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Slump Estimation of Cylindrical Segment Grains of a Typical Rocket Motor under Vertical Storage Condition

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Abstract: Structural integrity under inertia loading was considered to be one of the design conditions for a solid propellant rocket motor. Grain structural integrity evaluations were generally based upon mechanical properties which were evaluated from tests of specimens removed from carton samples of propellant. Significant deviations were noticed while testing specimens from carton samples and from dissecting grains due to manufacturing process. Main grain to carton correlation factors established for a HTPB-based propellant and their adequacy examined measuring slumps in cylindrical grain segments. A simple methodology presented to carryout viscoelastic finite element analysis for slump estimations in rocket grains under vertical storage condition. Finite element analysis was carried out on a cylindrical segment grain of a typical steel casing rocket motor using the eight-node quadrilateral axi-symmetric Hermann element of the MARC computer program. A mathematical model proposed to represent the time dependent master stress relaxation modulus of a HTPB-based propellant grain essential for estimation of slump displacements in cast segment grains. Measured slump displacements in rocket motors of different segment grains at different storage times were found to be reasonably in good agreement with the finite element analysis results.

Key words: Finite element analysis, rocket motor, cylindrical shell, steel casing, HTPB-based propellant, slump displacement

Introduction

Solid propellant grains in rocket motors are subjected to axial acceleration load during vertical storage, transportation and flight. Slump of a propellant grain is particularly pronounced in the case of long term vertical storage or short term high gravity loads during flight of relatively large solid booster motors.

The slump measurements of propellant grains in maraging steel rocket motors are examined in the present study. The motor has a diameter of 2.8 m and a length of 16.6 m (excluding nozzle). It is confined to contain 139 tons of case bonded composite HTPB based solid propellant to provide required propulsion performance. The casing consists of five individual welded segments, namely a Head End (HE) segment, a Nozzle End (NE) segment and three cylindrical Middle Segments (MS). The number of segments is chosen based on the feasibility of propellant casting, hardware fabrication limits and ease of transportation/handling, etc. These segments are connected to each other through the tongue and groove type of joint. Both the domes of HE and NE segments are tori-spherical shells having central circular opening. Interface between the dome/cylinder ends is machined out from a forging in the form of Y-rings. The end segments are identical in shape except that NE dome has wider opening

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and consequently smaller spherical portion. The motor grain is a five segmented grain, out of these, HE segment grain has star port and the rest four segments have circular port. The circular port grains have inner radius of 0.5 m and outer radius of 1.4 m. Because of large diameter and unsupported base, slump displacements are expected to be large in the middle segments because of the absence of dome support, under long term vertical storage condition. Since solid propellant material is viscoelastic in nature, it is essential to carry out the viscoelastic analysis for slump estimation of segment grains in a rocket motor.

The main problem that occurs in the viscoelastic stress analysis is that the stresses, strains and displacements are all functions of time. If this time variable could be removed by a transform operation, the resulting problem would be an equivalent problem in the theory of elasticity (called the associated elastic problem) in terms of the transform parameter, with the load and boundary conditions in the form of transforms of the original time-dependent functions. The inverse transformation of the solution of the associated elastic problem into the real time variable would give the solution to the original viscoelastic problem. Schapery (1961) proposed a direct method of inversion which is simple to apply and is given by:

$$f(t) = [s \bar{f}(s)]_{s=1/2t} \quad (1)$$

Where, $\bar{f}(s)$ is the Laplace transform of a function $f(t)$ and s is the transform parameter. To apply the method it is necessary to substitute $s = 1/2t$ for discrete values of t in the operational quantities involving the Laplace parameter, like $E(s) = s\bar{E}(s)$, $\nu(s) = s\bar{\nu}(s)$, etc. For each value of the transform parameter s , there are corresponding values of $E(s)$ and $\nu(s)$ which are the associated Young's modulus and Poisson's ratio to be used in the associated elastic solution. The finite element solution obtained for these values at $s = 1/2t$ gives directly the viscoelastic solution at time t . According to Schapery (1962), the relaxation modulus, E_{rel} can be expressed in the Prony series form as

$$E_{rel}(t) = E_{\infty} + \sum_{k=1}^n A_k e^{-\frac{t}{2\tau_k}} \quad (2)$$

Where, E_{∞} is the equilibrium modulus, τ_k are the relaxation times and A_k are constants. The Laplace transform for the relaxation modulus defined in Eq. 2, is

$$\bar{E}_{rel}(s) = \frac{E_{\infty}}{s} + \sum_{k=1}^n \frac{A_k}{s + \frac{1}{2\tau_k}} \quad (3)$$

