Trends in Applied Sciences Research

ISSN 1819-3579

Academic Journals Inc.

www.academicjournals.com

Trends in Applied Sciences Research 1 (1): 48-60, 2006
ISSN 1819-3579

Post-buckling of a Thin Film Strip Delamination in a Composite Laminate

[1]K.G. Muthurajan, [2]K. Sankaranarayanasamy,
[3]S.B. Tiwari and [3]B. Nageswara Rao
[1]Faculty of Mechanical Engineering, National Engineering College,
Kovilpatti-628 503, India
[2]Faculty of Mechanical Engineering, National Institute of Technology,
Tiruchirappalli-620 015, India
[3]Structural Analysis and Testing Group, Vikram Sarabhai Space Centre,
Trivandrum-695 022, India

Abstract: Studies are made to understand the delamination growth of general laminates with general loading conditions. Post-buckling solutions are obtained for a laminate with clamped ends applicable to thin film strip delamination in a base laminate under uniform membrane loads. The strain energy release rate at the crack-tip (G) is derived in terms of the critical equivalent base laminate strain at the onset of the buckling (ε^*_{cr}) and the applied equivalent strain (ε^*). It is also expressed in terms of the maximum amplitude (W_{max}) of the delaminated layer. A Griffith-type fracture criterion with constant specific fracture energy (G_c) of the material is used to govern the delamination growth. The stability characteristics of the delamination growth are discussed. The maximum amplitude (W_{max}) of the delaminated layer increases with the applied load without enhancement in the length of the delamination for the values of G less than G_c. Initiation of the delamination growth can be expected when the value of G is very close to G_c.

Key words: Buckling load, composite laminate, delamination, strain energy release rate

Introduction

Delamination represents a common and characteristic flaw in composite laminates that may be introduced during processing or subsequent service conditions. The local instability of composite laminates in the vicinity of interlaminar defects and the potential for delamination initiation and growth may induce significant strength reduction under compressive loadings. Therefore, a fundamental understanding of the mechanisms governing delamination initiation and growth is required to develop appropriate failure criteria to assess defect criticality.

Chai *et al*. (1981), Yin and Wang (1984) and Yin (1988) employed one-dimensional models to the problems of buckling induced delamination in composite plates. Bottega and Maewal (1983) and Yin (1985) examined the problems considering axisymmetric models, whereas, Chai and Babcock (1985) and Simites *et al*. (1985) utilized two-dimensional models. Davidson (1991) and Suemasu *et al*. (1995) have studied delamination in plates using the Rayleigh-Ritz method. Kardomateas (1993), Kardomateas and Pelegri (1994, 1996) and Bruno and Greco (2000) have employed the perturbation technique. Larsson (1991a, b) numerically solved the governing equations for delamination buckling and growth in circular and annular orthotropic plates.

Corresponding Author: Dr. B. Nageswara Rao, Structural Analysis and Testing Group,
Vikram Sarabhai Space Centre, Trivandrum-695 022, India
Tel: 91 471 2565831 Fax: 91 471 2564181

Chen (1991), Chattopadhyay and Gu (1994) and Kyoung and Kim (1995) have applied shear deformation theory for delamination buckling of composite laminates. Transverse shear deformation effects were found to significantly lower the delamination buckling load. Whitcomb (1981, 1986) and Whitcomb and Shivakumar (1989) have presented a finite element formulation for the instability related delamination growth. Srivatsa *et al.* (1993) have analyzed the problem using coarse mesh in their finite element model. Cochelin and Potier-Ferry (1991), Klug *et al.* (1996), Kim (1997), Kim and Hong (1997) and Hu *et al.* (1999) have applied higher order plate theories to examine the buckling induced elliptic and circular delaminations in composite laminates. Lee *et al.* (1995a, b) have studied these problems utilizing the layer-wise theories.

Perugini *et al.* (1999) have carried out two-dimensional finite element analyses of through-width delamination. Mukherjee *et al.* (1994), Riccio *et al.* (2000) and Shen *et al.* (2001) have carried out three-dimensional finite element analysis of instability related delamination growth in laminated composites. Nilsson *et al.* (1995, 2001a and b) have made comparison of finite element simulation with experiments. These computational models can be expensive for use at the design stage. Although the laminated structure has been modelled in several numerical studies using finite element methods, these studies do not undertake parametric analysis to determine the effects of layer structure and stacking sequence upon the buckling load, the post-buckling deformation and the associated stress intensity factors or energy release rates. It is well known that bending-stretching coupling reduces the buckling loads of composite laminates. It is also known that since delamination often occurs between two dissimilar layers or two identical layers with different orientations, the singularities of the stress at the tip of delamination are generally more often complex than those associated with cracks in a homogeneous medium.

