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On Treating Multiobjective Cutting Stock Problem in the Aluminum Industry under Fuzzy Environment

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Abstract: In this study we present a solution algorithm for solving multiobjective cutting stock problem in the aluminum industry under fuzziness, which in its general form, is an integer nonlinear problem. It is considered that the scrap is a fuzzy parameter and the concept of α-level set together with the definition of the fuzzy scrap and its membership function are introduced. A practical problem example of the method implementation for the solution algorithm is proposed.

Key words: Multiobjective mixed-integer nonlinear programming problems, cutting stock problem, fuzzy numbers, α-level set

INTRODUCTION

The problem of one-dimensional stock cutting occurs in many industrial processes and it has attracted an increasing attention of researchers from all over the world (Bishoff and Wawsher, 1995; Ezzat, 2003; Ferreira *et al*., 1990; Goulimis, 1990; Gradisar *et al*., 1997; Gradisar and Trkman, 2005; Haessler and Vonderembse, 1979; Hughes, 1989; Stadtler, 1990; Sweeney and Paternoster, 1992; Weng and Hung, 2003; Weng *et al*., 2003 and 2004). Most standard problems related to one-dimensional stock cutting are known to be NP-complete. However, in many cases the problems can be modeled by means of mathematical programming and a solution can be found by using approximate methods and heuristics.

In this study, a company producing high grade aluminum has orders for various kinds of cylindrical logs of the metal which is used for extrusion purposes. The kinds of logs differ from each other in dimension (diameter and length) and in the (small) amounts of alloying materials added to the aluminum to give specific properties to the finished product according to customer's requirements.

A continuous flow of molten aluminum, produced from the smelter at the rate of S tons per day, passes to holding furnaces where high quality scrap from previous runs and/or alloying materials may be added before the metal is cast into long cylindrical rods by a direct casting process. This involves releasing the molten metal into a number of circular moulds, of the same diameter, lying on a casting table and surrounded by a water-cooled jacket. As the molten aluminum cools, it solidifies around the side of each mould. The base of the table is lowered, allowing more metal to enter, until a certain depth is reached and the rods produced have the required length.

This simultaneous casting of several rods is termed a drop and the depth of casting, the drop length.

The butt ends of the rods are removed and the remainder swan into logs. Each rod is cut into logs of the same length in order to minimize the frequency of setting the saw. Rods cast surplus to requirements for sawing and logs cut surplus to orders, may enter inventory or, together with the butt ends, be treated as scrap to be recycled at additional cost.

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Given customers orders for one month, the problem is to find the most efficient pattern of casting and sawing that meets the demand.

PROBLEM STATEMENT

For aluminum cast into rods of a given diameter, let

L_i = Cast lengths of the aluminum rods
l_j = Lengths of the aluminum logs on order
D_j = Monthly demand for logs of length l_j.

When a rod of length L_i is cut into logs of order length l_j, let

n_{ij} = No. of logs obtained
$\tilde{r}_{ij}$ = Length left over (fuzzy scrap).

Clearly

$$n_{ij} = \frac{L_i}{\in l_j}$$

where [ ] denotes integer part and

$$L_i = n_{ij}\, l_j + \tilde{r}_{ij}, \qquad 0 \le \tilde{r}_{ij} \prec l_j.$$

Further, let
M = No. of rods produced per drop (fixed)
x_i = No. of drops to length L_i (drop variables)
z_{ij} = No. of rods of length L_i cut into logs of length l_j (sawing variables).

The following constraints apply:

(i) The No. of rods of length L_i that are swan cannot exceed the number produced, hence

$$\sum_{j} z_{ij} \le Mx_i \qquad \forall i,$$

(ii) To meet demand,

$$\sum_{j} n_{ij} z_{ij} \ge D_j \qquad \forall j,$$

(iii) Restrictions on the casting lengths

$$L_{min} \le L_i \le L_{max} \qquad \forall i,$$

where L_{min} and L_{max} are the minimum and maximum casting lengths,

(iv) Non-negativity

$$x_i \geq 0, \text{ and integer} \qquad \forall i,$$
$$z_{ij} \geq 0 \text{ and integer} \qquad \forall i, j.$$

The company is interested in maximizing the 'recovery' of the amount of aluminum that is cast, that is maximizing the recovery function R defined by

$$R = \frac{\text{Amount sold}}{\text{Amount cast}} = \frac{\sum_j l_j D_j}{\sum_i M x_i L_i}.$$

The numerator $\sum_j l_j D_j$ is a constant k (say), hence maximizing R is equivalent to minimizing the denominator $\sum_i M x_i L_i$.