The operational modulus, $E(s)$ is given by:

$$E(s) = s\bar{E}_{rel}(s) = E_{\infty} + \sum_{k=1}^n \frac{sA_k}{s + \frac{1}{2\tau_k}} \quad (4)$$

Using the bulk modulus (K) and the operational modulus $E(s)$, we can write the operational Poisson's ratio in the form:

$$\nu(s) = \frac{1}{2} \left[1 - \frac{E(s)}{3K} \right] \quad (5)$$

At any time t , substituting the value of $s = 1/2t$ in Eq. 4 and 5, the value of operational modulus $E(s)$ and the corresponding value of Poisson's ratio $\nu(s)$ can be obtained. The values of operational

modulus $E(s)$ and the Poisson's ratio, $\nu(s)$ are used for E and ν in the associated elastic finite element solutions to get time dependent viscoelastic solutions. Renganathan *et al.* (2000a, b, 2002) have verified the applicability of the above procedure through finite element solutions using the commercial MARC software package, which has capabilities for viscoelastic analysis.

In this study, the above procedure of combining the finite element method and an approximate inversion technique has been applied for slump estimation to the cylindrical segment grains under vertical storage condition. The applicability of the above procedure has also been examined by supplying the actual master relaxation modulus data of the propellant material to the MARC computer Program for the viscoelastic analysis of the grain. A comparative study is also made between theoretical and measured slump values for the cylindrical segment grains. A mathematical model is generated for representing master relaxation modulus curve using carton properties, to estimate the above slump values at different storage times.

Cylindrical Segment Grain Analysis for Vertical Storage

The cylindrical segment grain is basically a thick cylinder with circular port. The ratio of grain inner radius (a) to outer radius (b) is 0.36 and the ratio of its Length (L) to the Diameter (D) is 1.14. The middle segment grain is analysed for vertical storage without base support.

The basic input for the analysis considered is as follows:

Grain inner radius (a) = 50.0 cm
 Grain outer radius (b) = 138.9 cm
 Casing thickness = 0.78 cm

Young's modulus (E_c), Poisson's ratio (ν_c) and density (ρ_c) of the maraging steel casing material are:

$$E_c = 1900000 \text{ KSC}, \nu_c = 0.3 \text{ and } \rho_c = 0.0178 \text{ kg cm}^{-3}$$

Bulk modulus (K) and density (ρ_p) of the propellant are:

$$K = 35300 \text{ KSC}, \rho_p = 0.00178 \text{ kg cm}^{-3} \text{ and Load} = 1 \text{ g}$$

The Master Stress Relaxation Modulus (MSRM) curve, which accounts for variation of modulus with time for a HTPB-based propellant material, is considered in the present study. Schapery's method of fitting a Prony series by collocation method has been employed for the experimental MASRM curve. The first 16 values of τ_k (in seconds) and A_k (in KSC) in Eq. 2 are (Renganathan *et al.*, 2000b): $\tau_k = 10^{k-9}$ for $k = 1, 2, \dots, 16$ and the corresponding values of A_k are 1.17, 158.8, 387.3, 530.2, 225.6, 139.3, 52.2, 45.6, 13.9, 11.9, 4.46, 4.14, 0.26, 0.1, 0.445 and 0.655. The equilibrium modulus, $E_\infty = 20 \text{ KSC}$.

For slump estimation of the cylindrical segment grain using finite element method, the grain is discretised using 8 node quadrilateral axi-symmetric Hermann element of the MARC computer program. The finite element idealization of the grain is shown in Fig. 1.

Slump displacement values at time t are obtained by supplying the values of operational modulus $E(s)$ for E and the Poisson's ratio $\nu(s)$ for ν (where, $s = 1/2t$) in the finite element elastic analysis.