This research examines one dimensional delamination models with arbitrary laminated structure to obtain post buckling solutions for a laminate with clamped ends, which are applicable to a thin-film strip delamination in a base laminate subjected to uniform membrane loads. The energy-release rate at the crack-tip is evaluated to determine the stability characteristics of delamination growth.

Analysis

A thin laminated plate whose middle plane coincides with co-ordinate plane z = 0 of a rectangular co-ordinate system (x, y, z), is considered. The equilibrium equations of generally laminated plate are:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{1}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \tag{2}$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \tag{3}$$

Where,

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \kappa \end{Bmatrix} \tag{4}$$

The components in the matrices of the stress and moment resultants, viz.,

$N = \{N_x, N_y, N_{xy}\}^T$ and $M = \{M_x, M_y, M_{xy}\}^T$ are defined as:

$$(N_k, M_k) = \int_{-h/2}^{h/2} (1, z)\, \sigma_k dz \quad (k = x,y,xy)$$

N_x, N_y, N_{xy} are membrane forces per unit length, M_x, M_y, M_{xy} are the bending and twisting moments per unit length. The element A_{ij}, B_{ij} and D_{ij} (i, j = 1, 2, 6) in the 3×3 symmetric matrices of A, B and D in Eq. 4 are defined as:

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} (1, z, z^2)\, Q_{ij}\, dz \quad (i,j = 1,2,6).$$

The element A_{ij}, B_{ij} and D_{ij}, are, respectively, the membrane rigidities, coupling rigidities and flexural rigidities of the plate. h is the thickness of the laminate. Q_{ij} are the reduced stiffness coefficients, which can be related to the more familiar engineering moduli by $\sigma = \{Q_{ij}\}\varepsilon$. The components in the matrices of in-plane stress and strain are: $\sigma = \{\sigma_x, \sigma_y, \sigma_{xy}\}^T$ and $\varepsilon = \{\varepsilon_x, \varepsilon_y, \varepsilon_{xy}\}^T$. The matrices of strains (ε) and curvature changes (κ) are written by considering von Kármán type of geometric nonlinearity, as:

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\right)^2 \\ \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y}\right)^2 \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} + \dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y} \end{Bmatrix}, \tag{5}$$

$$\kappa = \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = -\begin{Bmatrix} \dfrac{\partial^2 w}{\partial x^2} \\ \dfrac{\partial^2 w}{\partial y^2} \\ 2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}, \tag{6}$$

u, v and w are the displacements at the reference plane (z = 0) along x, y and z directions. ε_x, ε_y, ε_{xy} are the reference surface strains. κ_x, κ_y, κ_{xy} are the curvature changes.

Eq. 4 is rewritten as:

$$\begin{Bmatrix} \varepsilon \\ M \end{Bmatrix} = \begin{bmatrix} A^* & B^* \\ -(B^*)^T & D^* \end{bmatrix} \begin{Bmatrix} N \\ \kappa \end{Bmatrix} \tag{7}$$

Where, $[A^*] = [A]^{-1}$, $[B^*] = -[A]^{-1}[B]$ and $[D^*] = [D] + [B]\,[B^*]$.

The Airy stress function (ϕ), which satisfies Eq. 1 and 2 is defined by

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial^2 \phi}{\partial y^2} \\ \dfrac{\partial^2 \phi}{\partial x^2} \\ -\dfrac{\partial^2 \phi}{\partial x \partial y} \end{Bmatrix} \tag{8}$$

The compatibility equation is derived from relation (5) as:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \tag{9}$$

Using Eq. 7 and 8 in Eq. 3 and 9, one obtains

$$L_2(\phi) - L_3(w) - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 + \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} = 0 \tag{10}$$

$$L_1(w) + L_3(\phi) - L(\phi, w) = 0 \tag{11}$$

where the differential operators are:

$$L_1 = D_{11}^* \frac{\partial^4}{\partial x^4} + 4D_{16}^* \frac{\partial^4}{\partial x^3 \partial y} + 2(D_{12}^* + 2D_{66}^*) \frac{\partial^4}{\partial x^2 \partial y^2} + 4D_{26}^* \frac{\partial^4}{\partial x \partial y^3} + D_{22}^* \frac{\partial^4}{\partial y^4}$$

$$L_2 = A_{22}^* \frac{\partial^4}{\partial x^4} - 2A_{26}^* \frac{\partial^4}{\partial x^3 \partial y} + (2A_{12}^* + A_{66}^*) \frac{\partial^4}{\partial x^2 \partial y^2} - 2A_{16}^* \frac{\partial^4}{\partial x \partial y^3} + A_{11}^* \frac{\partial^4}{\partial y^4}$$

$$L_3 = B_{21}^* \frac{\partial^4}{\partial x^4} + (2B_{26}^* - B_{61}^*) \frac{\partial^4}{\partial x^3 \partial y} + (B_{11}^* + 2B_{22}^* - 2B_{66}^*) \frac{\partial^4}{\partial x^2 \partial y^2} + (2B_{16}^* - B_{62}^*) \frac{\partial^4}{\partial x \partial y^3} + B_{12}^* \frac{\partial^4}{\partial y^4}$$

$$L(\phi, w) = \frac{\partial^2 \phi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 \phi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y}.$$

