



Trends in  
**Applied Sciences  
Research**

ISSN 1819-3579



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## Mean Value Engine Modeling and Validation for a 4-stroke, Single Cylinder Gasoline Engine

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**Abstract:** Mean Value Engine Model has been widely used in model based engine control development. In this paper, an engine model is presented for a 4-stroke single cylinder gasoline engine with respect of electrical throttle control development. The two-state nonlinear model mainly consists of two subsystems-the intake manifold and the crankshaft. The former subsystem represents dynamics of the intake manifold air mass flow, as well as the amount of fuel entered into the cylinder. The latter subsystem mainly accounts the dynamics of the crankshaft loading. Some empirical parameters in the model, such as the volumetric efficiency, are identified by using measured operating data from the engine. The steady state performance of the nonlinear engine model is then validated by experimental data of the engine. The model output shows relatively good agreement with the experiment measurements. It verifies that the MVEM model is appropriate for this application of automatic electronic throttle control development for maintaining a fixed speed.

**Key words:** Engine, mean value engine model, nonlinear

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### INTRODUCTION

Electronic throttle control technology has been implemented in gasoline engines since 1986 (Streib and Bischof, 1996). Without mechanical connection to the accelerator, the throttle plate can be controlled by the operator's together with current powertrain operating conditions, e.g., engine and vehicle speed, transmission gear, etc. Electrical controlled throttle simplifies the traction control, cruise control, idle speed control; consequently it improves the engine emissions, fuel economy and dynamic response. The advantage of electrical throttle makes it suitable for the control of small single cylinder gasoline engine such as mower where speed control is needed.

There are many different types of engine model such as a mapping model, a large cyclic simulation model, or a simplistic phenomenological transfer function model. A large cyclic simulation model is built by focusing on every detail in the engine working cycle including air flow, heat transfer and pressure wave propagation etc. (Heywood, 1988). A simplistic transfer function or a mapping model simplifies the nonlinear dynamics of the engine by a static model or a simple linear dynamics model. MVEM has been discussed in many literatures (Hendricks and Sorenson, 1990 a,b; Hendricks, 2000). Modeled properties in a MVEM are usually averaged over one or several cycles (Hendricks and Sorenson, 1990 a,b). As a result, it has a low order and few parameters to achieve easy tuning, which makes mean value engine model better for control development.

In this study, a two-state MVEM model is presented for the development of electronic throttle control. The objective engine for this model is an air-cooled single cylinder 4-stroke gasoline engine. Considering the application of speed control using electronic throttle control, intake manifold pressure

and crankshaft speed are taken as state variables in the model. The volumetric efficiency and crankshaft system friction are considered as functions of state variables and are calibrated using the engine experiment measurements. Model verification with engine experiment data shows a good agreement between the model output and the engine measurement in spite of the simplification assumption in the model. This indicates the simple MVEM model is suitable for the electronic throttle control.

### MODEL DESCRIPTION

Since the eventual objective of the model is to develop speed control of the engine using electronic throttle control, the throttle angle  $\theta_t$  is considered as system input and the engine speed  $n$  is considered as the output of the mean value engine model. The model mainly consists of an intake manifold, an air intake valve, a cylinder, an exhaust pipe and a crankshaft, as shown in Fig. 1.

#### Intake Manifold

The intake manifold subsystem consists of three components: the throttle, the intake manifold system and the cylinder intake valve. The intake manifold dynamics is modeled with the mass conservation law. Since the properties of the engine modeled in the MVEM are usually averaged over one or several cycles, it is reasonable to assume that air in the intake manifold can be treated as a perfect gas with constant specific heats. For the objective engine, it is reasonable to assume that heat transfer between the air and wall in the intake manifold is non-significant. Thereafter the air temperature  $T_{man}$  in the intake manifold is assumed to be of constant value as the ambient temperature. Based on above assumption, the mass conservation law in the intake manifold can be described as

$$\dot{m}_a = \dot{m}_i - \dot{m}_c \quad (1)$$

Taking derivative of the intake manifold air state equation and using (1), we will get

$$\dot{P}_{man} = (RT_{man} / V_{man})(\dot{m}_i - \dot{m}_c) \quad (2)$$

$\dot{m}_i$ , the air mass flow rate through the throttle valve, is determined by the throttle plate angle  $\theta_t$ , ambient air pressure and the intake manifold pressure (Aono and Kowatari, 2001) as in following equation