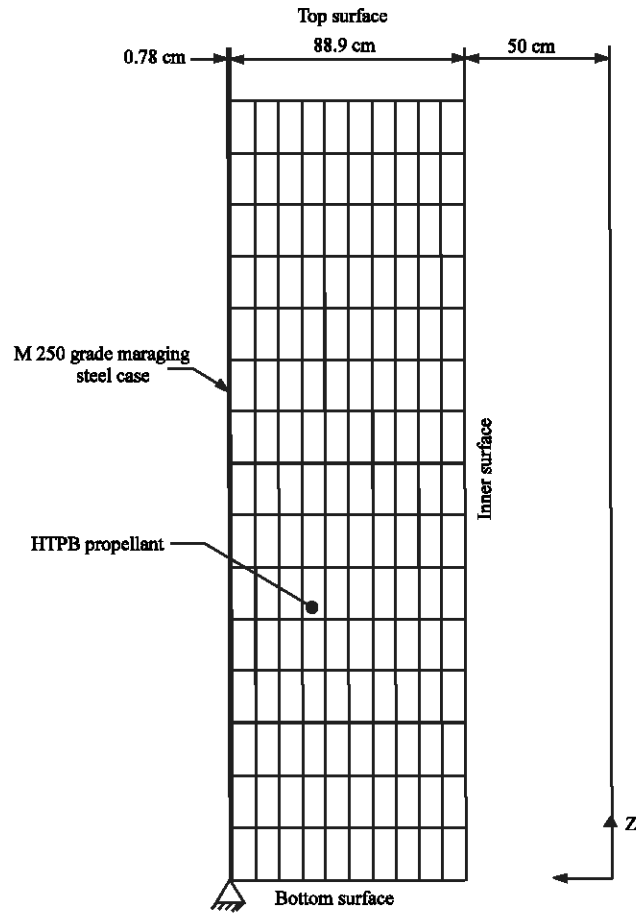


Fig. 1: Finite element idealization of a cylindrical segment grain

These values are also obtained from the direct finite element viscoelastic solution by using directly the Prony series itself of the MSRM curve, for comparison. The results obtained from the associated elastic solution are found to be in good agreement with those obtained from the viscoelastic solution (Renganathan *et al.*, 2000b). The slump displacement value is found to increase rapidly within an hour and later on increases slowly with time and approaches a value corresponding to the equilibrium modulus (E_∞).

The equilibrium deformed shape of the grain with the unsupported base obtained from MARC, is shown in Fig. 2. Fitzgerald and Hufferd (1971) as well as Anonymous (1973) presented the slump displacement expression:

$$w = \frac{\rho(1+\nu)}{E} \left[\frac{b^2 - r^2}{2} - a^2 \log \frac{b}{r} \right]$$

This expression corresponds to the case of an infinitely long cylindrical segment grain under 1 g vertical storage load. It varies only with r (across the web) and is constant along the length

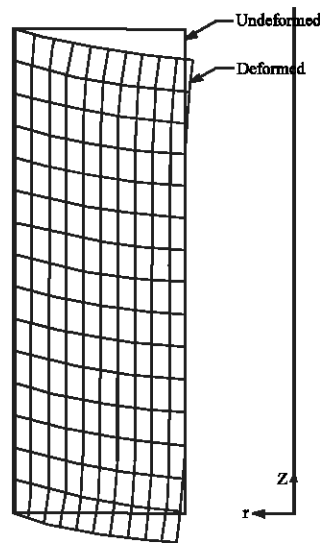


Fig. 2: Deformed shape of the grain with the unsupported base

(z-coordinate). But the present cylindrical segment grain has L/D ratio of 1.14. The effect of the finite length is examined through finite element solution. It can be seen from Fig. 2 that the slump displacements are almost uniform along the length of the grain, the actual values of the maximum slump displacements being 9.3 mm at the top and bottom surfaces and 8.5 mm at the mid length against 7.9 mm obtained from the above slump displacement expression.

Results and Discussion

Nine cylindrical Middle Segment (MS) grains were stored vertically with unsupported base and the slump displacements were measured at the bottom surface, at different storage times. Here, a mathematical model for representing MSRM curve is presented and slump values are estimated and compared with the measured values. Grain structural integrity evaluations are generally based upon mechanical properties data obtained from tests of specimens removed from carton samples of propellant. Each cylindrical mid segment grain carries 25 tons of HTPB propellant cast from 15 mixes. From each mix tensile specimens are drawn to evaluate mechanical properties. After considerable experience in testing specimens from carton samples and from dissecting grains, it has been found that significant deviations from the assumed behavior can be induced by the manufacturing process (Veit and Landuk, 1985). Main grain to carton correlation factors have been established for HTPB propellant. It is noted that Main grain to carton ratio on the average values of tensile strength, percentage of elongation and initial modulus are 1.13, 0.85 and 1.3, respectively. Since, slump estimations are modulus dependent, the above correction factor is applied to the average values of the initial modulus. Table 1 gives the average initial modulus values of the carton samples from 15 mixes for nine cylindrical mid-segments. In order to estimate slump at different storage times, Master Stress Relaxation Modulus (MSRM) curve corresponding to the batch of mixes is essential. Generation of such MSRM curves (Kruse, 1966) for each batch of mixes, is practically impossible because of large