Therefore, Eq. 10 and 11 are two coupled governing equations of arbitrarily laminated thin plates.

Buckling of the Delaminated Layer

The growth of a thin laminates strip delamination in a thick base laminate with clamped edges has been studied. The delamination model is illustrated in Fig. 1. It is assumed that the stiffness of the

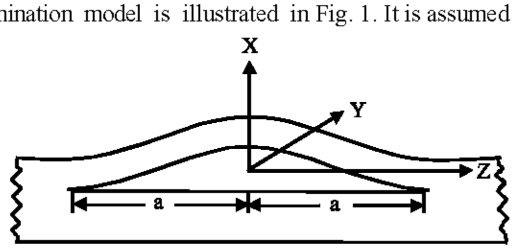

Fig. 1: Buckling of a thin-film strip delamination

delaminated layer is negligibly small compared to that of the base laminate, such that the membrane strains in the base laminate are not affected due to the buckling and growth of the thin delaminated layer. Since there is no out-of-plane transverse deflection in the base laminate, the deflection and slope of the delaminated layer at the delamination front are both zero. This simplified model is called 'thin-film delamination'.

A transverse deflection function w, is assumed as:

$$w = \frac{w_{max}}{2}\left\{1 + \cos\frac{\pi x}{a}\right\}, \tag{12}$$

which satisfies the geometric boundary conditions:

$$w = \frac{\partial w}{\partial x} = 0 \text{ at } x = \pm a. \tag{13}$$

Substituting Eq. 12 into Eq. 10 and using the in-plane boundary conditions:

$$N_x = \frac{\partial^2 \phi}{\partial y^2} = -P \quad \text{at } x = \pm a, \tag{14}$$

$$N_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y} = S \quad \text{at } x = \pm a, \tag{15}$$

The Airy stress function, ϕ is obtained as:

$$\phi = -P\frac{y^2}{2} - Sxy + \frac{B_{21}^*}{A_{22}^*}w = -P\frac{y^2}{2} - Sxy + \frac{B_{21}^*}{A_{22}^*}\frac{w_{max}}{2}\left\{1 + \cos\frac{\pi x}{a}\right\} \tag{16}$$

Substituting w and ϕ into Eq. 11 and applying Galerkin's method, one can get

$$P = \left(D_{11}^* + \frac{B_{21}^{*\,2}}{A_{22}^*}\right)\left(\frac{\pi}{a}\right)^2 = D_{eff}\left(\frac{\pi}{a}\right)^2, \tag{17}$$

where D_{eff} is the equivalent bending rigidity.

For a thin-film laminate having clamped edges at x = ± a, Eq. 12 represents the lowest buckling mode and Eq. 17 gives the corresponding axial buckling load.

Post-buckling of the Delaminated Layer

Problems of general laminates with different symmetric and unsymmetric lay-ups and more general loading conditions including axial compressive strain, transverse membrane strain and membrane shearing strain are considered. However, symmetric laminates with different fibre orientations and stacking sequences are commonly used in structures and as a consequence of delamination, an unsymmetric lay-up may appear in the delaminated layer. Hence the delamination growth of general laminates with general loading conditions is studied here.

Using the transverse deflection function (w) assumed in Eq. 12, the curvature changes (κ_x, κ_y, κ_{xy}) obtained from Eq. 6 are:

$$\kappa_x = \frac{w_{max}}{2}\left(\frac{\pi}{a}\right)^2 \cos\frac{\pi x}{a} \quad (18)$$

$$\kappa_y = 0 \quad (19)$$

$$\kappa_{xy} = 0 \quad (20)$$

Using the Airy stress function (ϕ) obtained in Eq. 16, the in-plane stress resultants (N_x, N_y and N_{xy}) found from Eq. 8 are:

$$N_x = -P \quad (21)$$

$$N_y = -\left(\frac{B^*_{21}}{A^*_{22}}\right)\left(\frac{\pi}{a}\right)^2 \frac{W_{max}}{2} \cos\frac{\pi x}{a} \quad (22)$$

