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Limit-Cycle Oscillation and Divergence Behavior of New Coupled Stability Effects in Aircraft Lateral Dynamics

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Abstract: Based on Hopf bifurcation, the behavior of limit cycle wing rock motion resulting from new couple of effects including dihedral effect derivative and directional stability derivative is presented in this study. To demonstrate the behavior of limit cycle oscillation and the divergence influence of state variables, a complete set of aircraft nonlinear equations of motion including these effects is solved. A candidate mechanism for the wing rock limit cycle is the inertia coupling between an unstable lateral-directional (Dutch roll) mode and a stable longitudinal (Short period) mode. The coupling mechanism is provided by the nonlinear interaction of motion related terms in the complete set of motion equations. The numerical results indicated that the situation of wing rock dynamics becomes more and more critical (divergent influence of roll and sideslip angles) as dihedral effect derivative increases and directional stability derivative decreases. The results also show that dihedral effect is the most significant stability derivative in the determination of wing rock dynamics. Furthermore, the weak influence of roll damping upon the behavior of limit cycle oscillation is demonstrated based on the method presented. A good agreement between the numerical results and the published work is obtained for limit cycle oscillation existence at different values of damping ratio.

Key words: Limit cycle oscillation, nonlinear dynamic system, dihedral effect, wing rock

INTRODUCTION

With the requirements for aircraft operating in moderate to high angle of attack regimes, the associated flight dynamic phenomena have been the subject of much interest recently. One such phenomenon is wing rock which for many aircraft is primarily a limit cycle oscillation in roll. The wing rock motion is both an annoying and dangerous characteristic of many of today's fighter aircraft. Therefore, it is important to know the most significant stability parameters affected in the determination of wing rock. The critical situation of this motion behavior is worthy of serious study. The wing rock motion is characterized as a periodic motion and needs to be avoided at all costs. This would be beneficial to the tactical pilot engaged in high angle of attack maneuvers and may serve to increase his time to place ordinance on the enemy target. Furthermore, these significant stability parameters play an important role in the determination of wing rock dynamics.

At some critical angle of attack, roll oscillation starts to grow in amplitude until it reaches maximum amplitude at which the airplane rocks back and forth continuously. The self-induced motion represents a limit cycle behavior. Nelson showed that wing rock motion is produced due to an unsteady flow field and strong vortical flow structures that can interact with various components of the aircraft (Nelson and Pelletier, 2003).

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From a dynamics point of view, aerodynamic stability derivatives show that wing rock is related to instability in the Dutch roll mode of motion of aircraft. Also, It is important to obtain the stability derivatives from flight data and in developing analytical departure criteria to be used in the design process of new aircraft configurations (Giovanni and Giulietti, 2004). Schmidt characterized the limit cycle of wing rock motion by inertia coupling while modifying the directional stability derivative and preserved time constants of the roll and spiral modes while altering only the Dutch roll mode (Schmidt, 1998).

Research has focused (either experimentally or numerically) on the development of simulation tools for wing rock to study physics of motion. Wind and water tunnel simulation of wing rock has faced many challenges that include model mounting, dependence of simulation on the mounting bearing friction and flow field visualization (Saad, 2000).

The object of this study is to characterize the behavior of a limit cycle oscillation wing rock resulting from new couple of effects including the effective dihedral derivative and the directional stability derivative. Effective dihedral is the change in rolling moment coefficient caused by the variation in sideslip angle. It is quite important to lateral stability, since it aids in damping both the Dutch roll and the spiral modes (Phillips, 2002). Directional stability is the change in yawing moment coefficient resulting from a change in sideslip angle. It determines the Dutch roll natural frequency and affects the spiral stability of aircraft. The present study consists of three steps:

- The couple of stability derivatives are modified in the plant matrix and the Dutch roll instability, roll and spiral modes are altered to demonstrate the behavior of limit cycle wing rock motion.
- The pole placement technique has been used to form the appropriate plant matrix basis of wing rock analysis.
- The complete set of nonlinear equations of motion including these effects was solved. To the best of author's knowledge, no attempts have been made to investigate the effect of this couple of stability derivatives on the behavior of limit cycle oscillation wing rock.

MATERIALS AND METHODS

Aerodynamic Model

Two baseline aircraft are considered in this analysis: A-4D, a Navy attack aircraft and Mig-21-Biz. The airframe fixed-coordinate system and the dimensional aerodynamic influence coefficients of A-4D are a hybrid from the model originally in accord with NASA convention in USA, 1973 (McRuer *et al.*, 1973) and updated at different flight conditions by (Schmidt, 1998). The aerodynamic model, longitudinal and lateral-directional stability derivatives of Mig-21 are a hybrid from model originally used in studies of SED (System Engineering Division) (Balakrishna *et al.*, 1986) and (Emad, 2003).

