Trends in
Applied Sciences Research

Academic Journals Inc.

Unsteady Flow of Two Immiscible Fluids under an Oscillatory Time-dependent Pressure Gradient in a Channel with One Porous Floor

[1]Sai K.S., [2]N.S. Swamy, [3]H.R. Nataraja, [4]S.B. Tiwari and [4]B. Nageswara Rao

[1]Department of Mathematics, S.V.H. College of Engineering,
Machilipatnam-521 002, India
[2]Department of Computer Science and Engineering, Vivekananda Institute of
Technology, Gudimavu, Bangalore-560 074, India
[3]Compressor Group, Gas Turbine Research Establishment, C.V. Raman Nagar,
Bangalore-560 093, India
[4]Structural Analysis and Testing Group, Vikram Sarabhai Space Centre,
Trivandrum-695 022, India

Abstract: This study presents an exact solution for the flow of two immiscible fluids under a general oscillatory time-dependent pressure gradient in a channel with one porous floor. The oscillatory behavior of the time-dependent pressure gradient is expressed in terms of Fourier series. At the interface, continuity of velocities and shear stresses is assumed. Equations governing the flow are solved using the slip condition at the permeable interface whereas the generalized Darcy's law in the porous region. The unsteady flow depends upon the Reynolds numbers of the fluids, slip parameter and porous parameter. Analytical expressions are provided for the mass flow rate and wall shearing stresses. Numerical results are presented considering water and mercury as the two immiscible fluids for the uniform pressure gradient as well as for the sinusoidal time-dependent pressure gradient. Since the formulation of the problem is general, it is possible to examine the unsteady flow of any two immiscible fluids under any specified oscillatory time-dependent pressure gradient. This study will be useful in learning how the pressure and viscous forces exert their influence to produce different flow patterns.

Key words: Immiscible fluids, oscillatory pressure gradient, wall shearing stress

Introduction

Studies are made on the fluid flow in a porous channel to understand some practical phenomena such as transpiration cooling and gaseous diffusion (Berman, 1953; Sellars, 1955; Yuan, 1956). Wang (1971) studied the interesting problem of pulsatile flow in a porous channel bounded by rigid walls. The pulsatile flow between permeable walls is important in understanding blood flow in the circulation system, where the nutrients are supplied to tissues of various organs and the waste products are removed. Vajravelu *et al.* (2003) have analyzed the pulsatile flow of a viscous fluid between two permeable beds and obtained the velocity field as well as the volume flux. The fluid in their mathematical formulation is driven by an unsteady pressure gradient. The fluid flow between the permeable beds is governed by Navier-Stokes equations and with the Darcy's law. Darcy's law is the

---
Corresponding Author: Dr. B. Nageswara Rao, Structural Analysis and Testing Group, Vikram Sarabhai Space Centre,
Trivandrum-695 022, India

most commonly used law to study the diffusion of one constituent through another. It is an approximation to a basic balance law (Rajagopal and Tao, 1995).

The problem of flow through a channel formed by two parallel walls has also been examined by a number of researchers. Such a problem when the lower wall is permeable finds applications in the study of hydrology, petroleum industry, agricultural engineering and many others. Russel and Charless (1959) have examined the effect of a less viscous liquid such as water on the laminar flow of a high viscous liquid. They have shown that the pressure gradient for the flow of the high viscous liquid can be reduced, if water is injected into the channel. Considering the flow of a lighter fluid with less viscosity over a heavier fluid with high viscosity in a parallel plate channel, Bird *et al.* (1960) have shown that the fluid having less viscosity flows more rapidly than that with high viscosity. Kapur and Shukla (1964) have extended the analysis of Bird *et al.* (1960) for the flow of two immiscible liquids under time-dependent pressure gradient. Their results indicate that the interface velocity increases and the skin-friction at the plates decrease with the Reynolds numbers for the flow of two immiscible liquids. Beavers and Joseph (1967) have performed experiments to study the effect of tangential velocity in Poiseuille flow with permeable bed. Richardson (1971), Taylor (1971), Rajasekhara *et al.* (1975), have studied the effect of slip velocity in Couette flow. In these studies, the flow was influenced by a constant pressure gradient. Practical situations, however, demand consideration of unsteadiness in the flow where the unsteadiness may be caused by time-dependent pressure gradient.