$$N_{xy} = S \quad (23)$$

Unlike N_x and N_{xy}, the transverse membrane force N_y is generally not a constant, if bending-stretching coupling is present.
Using Eq. 18-23 in Eq. 7, the strains of the middle surface are written in the form

$$\varepsilon_x = \varepsilon_\alpha + \varepsilon_\xi \cos\frac{\pi x}{a} \quad (24)$$

$$\varepsilon_y = \varepsilon_\beta \quad (25)$$

$$\varepsilon_{xy} = \varepsilon_\gamma + \varepsilon_\eta \cos\frac{\pi x}{a} \quad (26)$$

where $\varepsilon_\alpha = -A^*_{11}P + A^*_{16}S$; $\varepsilon_\beta = -A^*_{21}P + A^*_{26}S$; $\varepsilon_\gamma = -A^*_{61}P + A^*_{66}S$

$$\varepsilon_\xi = \left(-\frac{B^*_{21}}{A^*_{22}}A^*_{12} + B^*_{11}\right)\left(\frac{\pi}{a}\right)^2 \text{ and } \varepsilon_\eta = \left(-\frac{B^*_{21}}{A^*_{22}}A^*_{62} + B^*_{61}\right)\left(\frac{\pi}{a}\right)^2 \frac{w_{max}}{2}$$

Equation 25 indicates that ε_y is constant. The in-plane stress resultants are obtained from Eq. 4 using Eq. 18-20 for the curvature changes and Eq. 24-26 for the strains of the middle surface. These are substituted in Eq. 1 and 2 to obtain the following relations:

$$A_{11}\varepsilon_\xi + A_{16}\varepsilon_\eta + B_{11}\frac{w_{max}}{2}\left(\frac{\pi}{a}\right)^2 = 0 \quad (27)$$

$$A_{16}\varepsilon_\xi + A_{66}\varepsilon_\eta + B_{16}\frac{w_{max}}{2}\left(\frac{\pi}{a}\right)^2 = 0 \quad (28)$$

Using Eq. 18-28 in Eq. 4, one obtains the relations:

$$A_{11}\varepsilon_{\alpha} + A_{12}\varepsilon_{\beta} + A_{16}\varepsilon_{\gamma} = -P, \tag{29}$$

$$A_{16}\varepsilon_{\alpha} + A_{26}\varepsilon_{\beta} + A_{66}\varepsilon_{\gamma} = S, \tag{30}$$

as well as the moment resultants (M_x, M_y and M_{xy}), which are being used in Eq. 3 to obtain the relation:

$$B_{11}\varepsilon_{\xi} + B_{16}\varepsilon_{\eta} + D_{11}\frac{w_{max}}{2}\left(\frac{\pi}{a}\right)^2 - \frac{Pw_{max}}{2} = 0 \tag{31}$$

Using Eq. 17 for P in Eq. 31, one can write a relation:

$$B_{11}\varepsilon_{\xi} + B_{16}\varepsilon_{\eta} + (D_{11} - D_{eff})\frac{w_{max}}{2}\left(\frac{\pi}{a}\right)^2 = 0 \tag{32}$$

The above relations 27 to 32 will be useful later while simplifying the energy-release rate expression for a thin-film strip delamination.

The maximum amplitude W_{max} may be obtained from the kinematical relation for the total axial shortening:

$$2a\varepsilon_0 = -\int_{-a}^{a}\left(\varepsilon_x - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)dx,$$

which implies that

$$\left(\frac{\pi w_{max}}{4a}\right)^2 = \varepsilon_0 + \varepsilon_{\alpha}, \tag{33}$$

where ε_0 is the compressive axial strain in the base plate.

Evaluation of the Strain Energy Release Rate at the Crack-tip

The results of the buckling analysis of a laminate are applied to an across-the width, thin-film delamination in a thick base laminate, which is subjected only to the following constant membrane strains:

$$\varepsilon_x = -\varepsilon_0 \tag{34}$$

$$\varepsilon_y = \varepsilon_{\beta} \tag{35}$$

$$\varepsilon_{xy} = \varepsilon_{\gamma} \tag{36}$$

When these membrane strains are sufficiently large, the delaminated layer of length 2a buckles and deflects away from the base laminate. The stiffness of the delaminated layer, characterized by Eq. 4, is assumed to be small compared to the stiffness of the base laminate so that the buckling of the layer does not affect the existing state of membrane strain in the base laminate. Hence, the layer has zero-slope along the two end edges where it joins the base laminate. In above post-buckling deformation of the layer, ε_β and ε_γ are, respectively, the specified membrane strains for ε_y and ε_{xy} in the base laminate.