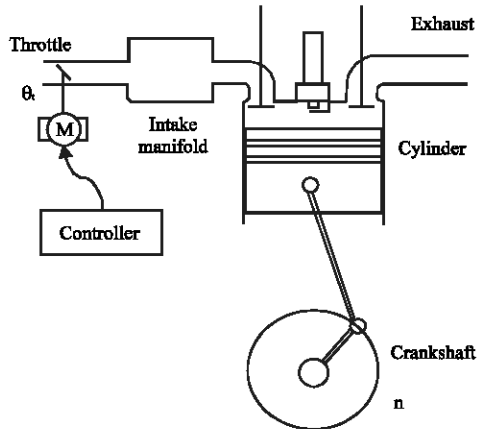


Fig. 1: Schematic diagram of the engine model

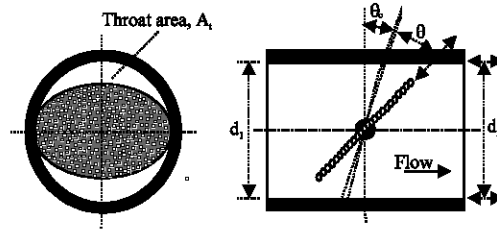


Fig. 2: The schematic diagram of butterfly throttle valve geometry

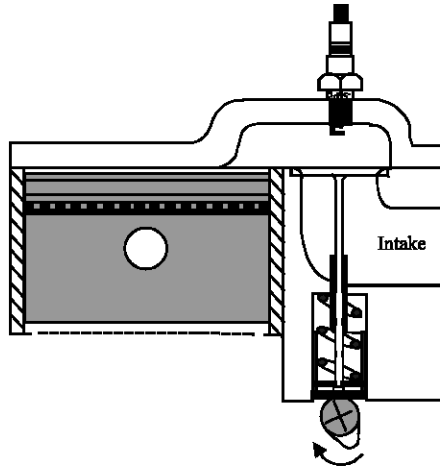


Fig. 3: Cylinder and air intake valve configuration of a side-valve engine

$$\dot{m}_t = C_d A_t \frac{P_{amb}}{\sqrt{RT_{amb}}} \sqrt{\frac{2k}{k-1}} \beta \quad (3)$$

where,

$$\beta = \begin{cases} \sqrt{\left(\frac{P_{man}}{P_{amb}}\right)^{\frac{2}{k}} - \left(\frac{P_{man}}{P_{amb}}\right)^{\frac{k+1}{k}}}, & \text{if } \left(\frac{P_{man}}{P_{amb}}\right) \geq \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \\ \sqrt{\left(\frac{2}{k+1}\right)^{\frac{2}{k-1}} - \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}, & \text{if } \left(\frac{P_{man}}{P_{amb}}\right) < \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \end{cases} \quad (4)$$

In above equations, the discharge coefficient  $C_d$  accounts for the effective air flow frictional losses through the throttle valve. Given constant ambient pressure and temperature and the maximum mass flow occurs at the critical pressure and temperature  $P_{amb}$  and  $T_{amb}$ , the maximum mass flow occurs at the throttle pressure ratio  $\left(\frac{P_{man}}{P_{amb}}\right) = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$ . The effective area, or throat area  $A_t$ , in (3) can be computed (Heywood 1988) as

$$A_t = \frac{\pi D^2}{4} \left(1 - \frac{\cos(\theta_i + \theta_c)}{\cos(\theta_c)}\right) \quad (5)$$

where,  $\theta_i$  is the input throttle angle,  $\theta_c$  is the idle throttle angle and  $\theta_c$  is the throttle closing angle, as shown in the geometry diagram of butterfly throttle valve in Fig. 2.