Sai (1980) has examined the unsteady behavior of Poiseuille flow of viscous and incompressible fluid in a plane channel formed by two walls of which the upper one is solid and the lower one a porous of infinite thickness. The pressure gradient is assumed to vary exponentially with time. He has applied the slip condition at the porous surface and the generalized Darcy's law in the porous region. It is found that the mass flow rate for the time-independent pressure gradient is greater than the mass flow rate for the time-dependent pressure gradient and velocity is found to attain maximum value at the porous interface. Sai and Agarwal (1980) have investigated the flow of two immiscible fluids with different densities and viscosities under constant pressure gradient in a parallel plate channel bounded by a rigid wall at the top and a permeable bed of infinite thickness at the bottom. Using the Darcy's law for the flow in the permeable medium and Beavers and Joseph's slip condition (Beavers and Joseph, 1967) at the permeable interface it has been shown that the fluid velocity and mass flux increase with the permeability of the bed. Sai (1989, 1990) has studied the unsteady flow of two viscous, incompressible and immiscible fluids in a long parallel channel of which the upper one is impervious and lower one is porous of infinite thickness by taking a pressure gradient of the form Pexp(ct), where P and c are constants. The porous medium is assumed to be homogeneous and isotropic so that its permeability is constant. The positive value of c is taken for a mathematical convenience. However, such a positive constant could model a non-autonomous system in which, time increasing pressure gradient can be maintained by a source. Beavers and Joseph's slip condition at the permeable interface and the generalized Darcy's law in the porous region have been used. The analysis reveals that the flow depends upon the Reynolds numbers for the upper and lower fluids, slip parameter and the porous parameter. The two layer flow problems are of wide industrial importance, and examples of their application include in-tube condensers, petroleum industry, a few types of water-heat boilers and in the area of ground water technology.

In view of the practical situation of generation of pressure where crude is being pumped up and the pumping suddenly stops (the so-called 'water-hammer' phenomenon), the flow is oscillatory and it has to be generated through a sinusoidal pressure gradient. These studies will have applications in petroleum industry, where crude with water above may be confined between boundaries of rigid strata above and porous bed below.
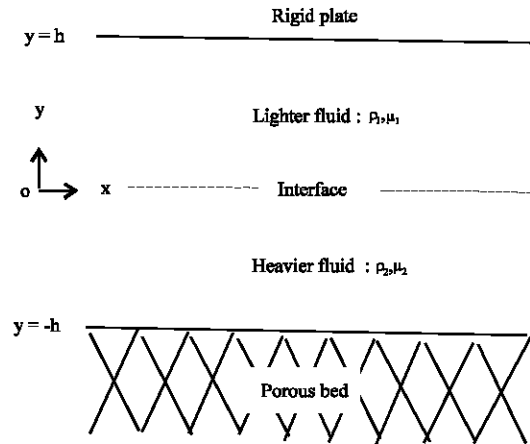
Fig. 1: Physical model

The objective of this study is to formulate the problem by considering the unsteady flow of two immiscible fluids under a general oscillatory time-dependent pressure gradient in a channel with one porous floor (Fig. 1). The oscillatory behavior of the time-dependent pressure gradient is expressed in terms of Fourier series. Exact solution of the equations governing the flow is obtained. Analytical expressions for the mass flow rate and wall shearing stresses are provided. Considering water and mercury as the two immiscible fluids, numerical results are presented for the case of the time-dependent sinusoidal pressure gradient. The present exact solution is useful to examine the unsteady flow of two immiscible fluids in a channel with one porous floor under any specified oscillatory time-dependent pressure gradient.