Although the obtained post-buckling solution from the classical lamination theory does not deliver the asymptotic stress field near the crack-tip, it can be used to determine the energy-release rate of delamination theory. Yin and Wang (1984) and Yin (1985; 1988) was pointed out that the energy release rate is not affected by superposition of a non-singular stress or deformation field. If one superimposes the following constant strain field:

$$\varepsilon_x = \varepsilon_0 \tag{37}$$

$$\varepsilon_y = -\varepsilon_\beta \tag{38}$$

$$\varepsilon_{xy} = -\varepsilon_\gamma \tag{39}$$

upon the post-buckling solution of the delaminated layer and upon the constant strain of the base laminate, then the base laminate becomes stress-free whereas the delaminated layer is now subjected to the strain field

$$\varepsilon_{xt} = \varepsilon_0 + \varepsilon_x + z\,\kappa_x = \varepsilon_0 + \varepsilon_\alpha + \left(\varepsilon_\xi + z\frac{W_{max}}{2}\left(\frac{\pi}{a}\right)^2\right)\cos\frac{\pi x}{a} \tag{40}$$

$$\varepsilon_{yt} = 0 \tag{41}$$

$$\varepsilon_{yxt} = \varepsilon_\eta \cos\frac{\pi x}{a} \tag{42}$$

Hence,

$$\sigma_x = Q_{11}\varepsilon_{xt} + Q_{16}\varepsilon_{xyt} \tag{43}$$

$$\sigma_{xy} = Q_{16}\varepsilon_{xt} + Q_{66}\varepsilon_{xyt} \tag{44}$$

The energy-release can be evaluated by means of the J-integral (Yin and Wang, 1984). After superimposing the constant strain field of Eq. 37-39, the base laminate becomes stress-free and consequently the two segments of the path that intersect the base laminate do not contribute to the J-integral. The contribution from the segment of the path which intersects the delaminated layer at x = 0 delivers the energy-release rate,

$$\begin{aligned} G = \int dJ = \frac{1}{2}\int_{-h/2}^{h/2}\left(\sigma_x\varepsilon_{xt} + \sigma_{xz}\varepsilon_{xyt}\right)_{x=0} dz \\ = \frac{A_{11}}{2}\left(\varepsilon_0 + \varepsilon_\alpha + \varepsilon_\xi\right)^2 + A_{16}\left(\varepsilon_0 + \varepsilon_\alpha + \varepsilon_\xi\right)\varepsilon_\eta + \frac{A_{66}}{2}\varepsilon_\eta^{\,2} \\ + B_{11}\left(\varepsilon_0 + \varepsilon_\alpha + \varepsilon_\xi\right)\frac{W_{max}}{2}\left(\frac{\pi}{a}\right)^2 + B_{16}\frac{W_{max}}{2}\left(\frac{\pi}{a}\right)^2\varepsilon_\eta \\ + D_{11}\frac{W_{max}}{2}\left(\frac{\pi}{a}\right)^4 \end{aligned} \tag{45}$$

Using the relations 27 to 32, one can simplify Eq. 45 in the form

$$G = \frac{A_{11}}{2}(\varepsilon_0 + \varepsilon_\alpha)^2 + \frac{w_{max}^2}{8}\left(\frac{\pi}{a}\right)^4 D_{eff} \tag{46}$$

Using Eq. 33 in Eq. 46, one can express the energy-release rate as

$$G = \frac{1}{2}(\varepsilon_0 + \varepsilon_\alpha)\left\{A_{11}(\varepsilon_0 + \varepsilon_\alpha) + 4D_{eff}\left(\frac{\pi}{a}\right)^2\right\} \tag{47}$$

The net effect of the three membrane strains on the base laminate can be represented by a single 'loading' parameter, ε^*, which is referred to as the 'equivalent base laminate strain' and is defined as (Chai *et al*., 1981):

$$\varepsilon^* = \varepsilon_0 - \frac{(A_{12}\varepsilon_\beta + A_{16}\varepsilon_\gamma)}{A_{11}}.$$

Using Eq. 17 and 29, one can write the loading parameter as

$$\varepsilon^* = \varepsilon_0 + \varepsilon_\alpha + \varepsilon_{cr}^* \tag{48}$$

Where the critical equivalent base laminate strain at the onset of the buckling is defined by

$$\varepsilon_{cr}^* = \frac{D_{eff}}{A_{11}}\left(\frac{\pi}{a}\right)^2 \tag{49}$$

Using Eq. 48 and 49 in Eq. 47, one can express

$$G = \frac{A_{11}}{2}(\varepsilon^* - \varepsilon_{cr}^*)(\varepsilon^* + 3\varepsilon_{cr}^*) \tag{50}$$

A Griffith-type fracture criterion is used to govern the delamination growth. If the equivalent strain, ε^*, applied on the base laminate is further increased from that of the buckling state, such that the energy release rate at the delamination front equals to the specific fracture energy (G_c) of the material, delamination growth occurs. Thereafter the equivalent strain (ε_0^*) to initiate delamination growth is maintained constant as the delamination growth proceeds. Eq. 50 preserves the formal simplicity of the known formula for a thin-film delamination in a homogeneous isotropic or specially orthotropic plate (Chai *et al*., 1981; Yin and Wang, 1984). The present result (50), which takes into account the laminated structure of the layer and membrane shear loading in the base laminate, generally includes contributions to the energy-release rate from all three modes of fracture.