Figure 3 is the schematic diagram of cylinder and air intake valve arrangement.

The air mass flow rate to the cylinder  $\dot{m}_c$  is determined by the intake manifold air pressure and the volumetric efficiency  $\eta_{vol}$  as in following equation (Heywood, 1988)

$$\dot{m}_c = \left( \frac{nV_d \eta_{vol}}{120RT_{man}} \right) P_{man} \quad (6)$$

$\eta_{vol}$  is considered as a function of  $n$  and  $P_{man}$  in order to compensate for residual gases in the cylinder, as in (7).

$$\eta_{vol}(n, P_{man}) = -0.13 + 3.5 \times 10^{-5} n - 5 \times 10^{-9} n^2 + 9 \times 10^{-6} P_{man} \quad (\text{Hendricks and Sorenson, 1990a}) \quad (7)$$

Using the above equations, the intake manifold subsystem is represented as a nonlinear state space model with the state variable of  $P_{man}$ , as in (2).

### **Crankshaft Dynamics Model**

The crankshaft dynamics model mainly consists of the cylinder and the crankshaft. Torque generated through cylinder combustion  $\tau_c$ , friction  $\tau_f$ , pumping loss  $\tau_p$  and the load torque  $\tau_l$  applied on the crankshaft, which is modeled as mechanical inertia  $J_{eng}$ , resulted in the governing dynamics Eq. 8 for this subsystem

$$\dot{\omega} = \frac{1}{J_{eng}} (\tau_c - \tau_f - \tau_p - \tau_l) \quad (8)$$

Since the MVEM engine properties are averaged over one or several cycles, the power generated in the cylinder and the friction and pumping loss are computed by the indicated mean effective pressure  $imep$  and the frictional mean effective pressure  $fimep$ . The indicated mean effective pressure  $imep$  is computed (Heywood, 1988) by

$$imep = \frac{120 \eta_f \dot{m}_f Q_{HV}}{nV_d} \quad (9)$$

where  $\eta_f$  is the fuel conversion efficiency and  $Q_{HV}$  is the fuel low heat value.

Fuel conversion efficiency  $\eta_f$  in (9) is usually a function of the compression ratio  $r_c$  and the air fuel ratio  $\lambda$  (Blair, 1999). Besides, the engine states such as speed, load and temperature etc. also affects  $\eta_f$ . However, since the objective for the engine model is to design a speed control through electronic throttle, it is reasonable to assume that air fuel mixture is generally stoichiometric and the influence of engine states can be neglected. Considering that the compression ratio  $r_c$  for a specific engine is approximately a constant,  $\eta_f$  is then assumed to be a constant. The variations of  $\eta_f$  in different working condition will be compensated by the controller.

With above assumption, the fuel mass flow rate  $\dot{m}_f$  is found as:

$$\dot{m}_f = \frac{\dot{m}_c}{AFR} \quad (10)$$

The mean friction and pumping losses can be considered as a function of the engine speed and intake manifold pressure, respectively. The frictional mean effective pressure  $f_{imep}$  is computed as a function of engine speed (Heywood, 1988) as shown in (11).

$$fmep = 10^4 \left( 9.7 + 1.5 \frac{n}{10^3} + 0.5 \frac{n^2}{10^6} \right), \text{ (Heywood 1988)} \quad (11)$$

The pumping loss mean effective pressure can be computed as

$$pmep = P_{exh} - P_{man} \quad (12)$$

where  $P_{exh}$  is the mean pressure after the exhaust valve. In the objective single cylinder engine, the exhaust manifold is short and the exhaust gas goes through a small muffler into the ambient environment. Considering that  $pmep$  is usually much small than  $imep$ , it is computed by approximating  $P_{exh}$  with  $P_{amb}$ . The error introduced by this approximation is then considered as perturbation to the plant and will be dealt with in the controller design.