The problem is now formulated as follows:

Find nonnegative integers x_i, z_{ij} and positive real numbers L_i so so as to solve:

P_1 :

$$\text{minimize } C = \sum_i M x_i L_i$$

subject to

$$\sum_j z_{ij} \leq M x_i \qquad \forall i,$$
$$\sum_j \left\lfloor \frac{L_i}{l_j} \right\rfloor z_{ij} \geq D_j \qquad \forall j,$$
$$L_{min} \leq L_i \leq L_{max} \qquad \forall i,$$

This is a mixed-integer optimization problem having a quadratic objective function subject to constraints, some linear and some quadratic, the latter involving integer parts.

A SIMPLIFIED PROBLEM

The quadratic terms and the appearance of integer parts of decision variables can be avoided if the drop length L_i is not a decision variable.

To this end, we introduce a heuristic to fix the possible values for L_i by considering the concept of a 'zero scrap drop' ZSD that is, a drop which yields no scrap when all the rods are cut into a given order length. Initially, we will consider only one ZSD drop length for each order length.

Let L_j = length of a ZSD drop that is cut into logs l_j. Then $L_j = \phi_j l_j$ with ϕ_j an integer such that $L_{min} \leq \phi_j l_j \leq L_{max}$. Clearly $M\phi_j$ is the number of logs of length l_j obtained from a ZSD drop.

The heuristic referred to above is to meet as much of the demand for logs as possible by ZSD's.

Let w_j = No. of ZSD drops cut into logs of length l_j. Then

$$w_j M\phi_j \leq D_j \leq (w_j + 1) M\phi_j \qquad \forall j.$$

We call $d_j = D_j - w_j M\phi_j$, the demand for logs not met by ZSD drops, the reduced demand for logs of length l_j. We note that:

$$0 \leq d_j \prec M\phi_j$$

and

$$w_j = \frac{D_j}{M\phi_j} \; .$$

The simplified problem is concerned with maximizing the recovery when further drops of length L_i-restricted to the set of ZSD drop lengths-are used to meet the reduced demand. The decision variables x_i, z_{ij} from now on refer to non ZSD drops with

x_i = The No. of additional drops of length L_i,
z_{ij} = The No. of rods of length L_i from non ZSD drops cut into lengths l_j

and

$$n_{ij} = \frac{L_i}{l_j} \; .$$

The recovery from the additional drop is maximized when $C = \sum_i Mx_iL_i$ is minimized. Hence the simplified problem is written as:

P_2:

$$\text{minimize } C' = \sum_i x_iL_i$$

subject to

$$\sum_j z_{ij} \leq Mx_i \qquad \forall i,$$

$$\sum_i n_{ij}z_{ij} \geq d_j \qquad \forall j,$$

x_i, z_{ij} non negative integers.

where:

$$d_j = D_j - Mw_jL_j / l_j$$

$$w_j = \frac{D_j}{M\phi_j} = \frac{D_jl_j}{ML_j}$$

$$n_{ij} = \frac{L_i}{l_j}$$

In practice, the set of ZSD drop lengths are specified at the outset. This means that the number x_i of additional drops, of length L_i is not a decision variable. Hence there is no objective function for problem P_1 since C' will be a constant whose value is known once the ZSD drop lengths are known. The recovery R will also be known once the ZSD drop lengths are specified and hence it cannot be used to differentiate between feasible solutions to the constraints of problem P_2.

To differentiate between them, another objective function, or set of objectives, is required. Suitable objectives to be minimized are:

(i) The value of the aluminum that goes inventory after sawing,
(ii) The value of the aluminum that goes to be recycled,
(iii) Some combination of (i) and (ii).

In view of what has been said, write the constraints of problem P_1 in the form:

$$\sum_{j} z_{ij} = s_i \qquad i = 1, 2, ..., n,$$

$$\sum_{i} n_{ij} z_{ij} - p_j = d_j \qquad j = 1, 2, ..., m,$$

where the right-hand side quantities $s_i = Mx_i$ are now constants and p_j is the number of logs of length l_j produced in excess of the demand for those logs.