Formulation

The problem investigated here is the fully developed laminar flow of two immiscible fluids under an oscillatory time-dependent pressure gradient:

$$-\frac{\partial p}{\partial x} = P_0 + \sum_{n=1}^{\infty} \{P_{cn} \cos(n\Omega t) + P_{sn} \sin(n\Omega t)\}, \qquad (1)$$

in a parallel plate channel bounded by a rigid and a permeable bed. Here Ω is the frequency of oscillations. The oscillatory behavior of the time-dependent pressure gradient in Eq. (1) is expressed in terms of Fourier series, which is convenient to represent the actual variation of $\frac{\partial p}{\partial x}$ with respect to time. In the present formulation, x-axis is in the direction of the flow whereas y-axis is perpendicular to the flow direction. The origin is chosen mid-way between the plates (Fig. 1). The equations governing the flow are:

$$\rho_j \frac{\partial u_j}{\partial t} = -\frac{\partial p}{\partial x} + \mu_j \frac{\partial^2 u_j}{\partial y^2}, \qquad (2)$$

Where, μ_j is the velocity, μ_j is the viscosity and ρ_j is the density for, $j = 1,2$ fluids. The no-slip condition at the solid wall is:

$$u_1 = 0 \qquad\qquad at \qquad y = h \qquad (3)$$

Continuity of velocities and shear stresses is assumed at the interface:

$$u_1 = u_2, \mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} \qquad\qquad at \qquad y = 0 \qquad (4)$$

The slip condition at the porous bed is:

$$\frac{\partial u_2}{\partial y} = \frac{\alpha}{\sqrt{\kappa}} (u_2 - u_3) \qquad\qquad at \qquad y = -h \qquad (5)$$

The velocity, u_3, in porous medium obeys the Darcy's law (Yih, 1965):

$$\frac{\rho_2}{\epsilon} \frac{\partial u_3}{\partial t} = -\frac{\partial p}{\partial x} - \frac{\mu_2}{\kappa} u_3, \qquad (6)$$

Where, ϵ and κ are the porosity and permeability of the medium. α is the non-dimensional quantity for the porous material.

Defining the Reynolds number for two fluids, $R_j = \dfrac{h^2 \Omega \rho_j}{\mu_j}$, the porous parameter, $\sigma = \dfrac{h}{\sqrt{\kappa}}$ and

$\eta = \dfrac{y}{h}$, the solution of Eq. (1) to (6) for the governing flow is obtained as

$$\frac{\mu_1 u_1}{h^2} = P_0(1-\eta)(\frac{\eta}{2} + s_2) + \sum_{n=1}^{\infty} \{u_{cn} \cos(n\Omega t) + u_{sn} \sin(n\Omega t)\}, \qquad (7)$$

$$\frac{\mu_2 u_2}{h^2} = P_0 \{\frac{\eta}{2}(1-\eta) + s_2(s_1 - \eta)\} + \sum_{n=1}^{\infty} \{v_{cn} \cos(n\Omega t) + v_{sn} \sin(n\Omega t)\}, \qquad (8)$$

$$\frac{\mu_2 u_3}{h^2} = \frac{P_0}{\sigma^2} + \sum_{n=1}^{\infty} \{w_{cn} \cos(n\Omega t) + w_{sn} \sin(n\Omega t)\}, \qquad (9)$$

Wall shearing stresses $\left(\tau_1 = (\mu_1 \dfrac{\partial u_1}{\partial y})_{y=h} \text{ and } \tau_2 = (\mu_2 \dfrac{\partial u_2}{\partial y})_{y=-h} \right)$ at the top wall and at the permeable bed are:

$$\frac{\tau_1}{h} = -P_0 \left(\frac{1}{2} + s_2 \right) + \sum_{n=1}^{\infty} m_1 \{r_{cn} \cos(n\Omega t) + r_{sn} \sin(n\Omega t)\}, \qquad (10)$$