Table 1: Average mechanical properties of carton samples of HTPB propellant and a comparison of estimated and measured maximum slump displacements of cylindrical segment grains

Seg. grain No.	Tensile strength (KSC)	% of elongation	Average value of initial modulus \bar{E}_i (KSC)	Coefficient of variation (%)	Corrected value of \bar{E}_i for main grain	Storage time t (h)	Operational modulus E(s) (KSC)	Estimated slump (mm) and its variation at bottom inner Surface	Measured slump (mm) at bottom inner surface
1	7.96	30.9	54.4	10.7	70.72	24	29.47	6.5 +0.7 -0.6	7.00
2	7.79	45.1	33.0	5.4	42.90	48	18.26	10.5 +0.5 -0.5	9.17
3	8.04	39.6	36.2	8.3	47.06	48	19.94	9.6 +0.8 -0.7	6.99
4	7.34	40.8	39.0	12.7	50.70	24	21.47	8.9 +1.2 -1.0	8.70
5	6.77	35.0	32.0	10.2	41.60	16	17.90	10.7 +1.1 -0.9	7.20
6	6.85	38.6	38.3	24.0	49.79	54	21.00	9.1 +2.7 -1.7	12.4
7	8.12	34.0	44.2	6.1	57.46	46	24.10	7.9 +0.5 -0.4	6.50
8	7.43	37.6	42.4	13.4	55.12	720	22.69	8.4 +1.3 -1.0	8.20
9	7.70	33.0	49.0	14.9	63.70	166	26.40	7.2 +1.3 -0.9	6.60

number of specimen tests requirement at different strain rates and temperatures as well as variation of mechanical properties from mix to mix. Therefore, for viscoelastic analysis, the MSRM curves for the nine segments are generated using the MSRM curve of the HTPB-based propellant (Renganathan *et al.*, 2000b) and the average initial modulus value from carton samples, in the form:

$$E_{rel}(t) = f_1 E_{\infty} + f_2 \sum_{k=1}^n A_k e^{-\frac{t}{2\tau_k}} \quad (6)$$

Where, the weightage factors f_1 and f_2 in Eq. 6 are:

$$f_1 = \frac{\bar{E}_i}{E_i}, \quad f_2 = \frac{E(0) - f_1 E_{\infty}}{E(0) - E_{\infty}} \text{ and } E(0) = E_{\infty} + \sum_{k=1}^n A_k,$$

is the glassy modulus; E_i is the initial modulus at room temperature corresponding to the time, $t = 1$ second in the MSRM curve and \bar{E}_i is the average initial modulus value of the cylindrical mid segment grain. In the typical MSRM curve for the HTPB-based propellant utilized by Renganathan *et al.* (2000b), the glassy modulus, $E(0) = 1596$ KSC and the initial modulus, $E_i = 50$ KSC. The weightage factors f_1 and f_2 in Eq. 6 for the relaxation modulus, $E_{rel}(t)$ of the cylindrical mid segments are selected in such a way that there is no change in the value of the glassy modulus $E(0)$ which is expected to be same for all segment grains with a particular HTPB formulation and maintaining a constant proportion of the initial modulus and the equilibrium modulus.

The MSRM curves for three specified values of \bar{E}_i (60, 50 and 30 KSC) generated from Eq. 6 are shown in Fig. 3, for illustration. After applying the Main grain to carton correlation factor to the average values of the initial modulus, \bar{E}_i and using Eq. 1 and 6, the values of operational modulus, $E(s)$ corresponding to the storage time of the measured slump displacements of the cylindrical mid segment grains, are evaluated and presented in Table 1. Using the values of the operational modulus and Poisson's ratio, the maximum slump displacements at the corresponding storage times, are computed