If k_j is a measure of the value of a log of length l_j that goes to inventory and $\tilde{c}_{ij}$ is a measure of the value of a butt end of fuzzy length $\tilde{r}_{ij}$ that goes to be recycled, two new possible objectives to be minimized are $\sum_{j} k_j p_j$ and $\sum_{ij} \tilde{c}_{ij} z_{ij}$. These may either be included separately in a multiobjective or combined into a simple objective.

In the earlier case, a multiobjective mixed-integer nonlinear programming cutting stock problem having t-objective functions and with fuzzy scrap $\tilde{r}_{ij}$ can be formulated as follows:

$$\text{(FMCS)}: \qquad \text{minimize} \quad C = (C_1(z,p), C_2(z,p), ..., C_t(z,p)),$$
$$\text{subject to}$$

$$\sum_{j} z_{ij} = s_i,$$
$$\sum_{i} n_{ij} z_{ij} - p_j = d_j,$$
$$i \in \{1, 2, ..., n\}, \quad j \in \{1, 2, ..., m\},$$
$$x_i, z_{ij} \geq 0, \quad \text{non negative integers.}$$

where each objective function in problem (FMCS) has the form:

$$C(z,p) = \sum_{ij} \tilde{c}_{ij} z_{ij} + \sum_{j} k_j p_j ,$$

and

s_i : The number of rods of length L_i available to meet the reduced demand d_j for logs of length l_j,
z_{ij} : The number of rods L_i swan into logs of length l_j,
$\tilde{c}_{ij}$: The value of the fuzzy scrap aluminum $\tilde{r}_{ij}$ when a rod of length L_{ij} is swan into logs of length l_j,
p_j : The over production of logs of length l_j,
k_j : The inventory value of a log of length l_j,
n : The number of different ZSD drop lengths,
m : The number of different lengths of logs in demand.

FUZZY CONCEPTS

Fuzzy set theory has been developed for solving problems in which descriptions of activities and observations are imprecise, vague and uncertain. The term fuzzy refers to the situation in which there are no well-defined boundaries of the set of activities or observations to which the descriptions apply.

A fuzzy set is a class of objects with membership grades. A membership function, which assigns to each object a grade of membership, is associated with each fuzzy set. Usually the membership grades are in [0,1]. When the grade of membership for an object in a set is one, this object is absolutely in that set; when the grade of membership is zero, the object is absolutely not in that set. Borderline cases are assigned numbers between zero and one.

Some techniques for solving fuzzy multiobjective programming problems should be reviewed (Cadenas and Verdegay, 2000; Chen and Fu, 2005; Li and Yu, 2000; Liang, 1999; Mohan and Nguyen, 1998; Ram and Vlach, 2002).

In the following, it is assumed that $\tilde{r}_{ij}$: the lengths left from cutting rods of length L_i into logs l_j are fuzzy scrap and those parameters are characterized by fuzzy numbers.

Now, some necessary definitions from the fuzzy set theory are introduced and the reader is referred to (Dubois and Prade, 1980; Sakawa and Yano, 1989).

A fuzzy number is defined differently by many authors and the most frequently used definition is the following one.

Definition 1: (Dubois and Prade, 1980)

A real fuzzy number $\tilde{r}_{ij}$ is a convex continuous fuzzy subset of the real line R whose membership function, denoted by $\mu_{\tilde{r}_{ij}}(r_{ij})$ and is defined as:

(1) $\mu_{\tilde{r}_{ij}}(r_{ij})$ a continuous mapping from R to the closed interval [0, 1],

(2) $\mu_{\tilde{r}_{ij}}(r_{ij})=0 \qquad \forall r_{ij} \quad (- \quad ,q_1]$,

(3) $\mu_{\tilde{r}_{ij}}(r_{ij})$ is strictly increasing on $[q_1, q_2]$,

(4) $\mu_{\tilde{r}_{ij}}(r_{ij})=1 \qquad \forall r_{ij} \quad [q_2,q_3]$,

(5) $\mu_{\tilde{r}_{ij}}(r_{ij})$ is strictly decreasing on $[q_3, q_4]$,

(6) $\mu_{\tilde{r}_{ij}}(r_{ij})=0 \qquad \forall r_{ij} \quad [q_4,+ \quad]$.