197

Table 1: Mass flow rate and wall shearing stresses for the oscillatory pressure gradient by considering water and mercury as two immiscible fluids $(-\frac{\partial p}{\partial x} = P\cos(\Omega t))$

Ωt	Reynolds number, R_1	Mass flow rate, $\frac{\mu_2 Q}{Ph^3}$	Wall shearing stress, $\frac{\tau_1}{Ph}$	Wall shearing stress, $\frac{\tau_2}{Ph}$
0	0.25	0.6100	-0.9758	2.0670
	0.50	0.2121	-0.7094	1.7100
	0.75	0.1130	-0.6335	1.5350
	1.00	0.0734	-0.5950	1.3610
$\pi/2$	0.25	-0.9378	0.0723	1.1350
	0.50	-0.3280	-0.0830	0.1645
	0.75	-0.1747	-0.1101	-0.4039
	1.00	-0.1019	-0.1243	-0.8293

$$\frac{\tau_2}{h} = P_0\left(\frac{3}{2} - s_2\right) + \sum_{n=1}^{\infty} m_2 \{s_{cn}\cos(n\Omega t) + s_{sn}\sin(n\Omega t)\}, \tag{11}$$

The mass flow rate $\left(Q = \int_{-h}^{h} u\,dy = \int_{-h}^{0} u_2\,dy + \int_{0}^{h} u_1\,dy\right)$ is:

$$\frac{\mu_2 Q}{h^3} = P_0\{2s_1s_2 - \frac{1}{6}(1+s_1)\} + \frac{1}{2}\sum_{n=1}^{\infty}\{q_{cn}\cos(n\Omega t) + q_{sn}\sin(n\Omega t)\}, \tag{12}$$

All the constants in Eq. (7) to (12) are given in the Appendix. It should be noted that when $\alpha \to o$ the solution of the problem becomes independent of ϵ and σ.

Numerical Results

Analytical solution is obtained for Eq. (1) to (6) governing the unsteady flow of two immiscible fluids under an oscillatory time-dependent pressure gradient in a channel with one porous floor. The unsteady flow depends upon the Reynolds numbers of fluids, slip parameter and the porous parameter. The solution of the problem for any oscillatory time-dependent pressure gradient can be obtained after determining the coefficients in the Fourier series of Eq. (1). The interface velocity of the two immiscible fluids is obtained from Eq. (7) or Eq. (8) at $\eta = 0$. Wall shearing stresses at the top wall and at the permeable bed are obtained from Eq. (10) and (11). The mass flow rate is obtained using Eq. (12).

Numerical results are obtained by considering water and mercury as two immiscible fluids. The ratios of viscosities and densities for the fluids at room temperature are: $\frac{\mu_2}{\mu_1} = 1.55$ and $\frac{\rho_2}{\rho_1} = 13.6$.

Values of the slip parameter (α) and the porosity (ϵ) are specified by assuming $\alpha\sigma$ and $\epsilon\sigma^2$ as unity. The mass flow rate (Q), and wall shearing stresses (τ_1, τ_2) are evaluated for the porous parameter, $\sigma = 100$.