Using Eq. 33 in Eq. 47, the energy release rate can also be expressed in terms of the maximum amplitude (W_{max}) as

$$G = \frac{w_{max}^2}{512}(A_{11}w_{max}^2 + 64D_{eff})\left(\frac{\pi}{a}\right)^4 \tag{51}$$

Stability Characteristics of the Delamination Growth

The post buckling solutions for a laminate with clamped ends are applied to a thin strip delamination in a base laminate subjected to uniform membrane loads. The energy release rate (G) at the crack tip is evaluated to determine the stability characteristics of delamination growth. From the present post-buckling solution of laminated strip delamination for the thin-film model, the lowest value of equivalent base laminate strain (ε^* ol) can be obtained from the conditions:

$$G = G_c \text{ and } \frac{dG}{da} = 0 \quad \text{at } a = a_c \tag{52}$$

The stains (ε^*_{ol}), the delamination length (a_c) and the corresponding maximum amplitude (W^*_{max}) are obtained from the above conditions as follows:

$$\varepsilon_{ol}{}^* = \sqrt{\frac{3}{2}\frac{G_c}{A_{11}}},\ a_c = \pi\sqrt{\frac{3D_{eff}}{A_{11}}}\left(\frac{2}{3}\frac{A_{11}}{G_c}\right)^{1/4} \text{ and } W^*_{max} = \sqrt{\frac{32D_{eff}}{A_{11}}} \tag{53}$$

The limit strain (ε^*_{olim}) beyond which the strain energy release rate, G, exceeds the specific fracture energy (G_c), as $a\to\infty$, is obtained from the conditions: G (a_{lim}) $\cong G_c$ and $G\to G_c$ as $a\to\infty$. The strain (ε^*_{olim}), the delamination length (a_{lim}) and the corresponding maximum amplitude (W_{maxl}) are obtained from these conditions as follows:

$$\varepsilon^*_{o\lim} = \sqrt{\frac{2G_c}{A_{11}}},\ a_{\lim} = \pi\sqrt{\frac{3}{2}\frac{D_{eff}}{A_{11}}}\left(\frac{A_{11}}{2G_c}\right)^{1/4} \text{ and } W_{max1} = \sqrt{\frac{8D_{eff}}{A_{11}}} \tag{54}$$

The equivalent base laminate strains required to initiate delamination growth is an increasing function of the specific fracture energy, G_c.

Results and Discussion

The effect of laminated structure upon the buckling strength of the delamination is examined by unsymmetric angle ply as well as cross-ply delaminations of E glass material. The material properties considered for E glass material are: E_{11} = 60725 MPa; E_{22} = 24810 MPa; ν_{12} = 0.23 and G_{12} = 12115 MPa.

Table 1 gives the critical value of ε^* at the onset of buckling ε^*_{cr} for a two layered angle-ply (θ /-θ) strip delamination for different values of θ.

Figure 2 and 3 show the variation of the energy release rate (G) with axial compressive strain ε^*. It is noted that G increases with ε^*. The strain energy release rate, G increases with the strain parameter ε^* of the base laminate.

If the value of G for any ε^* is less than the fracture toughness (G_c) of the material, then there is no change in the length of the delamination other than enhancing the maximum deflection of the

Table 1: The critical value of ε^* at the onset of buckling ε^*_{cr} for a two layered angle-ply (θ /-θ) strip delamination

θ (degree)	0	30	45	60	75	90
$\varepsilon^*_{cr}\times10^3$	0.32899	0.29336	0.29924	0.31686	0.32716	0.32899