Definition 2: (Sakawa and Yano, 1989)

The α-level set of the fuzzy numbers $\tilde{r}_{ij}$ is defined as the ordinary set $L_\alpha(\tilde{r}_{ij})$ for which the degree of their membership function exceeds the level $\alpha\in[0, 1]$:

$$L_\alpha((\tilde{r}_{ij})=\left\{r_{ij}\in R: \mu_{\tilde{r}_{ij}}(r_{ij})\geq\alpha, \quad i=1,2,...,n; j=1,2,...,m\right\}.$$

It is clear that the level sets have the following consequence:

$$\alpha_1\leq\alpha_2 \text{ if only if } L_{\alpha 1}(\tilde{r}_{ij})\supset L_{\alpha 2}(\tilde{r}_{ij}).$$

Now, for a certain degree $\alpha\in[0,1]$ the (FMCS) problem can be converted into a non-fuzzy α-multiobjective mixed-integer nonlinear cutting stock programming problem, denoted by (α-MMINLCS) and can be written as follows:

$$(\alpha-\text{MMINLCS}): \qquad \text{minimize}\, C=(C_1(z,p), C_2(z,p),...,C_t(z,p)),$$
$$\text{subject to}$$

$$\sum_{j} z_{ij} = s_i,$$

$$\sum_{i} n_{ij} z_{ij} - p_j = d_j,$$

$$i \in \{1, 2, \ldots, n\} \;,\; j \in \{1, 2, \ldots, m\},$$

$$x_i, z_{ij} \geq 0, \quad \text{non negative integers.}$$

$$r_{ij} \in L_\alpha(\tilde{r}_{ij}).$$

where

$$C(z, p) = \sum_{ij} c_{ij} z_{ij} + \sum_{j} k_j p_j$$

In problem (α-MMINLCS) above it should be noted that the parameters c_{ij}, the measure of the value of a butt end of length r_{ij}, are treated as decision variables rather than constants.

Problem (α-MMINLCS) can be rewritten in the following equivalent form:

$$(\alpha - \text{MMINLCS}): \qquad \text{minimize } C = (C_1(z, p), C_2(z, p), \ldots, C_t(z, p)),$$

subject to

$$\sum_{j} z_{ij} = s_i,$$

$$\sum_{i} n_{ij} z_{ij} - p_j = d_j,$$

$$i \in \{1, 2, \ldots, n\} \;,\; j \in \{1, 2, \ldots, m\},$$

$$x_i, z_{ij} \geq 0, \quad \text{non negative integers.}$$

$$u_{ij} \leq r_{ij} \leq U_{ij},$$

where

$$C(z, p) = \sum_{ij} c_{ij} z_{ij} + \sum_{j} k_j p_j$$

provided that u_{ij}, U_{ij} are lower and upper bounds on the variables r_{ij}, respectively.

The solution concept of problem (FMCS) can be stated, via the definition of problem (α-MMINLCS), in the following manner:

Definition 3: (Sakawa and Yano, 1989)

A point (z^*_{ij}, p^*_j) is said to be an α-Pareto optimal solution to problem (α-MMINLCS), if and only if there does not exist another (z_{ij}, p_j), $r_{ij} \in L_\alpha(\tilde{r}_{ij})$ such that:

$$C_t(z_{ij}, p_j) \leq C_t(z^*_{ij}, p^*_j), \qquad \forall t.$$

with strictly inequality holding for at least one t, where the corresponding values of parameters $r^*_{ij} \in L_\alpha(\tilde{r}_{ij})$ are called α-level optimal parameters.

To find an α-Pareto optimal solution to problem (α-MMINLCS), a weighted objective function is minimized by multiplying each objective function in problem (α-MMINLCS) by a weight, then adding them together (Chankong and Haimes, 1983). This leads to find a solution of the following problem P(λ):

$$P(\lambda): \qquad \text{minimize} \ \sum_t \lambda_t C_t(z,p),$$

$$\text{subjet to}$$

$$\sum_j z_{ij} = s_i,$$

$$\sum_i n_{ij} z_{ij} - p_j = d_j,$$

$$i \in \{1,2,\dots,n\} \ , \ j \in \{1,2,\dots,m\},$$

$$x_i, z_{ij} \geq 0, \ \text{non negative integers.}$$

$$u_{ij} \leq r_{ij} \leq U_{ij},$$

such that $\lambda_t \geq 0, \ \forall t$ and $\sum_t \lambda_t = 1.$

It should be noted from (Sakawa and Yano, 1989) that (z^*_{ij}, p^*_j) is an α-Pareto optimal solution to problem (α-MMINLCS) or problem P(λ) with the corresponding α-level optimal parameters $r^*_{ij} \in L_\alpha(\tilde{r}_{ij})$ if there exists $\lambda^*_t \geq 0$ such that (z^*_{ij}, p^*_j) solves P(λ*) and either one of the following conditions holds:

(i) $\lambda^*_t > 0$ for all t,
(ii) (z^*_{ij}, p^*_j) is the unique minimizer of problem P(λ*).