For the case of uniform pressure gradient $\left(-\frac{\partial p}{\partial x} = P\right)$, the non-dimensional mass flow rate, $\frac{\mu_2 Q}{Ph^3} = 1.134$; the shear stress at the rigid plate, $\frac{\tau_1}{Ph} = -1.204$ and the shear stress at the porous bed,
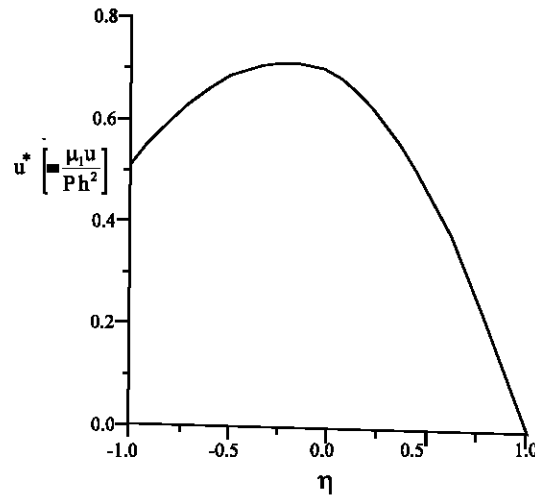
Fig. 2: Velocity $u^* \left(= \dfrac{\mu_1 u}{Ph^2} \right)$ across the flow of two immiscible fluids (water and mercury) for the

uniform pressure gradient : $-\dfrac{\partial p}{\partial x} = P$

$\dfrac{\tau_2}{Ph} = 0.7961.$ Shear stress (τ_2) at the porous bed ($\eta = -1$) clearly indicates the positive velocity

gradient, which implies increasing nature of the velocity near the porous bed. Shear stress (τ_1) at the rigid plate ($\eta = 1$) shows the negative velocity gradient, which implies decreasing nature of the velocity near the rigid plate. Due to difference in fluid properties, the velocity gradient may not vanish at the interface ($\eta = 0$) and hence, the interface velocity may not be maximum. Figure 2 shows the velocity distribution across the flow. Because of the slip condition at the porous bed ($\eta = -1$) and no-slip condition at the rigid plate ($\eta = 1$), the velocity increases from the porous bed gradually to a maximum value and start decreasing and finally vanishes at the rigid plate. The interface velocity is found to be lower than the maximum attained velocity in the heavier fluid.

Table 1 gives the mass flow rate and wall shearing stresses for the oscillatory pressure gradient, $-\dfrac{\partial p}{\partial x} = \rho \cos(\Omega t)$. At the instant of time, t = 0, the mass flow rate, the shear stress at the porous bed as well as the magnitude of the shear stress at rigid plate decrease with increasing the Reynolds number. At the instant of time, $t = \dfrac{\pi}{2\Omega}$, the magnitude of the mass flow rate decreases with increasing the Reynolds number. The positive and negative values of the wall shearing stresses at the porous bed and the rigid plate are mainly due to the oscillatory pressure gradient. Figure 3 shows the contour plot of velocity for the Reynolds number ($R_1 = 1$), which shows clearly the oscillatory behaviour.

Conclusions

This study presents an exact solution for the unsteady flow of two immiscible viscous incompressible fluids under a general oscillatory time-dependent pressure gradient in a channel with
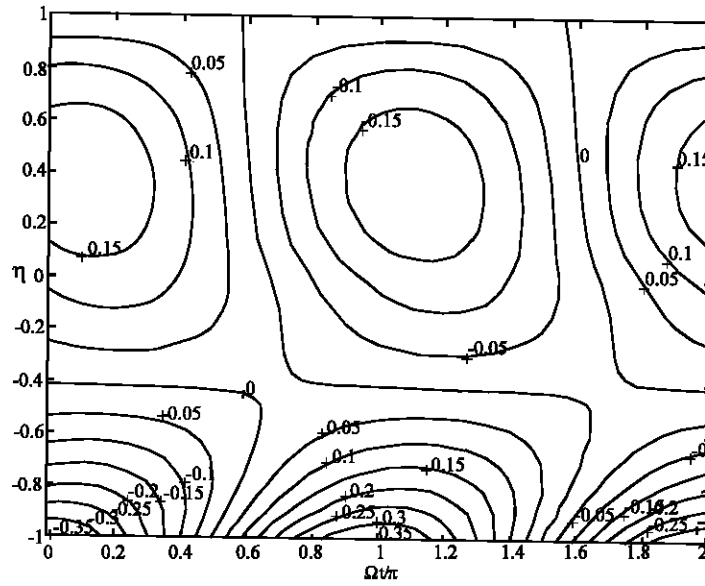
Fig. 3: Contour plot for the velocity $u^*\left(=\dfrac{\mu_1 u}{P h^2}\right)$ for the oscillatory pressure

