunn, J.E. and R.L. Fosdick, 1974. Thermodynamics, stability and boundedness of fluids of complexity2 and fluids of second-grade. Arch. Rational Mech. Anal., 56: 191-252.

Foote, J.R., P. Puri and R.K. Kythe, 1987. Some exact solutions of the Stokes problem for an elastic-viscous fluid. Acta Mechanica, 68: 223-230.

Fosdick, R.L. and K.R. Rajagopal, 1979. Anomalous features in the model of second-order fluids. Arch. Rational Mech. Anal., 7: 145-152.

Hayat, T., S. Asghar and A.M. Siddiqui, 1998. Periodic unsteady flows of a non-Newtonian fluid. Acta Mechanica, 131: 169-175.

Hayat, T., S. Asghar and A.M. Siddiqui, 1999. On the moment of a plane disk in a non-Newtonian fluid. Acta Mechanica, 136: 125-131.

Hinch, E.J., 1992. Perturbation Methods. Cambridge University Press, Cambridge.

Rajagopal, K.R., 1982. A note on unsteady unidirectional flows of a non-Newtonian fluid. Intl. J. Non-Linear Mech., 17: 369-373.

Rajagopal, K.R. and T.Y. Na, 1983. On Stokes' problem for a non-Newtonian fluid. Acta Mechanica, 48: 233-239.

Rivlin, R.S. and J.L. Erickson, 1955. Stress deformation relations for isotropic materials. J. Rational Mech. Anal., 4: 323-425.