Obviously, problem P(λ) above is a mixed-integer nonlinear programming problem with single-objective function and can be solved using LINGO software along with the branch-and-bound method (Taha, 1975).

In the following, an algorithm is described in finite steps to solve multiobjective mixed-integer nonlinear programming cutting stock problem (FMCS) with fuzzy scrap $\tilde{r}_{ij}$.

This proposed solution algorithm of the problem of concern can be summarized as follows:

SOLUTION ALGORITHM

Step 0

Start with a degree α = α* = 0

Step 1

Determine the points (q_1, q_2, q_3, q_4) for the fuzzy parameters $\tilde{r}_{ij}$ in problem (FMCS) with the corresponding membership function $\mu_{\tilde{r}_{ij}}(r_{ij})$ satisfying assumptions (1)-(6) in Definition 1.

Step 2

Convert problem (FMCS) into the non-fuzzy version of problem (α-MMINLCS).

Step 3

Use the nonnegative weighted sum approach (Chankong and Haines, 1983) to formulate problem P(λ) at certain $\lambda_t \geq 0, \ \forall t$ and $\sum_t \lambda_t = 1.$

Step 4

Find the α-optimal solution of the problem $P(\lambda^*)$ using the LINGO software along with the branch-and-bound method (Taha, 1975).

Step 5

Set $\alpha = (\alpha^* + \text{step}) \in [0, 1]$ and go to step 1.

Step 6

Repeat again the above procedure until the interval [0, 1] is fully exhausted. Then, stop.

PRACTICAL PROBLEM EXAMPLE

Suppose a factory has an order (dj = 400 pieces), where the rods are of the length (L_i = 5 m) for i = 1 and swan into logs of length (l_1 = 50 cm, l_2 = 55 cm, l_3 = 70 cm).
where $n_{ij} = L_i/l_j$, then $n_{11} = 10$, $n_{12} = 9$, $n_{13} = 7$. Also, the inventory values are given as $k_1 = 500$, $k_2 = 600$. The number of rods produced from one drop is (M = 20 rods). There is an additional drop determined by (x_i = 30) for i = 1. It is assumed that the constraint of the over production is $500 \le P_j \le 600$.

In order to minimize the scarp and the inventory, the following multiobjective mixed-integer nonlinear cutting stock problem can be formulated as:

$$\text{minimize} \quad C = (C_1(z,p), C_2(z,p), C_3(z\,p)),$$
$$\text{subject to}$$

$$\sum_j z_{ij} = s_i,$$
$$\sum_i n_{ij}\, z_{ij} - p_j = d_j,$$
$$i \in \{1, 2, ..., n\} \;,\; j \in \{1, 2, ..., m\},$$
$$x_i, z_{ij} \ge 0, \quad \text{non negative integers.}$$

where

$$C_1(z,p) = \tilde{r}_{11}\, z_{11} + k_1\, p_1,$$
$$C_2(z,p) = \tilde{r}_{12}\, z_{12} + k_2\, p_2,$$
$$C_3(z,p) = \tilde{r}_{13}\, z_{13} + k_3\, p_3.$$

We assume that the membership function of the fuzzy parameters has the following trapezoidal form:

$$\mu_{\tilde{r}_{ij}}(r_{ij}) = \begin{cases} 0, & r_{ij} \le q_1, \\ 1 - (\frac{r_{ij} - q_2}{q_1 - q_2})^2 & q_1 \le r_{ij} \le q_2, \\ 1, & q_2 \le r_{ij} \le q_3, \\ 1 - (\frac{r_{ij} - q_3}{q_4 - q_3})^2 & q_3 \le r_{ij} \le q_4, \\ 0, & r_{ij} \ge q_4. \end{cases}$$