gradient: $-\dfrac{\partial p}{\partial x} = P\cos(\Omega t)$ and $R_1 = 1$

one porous floor. Assuming water and mercury as the two immiscible fluids, numerical results are presented for the case of time-dependent sinusoidal pressure gradient. Since the formulation of the problem is general, the solution of the problem can be easily obtained for any immiscible fluids under any specified oscillatory time-dependent pressure gradient. The present analytical solution can provide not only a check against the computer experiments of the problem, but also provide a means of parametric study which is useful in learning how the pressure and viscous forces exert their influence to produce different flow patterns.

Appendix

Constants in Eq. (7) to (9) for velocity components are:

$u_{cn} = P_{cn}U_{11} + P_{sn}U_{12}$;

$u_{sn} = P_{sn}U_{11} - P_{cn}U_{12}$;

$v_{cn} = P_{cn}U_{21} + P_{sn}U_{22}$;

$v_{sn} = P_{sn}U_{21} - P_{cn}U_{22}$;

$w_{cn} = P_{cn}U_{31} + P_{sn}U_{32}$;

$w_{sn} = P_{sn}U_{31} - P_{cn}U_{32}$;

$U_{11} = A_{11}\beta_1(m_1\eta) - A_{14}\beta_2(m_1\eta) + A_{13}\beta_3(m_1\eta) - A_{12}\beta_4(m_1\eta)$;

$U_{12} = A_{12}\beta_1(m_1\eta) + A_{13}\beta_2(m_1\eta) + A_{14}\beta_3(m_1\eta) + A_{11}\beta_4(m_1\eta) - \dfrac{1}{2m_1^2}$;

$$U_{22} = A_{22}\beta_1(m_2\eta) + A_{23}\beta_2(m_2\eta) + A_{24}\beta_3(m_2\eta) + A_{21}\beta_4(m_2\eta) - \frac{1}{2m_2^2};$$

$$U_{21} = A_{21}\beta_1(m_2\eta) - A_{24}\beta_2(m_2\eta) + A_{23}\beta_3(m_2\eta) - A_{22}\beta_4(m_2\eta);$$