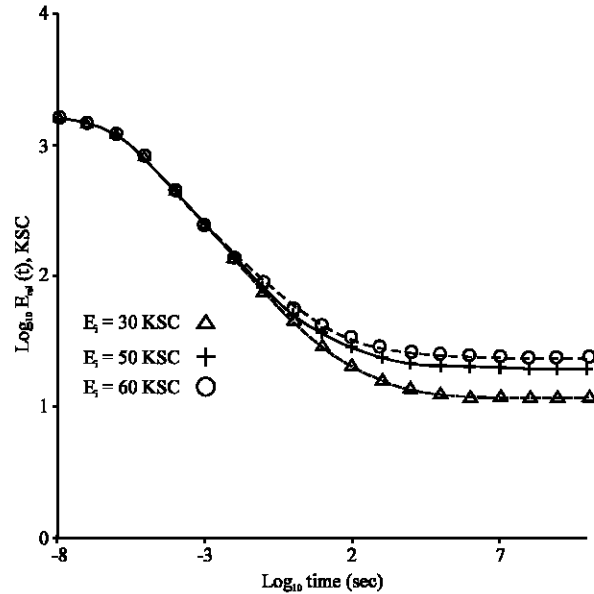


Fig. 3: Typical MSRM curves generated from Eq. 6 for the specified values of initial modulus, \bar{E}_i

using the finite element idealization (Fig. 1) and presented in Table 1 along with the measured values, for comparison. Table 1 shows that the measured and estimated slump, are reasonably in good agreement with each other. The discrepancy between the estimated and measured slump values seen in certain cases could be mainly due to the variation of modulus values within the grain. The dispersion of the initial modulus data relative to its mean represented by a normalized parameter, namely, the coefficient of variation, $cv (= 100 * \text{Standard deviation}/\text{mean})$, is also presented in Table 1 along with the average value of the initial modulus. In addition, in the estimated slump at bottom inner surface of the segment grains are given in Table 1. Most of the measured slump values are close to those estimated by using the average values of the initial modulus. For segment grain Numbers 1 and 6, the estimated slump values based on average initial modulus, are seen to be un-conservative. However, the measured values for these segment grains are found closer to the upper bound estimations.

Conclusions

A mathematical model for the generation of the Master Stress Relaxation Modulus (MSRM) curve based on the average initial modulus value obtained from carton samples, is presented and used for the estimation of slump displacements at different storage times, for the cast segment grains stored vertically without base support. Estimated slump values for several cylindrical segment grains match well with measured values at different storage times.

References

- Anonymous, 1973. Solid propellant grain structural integrity analysis. National Aeronautics and Space Administration, Lewis Research Center (Design Criteria Office), Cleveland, Ohio, Report No. NASA- SP- 8073.
- Fitzerald, J.E. and W.L. Hufferd, 1971. Handbook for the Engineering Structural Analysis of Solid Propellants. Prepared in cooperation with the Johns Hopkins University, Applied Physics Lab, Chemical Propulsion Information Agency, Silver Spring, MD, CPIA Publication No. 214.

- Kruse, R.B., 1966. Laboratory characterization of solid propellant mechanical properties. Thiokol Chemical Corporation, Alabama.
- Renganathan, K., B. Nageswara Rao and M.K. Jana, 2000a. A multi-layer thick cylindrical shell under internal pressure and thermal loads applicable to solid propellant rocket motors. *Forschung in Ingenieurwesen*, 66: 57-66.
- Renganathan, K., B. Nageswara Rao and M.K. Jana, 2000b. An efficient axisymmetric hybrid-stress-displacement formulation for compressible/nearly incompressible material. *Intl. J. Pressure Vessels Piping*, 77: 651-667.
- Renganathan, K., 2002. Studies on failure behavior of a solid propellant rocket motor. Department of Mathematics, Institute of Post Graduate Studies and Research, Jawaharlal Nehru Technological University, Hyderabad, India.
- Schapery, R.A., 1961. Two simple approximate methods of Laplace transform inversion for viscoelastic stress analysis. Graduate Aeronautical Laboratories, California Institute Technology SM, pp: 61-23.
- Schapery, R.A., 1962. Approximate methods of transform inversion for viscoelastic stress analysis. *Proceeding of 4th U.S. National Congress of Applied Mechanics*, Vol.2, ASME, pp: 1075-1085.
- Veit, P.W. and L.G. Landuk, 1985. Experimental evaluation of as-processed propellant grains. *J. Propulsion*, 1: 494-497.