For a four-stroke engine at given load, using (8), (9), (11) and (12) the dynamics equation of the crankshaft subsystem becomes

$$\dot{\omega} = \frac{1}{J_{eng}} \left[ \frac{V_d (imep - fmep - pmep)}{4\pi} - \tau_{load} \right] \quad (13)$$

### Summary of the Model

As described before, the engine model takes the throttle angle as its input variable. The state variables are the engine  $\omega$  speed and the intake manifold pressure  $P_{man}$ . For the speed control the engine speed  $\omega$  is the model output. The ambient pressure and temperature, the engine geometry parameters and working process parameters such as throttle discharge coefficient  $C_d$  and volumetric efficiency  $\eta_{vol}$  are considered as adjustable model parameters. The values of these parameters may be calibrated with the objective engine.

Based on the dynamics equations discussed above, the engine can be represented as the following nonlinear state space model

$$\dot{P}_{man} = -\frac{V_d \eta_{vol}}{4\pi V_{man}} \omega P_{man} + \frac{RT_{man}}{V_{man}} \dot{m}_t \quad (14)$$

$$\dot{\omega} = \frac{V_d}{4\pi J_{eng}} \left[ \frac{\eta_f Q_{HV} \eta_{vol} P_{man}}{RT_{man} \cdot AFR} - fmep - (P_{amb} - P_{man}) \right] - \tau_{load} \frac{1}{J_{eng}} \quad (15)$$

where  $\dot{m}_t$  is a function of the model input  $\theta_t$  and state variable  $P_{man}$ .

### MODEL VERIFICATION

The target engine for this application is a 4-stroke air-cooled single cylinder engine manufactured by Briggs and Stratton (Tyler and Belf, 1974). The engine nominal output is 3.5 hp at the rate of 3600 RPM and 6.6 Nm at the rate of 2800 RPM. It has a bore diameter of 65.09 mm and a stroke displacement of 44.45 mm. The cylinder displacement volume  $V_d$  is about  $1.4791 \times 10^{-4} \text{ m}^3$ . Compression ratio  $r_c$  is about 8.94. For regular gasoline, the ideal air-to-fuel ratio is  $AFR = 14.8$ . The fuel heating value  $Q_{HV} = \text{J kg}^{-1}$  (Heywood, 1988). Consider air as ideal gas, the ratio of specific heats  $k = 1.4$  and the ideal gas constant  $R = 287 \text{ J/(K} \cdot \text{kg)}$ .  $C_d$  is selected to be 0.5 (Aono and Kowatari, 2001) and is selected to be 0.21.

Using above parameter values, simulation results of the MVEM model is compared to the engine experimental measurements. Figure 4-8 show the comparisons of the experimented data and the simulation results for static working conditions. Several engine speeds corresponding to different

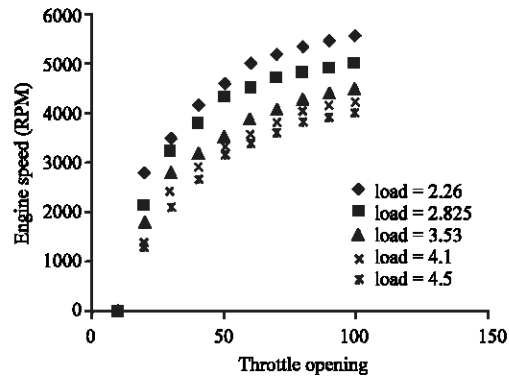


Fig. 4: Measured engine speed characteristics curve at different load torque

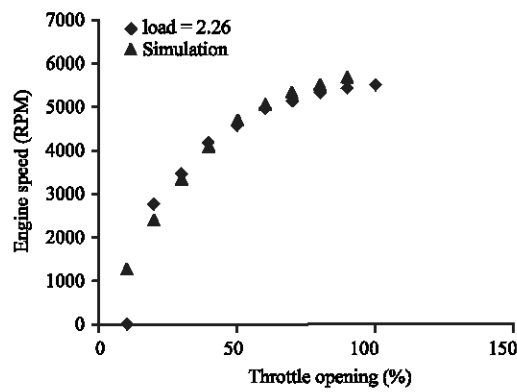


Fig. 5: Performance comparison between the MVEM model and engine experiments at a constant load of 2.26 Nm