$$U_{31} = \frac{\varepsilon^2\sigma^2}{U_{33}};$$

$$U_{32} = -2\frac{\varepsilon}{U_{33}}m_2^2;$$

$$U_{33} = \varepsilon^2\sigma^4 + 4m_2^2;$$

$$A_{11} = -\frac{(e_1 d_1 + e_2 d_2)}{d_{12}};$$

$$A_{12} = \frac{(e_1 d_2 - e_2 d_1)}{d_{12}};$$

$$A_{13} = \frac{(f_1 d_1 + f_2 d_2)}{d_{12}};$$

$$A_{14} = \frac{(f_2 d_1 - f_1 d_2)}{d_{12}};$$

$$A_{21} = s_1 A_{11};$$

$$A_{22} = s_1\left(A_{12} - \frac{1}{2m_1^2}\right) + \frac{1}{2m_2^2};$$

$$A_{23} = \sqrt{s_3}\,A_{13};$$

$$A_{24} = \sqrt{s_3}\,A_{14};$$

$$d_{12} = d_1^2 + d_2^2;$$

$$d_1 = b_1\beta_1(m_1) - a_2\beta_2(m_1) + a_1\beta_3(m_1) - b_2\beta_4(m_1);$$

$$d_2 = b_2\beta_1(m_1) + a_1\beta_2(m_1) + a_2\beta_3(m_1) + b_1\beta_4(m_1);$$

$$e_1 = c_1\beta_3(m_1) - c_2\beta_2(m_1) + \frac{b_2}{2m_1^2};$$

$$e_2 = c_2\beta_3(m_1) + c_1\beta_2(m_1) - \frac{b_1}{2m_1^2};$$

$$f_1 = c_1\beta_1(m_1) - c_2\beta_4(m_1) - \frac{a_2}{2m_1^2};$$

$$f_2 = c_1\beta_4(m_1) + c_2\beta_1(m_1) + \frac{a_1}{2m_1^2};$$

$$a_1 = s_1 m_2\{\beta_3(m_2) - \beta_2(m_2)\} + \alpha\sigma\beta_1(m_2);$$

$$a_2 = s_1 m_2\{\beta_3(m_2) + \beta_2(m_2)\} + \alpha\sigma\beta_4(m_2);$$

$$b_1 = \sqrt{s_3}\,[m_2\{\beta_1(m_2) - \beta_4(m_2)\} + \alpha\sigma\beta_3(m_2)];$$

$$b_2 = \sqrt{s_3}\,[m_2\{\beta_1(m_2) + \beta_4(m_2)\} + \alpha\sigma\beta_2(m_2)];$$

$$c_1 = c_{11}\{a_2 - (1 - s_1)\alpha\sigma\beta_4(m_2)\} - \alpha\sigma U_{31};$$

$$c_2 = c_{11}\{a_1 - (1 + s_1)\alpha\sigma\beta_1(m_2)\} - \alpha\sigma\left(\frac{1}{2m_2^2} + U_{32}\right);$$

$$c_{11} = \frac{1}{2s_1}\left(\frac{s_1}{m_1^2} - \frac{1}{m_2^2}\right);$$

$$\sigma = \frac{h}{\sqrt{\kappa}}, \quad R_j = \frac{h^2 \Omega \rho_j}{\mu_j}, \quad m_j = \sqrt{\frac{nR_j}{2}};$$

$$s_1 = \frac{\mu_2}{\mu_1}; \quad s_2 = \frac{\frac{3}{2} + \frac{\alpha}{\sigma} + \alpha\sigma}{1 + (1 + s_1)\alpha\sigma}; \quad s_3 = \frac{\mu_2 \rho_1}{\mu_1 \rho_2};$$

$$\beta_1(\xi) = \cosh(\xi)\cos(\xi);$$

$$\beta_2(\xi) = \cosh(\xi)\sin(\xi);$$

$$\beta_3(\xi) = \sinh(\xi)\cos(\xi);$$

$$\beta_4(\xi) = \sinh(\xi)\sin(\xi).$$

Constants in Eq. (10) and (11) for wall shearing stresses are:

$$r_{cn} = P_{cn}V_{11} + P_{sn}V_{12};$$

$$r_{sn} = P_{sn}V_{11} - P_{cn}V_{12};$$

$$s_{cn} = P_{cn}V_{21} + P_{sn}V_{22};$$

$$s_{sn} = P_{sn}V_{21} - P_{cn}V_{22};$$

$$V_{11} = (A_{13} - A_{14})\beta_1(m_1) - (A_{11} + A_{12})\beta_2(m_1) + (A_{11} - A_{12})\beta_3(m_1) - (A_{13} + A_{14})\beta_4(m_1);$$

$$V_{12} = (A_{13} + A_{14})\beta_1(m_1) + (A_{11} - A_{12})\beta_2(m_1) + (A_{11} + A_{12})\beta_3(m_1) + (A_{13} - A_{14})\beta_4(m_1);$$

$$V_{21} = (A_{23} - A_{24})\beta_1(m_2) + (A_{21} + A_{22})\beta_2(m_2) - (A_{21} - A_{22})\beta_3(m_2) - (A_{23} + A_{24})\beta_4(m_2);$$

$$V_{22} = (A_{23} + A_{24})\beta_1(m_2) - (A_{21} - A_{22})\beta_2(m_2) - (A_{21} + A_{22})\beta_3(m_2) + (A_{23} - A_{24})\beta_4(m_2).$$