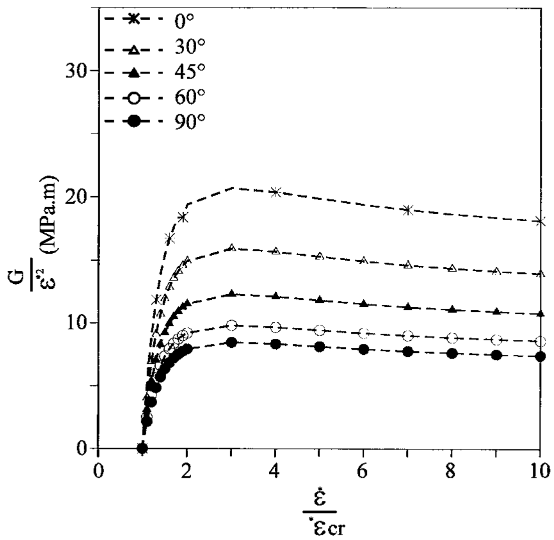

Fig. 2: Energy-release rate versus compressive load parameter for a two layered angle-ply (θ /-θ) thin film strip delamination (2 a = 50 mm and h = 0.5 mm)

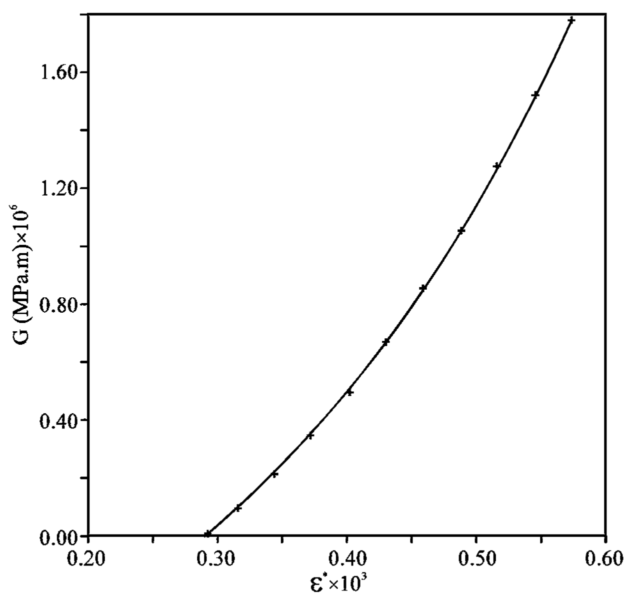

Fig. 3: Energy release rate versus compressive load Parameter for a two layered cross-ply thin film strip delamination (2 a = 50 mm, h = 0.5 mm and ε^*_{cr} = 0.00028549)

delamination. Initiation of delamination growth takes place when G is approaching G_c. Further increasing in the strain energy release rate (G) may cause growth in the delamination.

The combination of the present analytical solution for post-buckling with calculation of the energy release rate at the crack-tip from Eq. 50 or 51 and the measured specific fracture energy of the material will be useful to describe the stability characteristics of delamination growth.

Conclusions

The post buckling solutions for a laminate with clamped ends are applied to a thin-film strip delamination in a base laminate subjected to uniform membrane loads. The energy release rate at the crack trip is evaluated to determine the stability characteristics of delamination growth. In the case of one-dimensional delamination models, the total energy release rate can be obtained by evaluating the J-integral of the post buckling solution based on lamination theory, without knowing the nature or asymptotic form of the interlaminate stress between two dissimilar or differently oriented layers represented by the delamination. Bending stretching coupling reduces the buckling load and enhances the post-buckling deformation of the delaminated layer.

References

Bottega, W.J. and A. Maewal, 1983. Delamination buckling and growth in laminates. Trans. ASME J. Applied Mechanics, 50: 184-189.

Bruno, D. and F. Greco, 2000. An asymptotic analysis of delamination buckling and growth in layered plates. Intl. J. Solids and Struc., 37: 6239-6276.

Chai, H., C.D. Babcock and W.G. Knauss, 1981. One dimensional modeling of failure in laminated plates by delamination buckling. Intl. J. Solids and Struc., 17: 1069-1083.

Chai, H. and C.D. Babcock, 1985. Two dimensional modelling of compressive failure in delaminated laminates. J. Composite Materials, 19: 67-98.

Chattopadhyay, A. and H. Gu, 1994. New higher order plate theory in modelling delamination buckling of composite laminates. AIAA J., 32: 1709-1716.

Chen, H.P., 1991. Shear deformation theory for compressive delamination buckling and growth. AIAA J., 29: 813-819.

Cochelin, B. and M. Potier-Ferry, 1991. A numerical model for buckling and growth of delamination in composite laminates. Computational Methods in Applied Mechanics and Engineering, 89: 361-380.

Davidson, B.D., 1991. Delamination buckling: Theory and experiment. J. Composite Materials, 25: 1351-1378.

Hu, N., H. Fukunaga, H. Sekine and M.A. Kouchakzadeh, 1999. Compressive buckling of laminates with an embedded delamination. Composites Sci. Technol., 59: 1247-1260.