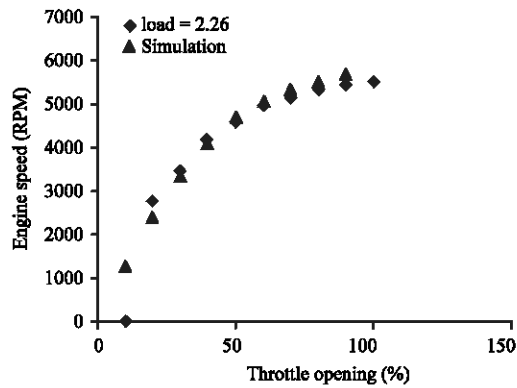


Fig. 6: Performance comparison between the MVEM model and engine experiments at a constant load of 2.825 Nm

throttle opening angles are measured at a few constant load torques. From Fig. 5 to 7, model outputs show good agreement with engine experiment measurements for a wide throttle opening range. When the throttle angle is very small and when the load torque is high, the constant fuel conversion efficiency

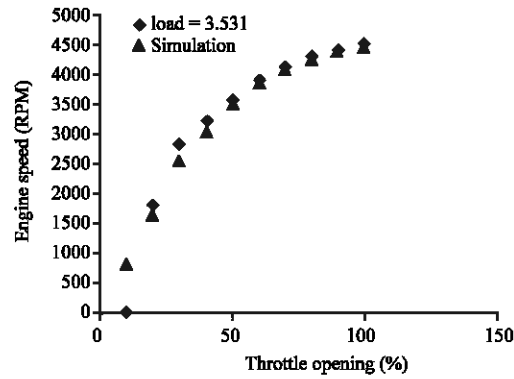


Fig. 7: Performance comparison between the MVEM model and engine experiments at a constant load of 3.531 Nm

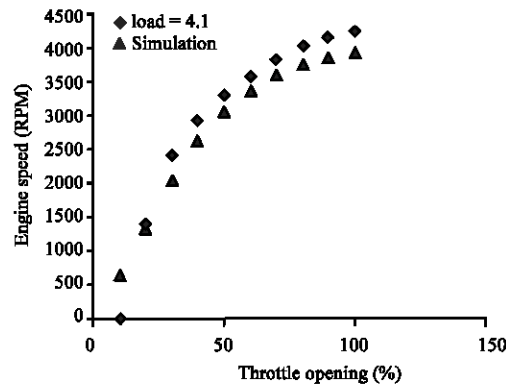


Fig. 8: Throttle opening versus engine speed with loads of 4.1 Nm in measured engine data and MVEM model simulation

and the stoichiometric air fuel ratio assumption is violated. This consequently introduced higher error between the model outputs and the experiment measurements. However, the agreement between the model and the engine experimental data during mid-range throttle opening is enough for the speed control development through the electronic throttle control.

### CONCLUSIONS

A two-state dynamic mean value engine model has been presented for a 4-stroke air-cooled single cylinder gasoline engine. The engine model takes the throttle angle as its input. The working properties of the engine are represented by the state variables: The engine speed and the intake manifold pressure. The engine speed is the model output. Model simulation results show good agreement with the experimental measurements from the engine in the operating region where the air fuel ratio is close to stoichiometric and fuel conversion efficiency is approximately constant. This indicates the MVEM model is qualified for the application of electronic throttle control development for the automatic speed control of the engine.



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