Constants in Eq. (12) for the mass flow rate are:

$$q_{cn} = \frac{s_1}{m_1}(P_{cn}W_{11} + P_{sn}W_{12}) + \frac{1}{m_2}(P_{cn}W_{21} + P_{sn}W_{22});$$

$$q_{sn} = \frac{s_1}{m_1}(P_{sn}W_{11} - P_{cn}W_{12}) + \frac{1}{m_2}(P_{sn}W_{21} - P_{cn}W_{22});$$

$$W_{11} = V_{12} - A_{13} - A_{14};$$

$$W_{12} = -V_{11} + A_{13} - A_{14} - \frac{1}{m_1};$$

$$W_{21} = -V_{22} + A_{23} + A_{24};$$

$$W_{22} = V_{21} - A_{23} + A_{24} + \frac{1}{m_2}.$$

References

Beavers, G.S. and D.D. Joseph, 1967. Flow over a naturally permeable bed. J. Fluid Mechanics, 30: 197-207.

Berman, A.S., 1953. Laminar flow in channels with porous walls. J. Applied Phys., 24: 1232-1235.

Bird, R.B., W.E. Stewart and E.N. Lightfoot, 1960. Transport Phenomena. John Wiley and Sons, New York.

Kapur, J.N. and J.B. Shukla, 1964. On the unsteady flow of two incompressible immiscible fluids between two plates. J. Applied Mathematics and Mechanics., 44: 268-269.

Rajagopal, K.R. and L. Tao, 1995. Mechanics of Mixtures. Series on Advances in Mathematics for Applied Scviences, World Scientific Publishing Co., Inc, River Edge, NJ, 35

Rajasekhara, B.M., N. Rudraiah and K. Ramaiah, 1975. Couette flow over a naturally permeable bed. J. Math. Phys. Sci., 9: 49-56.

Richardson, S., 1971. A model for the boundary condition of a porous material. J. Fluid Mechanics, 49: 327-337.

Russell, T.W.R., and M.E. Charles, 1959. The effect of the less viscous liquid in the laminar flow of two immiscible liquids. Can. J. Chem. Eng., 37: 18-24.

Sai, K.S., 1980. On the unsteady flow of incompressible fluid over a naturally permeable bed. J. Math. Phys. Sci., 14: 599-609.

Sai, K.S. and J.P. Agarwal, 1980. Flow of incompressible immiscible fluids over a naturally permeable bed. Proceedings of the International Symposium on the Non-linear Continuum Mechanics, Kharagpur, India, pp: 105-115.

Sai, K.S., 1989. Some flow problems of single and two fluid systems with porous boundaries. Ph. D Thesis, Indian Institute of Technology, Kharagpur, India.

Sai, K.S., 1990. Unsteady flow of two immiscible viscous fluids over a naturally permeable bed. Defense Sci. J., 40: 183-189.

Sellars, J.R., 1955. Laminar flow in channels with porous walls at high suction reynolds numbers. J. Applied Phys., 26: 489-490.

Taylor, G.I., 1971. A model for the boundary condition of a porous material. J. Fluid Mechanics, 49: 319-326.

Vajravelu, K., K. Ramesh, S. Sreenadh and P.V. Arunachalam, 2003. Pulsatile flow between permeable beds. Intl. J. Non-Linear Mechanics, 38: 999-1005.

Wang, Y.C., 1971. Pulsatile flow in a porous channel. Trans. ASME J. Applied Mechanics, 38: 553-555.

Yih, C.S., 1965. Dynamics of Non-homogeneous Fluids. Macmillan Company, New York.

Yuan, S.W., 1956. Further investigation in laminar flow in channels with porous walls. J. Applied Phys., 27: 267-269.