Also, it is assumed that the fuzzy scraps $\tilde{r}_{ij}$ are given by the following fuzzy numbers shown below:

	Fuzzy No.			
Fuzzy scraps	q_1	q_2	q_3	q_4
$\tilde{r}_{11}$	5	10	15	20
$\tilde{r}_{12}$	10	20	30	35
$\tilde{r}_{13}$	2	5	7	10

For a certain degree $\alpha = \alpha^* = 0.36$ (say), it is easy to find:

$$6 \le r_{11} \le 19, \qquad 12 \le r_{12} \le 34, \qquad 2.6 \le r_{13} \le 9.4.$$

Therefore, the non-fuzzy multiobjective cutting stock problem can be written in the following form:

$$\text{minimize} \quad C = (r_{11} z_{11} + k_1 p_1; r_{12} z_{12} + k_2 p_2; r_{13} z_{13} + k_3 p_3),$$
$$\text{subject to}$$

$$z_{11} + z_{12} + z_{13} \le 600,$$
$$2 z_{11} + 15 z_{12} \le 300,$$
$$20 z_{12} \ge 200,$$
$$10 z_{11} + z_{12} \ge 100,$$

$$(10 z_{11} - p_1) + (9 z_{12} - p_2) + (7 z_{13} - p_3) = 400,$$

$$6 \le r_{11} \le 19,$$
$$12 \le r_{12} \le 34,$$
$$2.6 \le r_{13} \le 9.4.$$

Using the weighting method (Chankong and Haimes, 1983) by choosing $\lambda_{11} = \lambda_{12} = \lambda_{13} = 1/3$, then the cutting stock problem with a single-objective function will take the following simple form:

$$\text{minimize} \quad C = 1/3(r_{11} z_{11} + 500 p_1) + 1/3(r_{12} z_{12} + 550 p_2) + 1/3(r_{13} z_{13} + 600 p_3),$$

subject to

$$z_{11} + z_{12} + z_{13} \le 600,$$
$$2 z_{11} + 15 z_{12} \le 300,$$
$$20 z_{12} \ge 200,$$
$$10 z_{11} + z_{12} \ge 100,$$

$$(10 z_{11} - p_1) + (9 z_{12} - p_2) + (7 z_{13} - p_3) = 400,$$

$$6 \le r_{11} \le 19,$$
$$12 \le r_{12} \le 34,$$
$$2.6 \le r_{13} \le 9.4.$$

$$500 \le p_1 \le 600,$$
$$500 \le p_2 \le 600,$$
$$500 \le p_3 \le 600.$$

The above mixed-integer nonlinear programming problem can be solved using the LINGO software along with the branch-and-bound method (Taha, 1975) to obtain the following α-Pareto mixed-integer optimal solution:

$$r_{11} = 14, \quad r_{12} = 28, \quad r_{13} = 7.4,$$
$$z_{11} = 9, \quad z_{12} = 10, \quad z_{13} = 246,$$
$$p_1 = 502, \quad p_2 = 500, \quad p_3 = 500,$$
with
$$C = 276075.5$$

It should be noted that a systematic variation of the degree $\alpha \in [0,1]$ will yield another α-Pareto optimal solution and the effectiveness of various values of α-levels will be studied and reported later.

CONCLSIONS

In this study a solution algorithm for solving multiobjective cutting stock problem in the aluminum industry under fuzzy environment has been proposed. It has been considered that the scrap is the fuzzy parameter. The concept of α-level set together with the definition of this fuzzy parameter and its membership function have been introduced. A practical problem example of the method implementation for the solution algorithm has been presented.

In our opinion, many aspects and general questions remain to be studied and explored in the area of multiobjective cutting stock problem in the aluminum industry. There are, however, several unsolved problems should be discussed in the future. Some of these problems are:

- An algorithm is required for treating multiobjective cutting stock problem in the aluminum industry with fuzzy parameters in the resources (the right-hand side of the constraints).
- An algorithm is needed for dealing with multiobjective cutting stock problem in the aluminum industry with fuzzy parameters in the objective functions and in the resources.
- It is required to continue research in the area of large-scale multiobjective cutting stock problem in the aluminum industry under fuzzy environment.
- A parametric study on multiobjective cutting stock problem in the aluminum industry should be carried out for different values of α-level sets of the fuzzy parameters.

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