Kardomateas, G.A., 1993. The initial post-buckling and growth behaviour of internal delaminations in composite plates. Trans. ASME J. Applied Mechanics, 60: 903-910.

Kardomateas, G.A. and A.A. Pelegri, 1994. The stability of delamination growth in compressively loaded composite plates. Intl. J. Fracture, 65: 261-276.

Kardomateas, G.A. and A.A. Pelegri, 1996. Growth behaviour of internal delamination in composite beam/plates under compression: Effect of the end conditions. Intl. J. Fracture, 75: 49-67.

Kim, H.J., 1997. Post-buckling analysis of composite laminates with a delamination. Computers and Structures, 62: 975-983.

Kim, H.J. and C.S. Hong, 1997. Buckling and Post-buckling behavior of composite laminates with an embedded delamination. Composites Sci. Technol., 57: 557-564.

Klug, J., X.X. Wu and C.T. Sun, 1996. Efficient modelling of post-buckling delamination growth in composite laminates using plate elements. AIAA J., 34: 178-184.

Kyoung, W.M. and C.G. Kim, 1995. Delamination buckling and growth of composite laminated plates with transverse shear deformation. J. Composite Materials, 29: 2047-2068.

Larsson, P.L., 1991a. On delamination buckling and growth in circular and annular orthotropic plates. Intl. J. Solids and Struc., 27: 15-28.

Larsson, P.L., 1991b. On multiple delamination buckling and growth in composite plates. Intl. J. Solids and Struc., 27: 1623-1637.

Lee, J., Z. Gurdal and O.H. Jr. Griffith, 1995. Post-buckling of laminated composites with delamination. AIAA J., 38: 1963-1970.

Lee, J., O.H. Jr. Griffith and Z. Gurdal, 1995. Buckling and post-buckling of circular plates containing concentric penny-shaped delaminations. Trans. ASME J. Applied Mechanics, 56: 1053-1063.

Mukherjee, Y.X., S.N. Gulrajani, S. Mukherjee and A.N. Netravali, 1994. A numerical and experimental study of delaminated layered composites. J. Composite Materials, 28: 837-870.

Nilsson, K.F. and A.E. Gianna Kopoulos, 1995. A finite element analysis of configurational stability and finite growth of buckling driven delamination. J. Mechanics Phys. Solids, 43: 1983-2021.

Nilsson, K.F., L.E. Asp, J.E. Alpman and L. Nystedt, 2001. Delamination buckling and growth for delaminations at different depths in a slender composite panel. Intl. J. Solids and Struc., 38: 3039-3071.

Nilsson, K.F., L.E. Asp and A. Sjogren, 2001. On transition of delamination growth behavior for compression loaded composite panels. Intl. J. Solids and Struc., 38: 8407-8440.

Perugini, P., A. Riccio and F. Scaramuzzino, 1999. Influence of delamination growth and contact phenomena on the compressive behaviour of composite panels. J. Composite Materials, 33: 1433-1456.

Riccio, A., P. Perugini and F. Scaramuzzino, 2000. Modelling compression behaviour of delaminated composite panels. Computers and Structures, 78: 73-81.

Shen, F., K.H. Lee and T.E. Tay, 2001. Modelling delamination growth in laminated composites. Composites Sci. Technol., 61: 1239-1251.

Simitses, G.J., S. Sallam and W.L. Yin, 1985. Effect of delamination of axially loaded homogeneous laminated plates. AIAA J., 23: 1437-1444.

Srivasta, K.S., B.R. Vidyashankar, A.V. Krishna Murty and K. Vijaykumar, 1993. Buckling of laminated plates containing delaminations. Computers and Structures, 48: 907-912.

Suemasu, H., K. Gozu and K. Hayashi, 1995. Compressive buckling of rectangular composite plates with a free-edge delamination. AIAA J., 33: 312-319.

Whitcomb, J.D., 1981. Finite element analysis of instability related delamination growth. J. Composite Materials, 15: 403-426.

Whitcomb, J.D., 1986. Parametric analytical study of instability related delamination growth. Composites Sci. Technol., 25: 19-48.

Whitcomb, J.D. and K.N. Shivakumar, 1989. Strain energy release rate analysis of a laminate with a post-buckled delamination. J. Composite Materials, 23: 714-734.

Yin, W.L. and J.T.S. Wang, 1984. The energy-release rate in the growth of a one dimensional delamination. Trans. ASME J. Applied Mechanics, 51: 939-941.

Yin, W.L., 1985. Axisymmetric buckling and growth of circular delamination in a compressed laminate. Intl. J. Solids and Structures, 21: 503-514.

Yin, W.L., 1988. The effect of laminated structure on delamination buckling and growth. J. Composite Materials, 22: 502-517.