Aircraft Nonlinear Equations of Motion

The aircraft model in flight regime of interest is not a linear system; the behavior of nonlinear system must be considered. In nonlinear systems, the stable solution is not always an equilibrium point but may be a limit cycle or other periodic motion. Wing rock is one of these limit cycle motions. The motion investigations of this study involve equations in body-fixed axes and include both linearized and complete nonlinear format. In the linear model, which is based upon small perturbations about the initial equilibrium, the lateral-directional (Dutch roll, Spiral and Roll) and longitudinal (Short period and Phugoid) model responses are uncoupled. It was assumed that the aircraft was initially trimmed in level flight and the aircraft velocity remained constant during the ensuing motion. The uncoupled set of linear motion equations was (Schmidt, 1998).

$$\{X'_{Lat-Dir}\} = [A_{Lat-Dir}] \{X_{Lat-Dir}\} \quad (1)$$

$$\{X'_{Long}\} = [A_{Long}] \{X_{Long}\} \quad (2)$$

Where,

$\{X_{Lat-Dir}\}$ = Lateral-Directional state vector, $[\beta \ p \ \phi \ r]^T$.
 $\{X_{Long}\}$ = Longitudinal state vector, $[\alpha \ q]^T$

where, β is the sideslip angle, p is the roll rate, ϕ is the roll angle, r is the yaw rate, α is angle of attack and q is the pitch rate.

The lateral directional plant matrix was defined by:

$$[A_{Lat-Dir}] = \begin{bmatrix} Y_{\beta}/U & 0.0 & (g \cos \Theta_o)/U & -1 \\ L_{\beta} & L_p & 0.0 & L_r \\ 0.0 & 1 & 0.0 & 0.0 \\ N_{\beta} & N_p & 0.0 & N_r \end{bmatrix} \text{ and } [A_{Long}] = \begin{bmatrix} Z_{\alpha}/U & 1 \\ M'_{\alpha} & M'_{\dot{q}} \end{bmatrix}$$

where,

Y_{β} = The change in side force caused by a variation in sideslip angle.
 L_{β} = The dihedral effect.
 L_p = The roll damping.
 L_r = The change in rolling moment caused by yawing.
 N_{β} = The directional stability.
 N_p = The change in yawing moment caused by rolling.
 N_r = The yaw damping.
 U = The free stream velocity.
 Z_{α} = The normal force due to angle of attack.
 M_{α} = The longitudinal stability derivative.
 $M_{\dot{q}}$ = The pitch damping.
 $M'_{\alpha} = M_{\alpha} + [M_{\alpha} Z_{\alpha}]/U$.
 $M'_{\dot{q}} = M_{\dot{q}} + M_{\alpha}$.

The small angle assumption for motion about an initial level flight condition was removed when analyzing the full set of equations. In addition, Euler angle relations were introduced; i.e, Roll angle, ϕ became the Euler angle Φ whereas the pitch angle θ became the Euler angle Θ . The nonlinear relations listed below were added to corresponding terms in the linear Eq. 1-2, to obtain a full set of motion expressions. The NonLinear additions are denoted by (NL) with the subscript denoting the element of the linear expressions to which they apply. Thus,

$$(NL)_{\beta} = p\alpha + g(C_{\Theta}S_{\Phi} - C_{\Theta_o}\Phi)/U \quad (3)$$

$$(NL)_p = q r (I_Y - I_Z) / I_X \quad (4)$$

$$(NL)_{\Phi} = (q S_{\Phi} + r C_{\Phi}) T_{\Theta} \quad (5)$$

$$(NL)_r = q p (I_x - I_y) / I_z \quad (6)$$

$$(NL)_\alpha = -p \beta + g (C_\Theta C_\Phi - C_{\Theta_0}) / U \quad (7)$$

$$(NL)_q = p r (I_z - I_x) / I_y + M_\alpha (NL)_\alpha \quad (8)$$

In Eq. 3-8, trigonometric terms are shown as:

$$C_\Phi = \cos \Phi, \quad S_\Phi = \sin \Phi, \quad C_\Theta = \cos \Theta \quad \text{and} \quad T_\Theta = \tan \Theta$$

An Euler angle relationship for Θ was introduced by:

$$\dot{\Theta} = q C_\Phi - r S_\Phi \quad (9)$$

The nonlinear set of motion relations were obtained by including Eq. 2-8 as an additional set of terms added to Eq. 1-2. In addition, a seventh relation was added, Eq. 9, to account for the dynamics of the Euler angle Θ . As a result, the state vector for the nonlinear system became:

$$\{X_{NL}\} = [\beta \ p \ \Phi \ r \ \alpha \ q \ \Theta]^T \quad (10)$$

Pole Placement Technique

The closed-loop pole locations have a direct impact on time response characteristics such as rise time, settling time and transient oscillations. Root locus uses compensator gains to move closed-loop poles to achieve design specifications for SISO systems. However, the use of state-space techniques to assign closed-loop poles is known as pole placement. Pole placement requires a state-space model of the system defined as:

$$\begin{aligned} \dot{X} &= AX + bu \\ y &= CX + Du \end{aligned} \quad (11)$$

where,

u = The vector of control inputs.

X = The state vector.

y = The vector of measurements. Under state feedback $u = -KX$.

The closed-loop dynamics are given as

$$\dot{X} = (A - BK)X \quad (12)$$

and the closed-loop poles are the eigenvalue of $A - BK$. The gain matrix K is determined to assign these poles to any desired location in the complex plane (provided that (A, B) is controllable). We cannot implement the state-feedback law $u = -KX$ unless the full state X is measured. However, the state estimates ξ such that the law $u = -K\xi$ retains similar pole assignment and closed-loop properties. This is achieved by designing a state estimator (or observer) of the form

$$\dot{\xi} = A\xi + Bu + L(y - C\xi - Du) \quad (13)$$

The estimator poles are the eigenvalues of A-LC, which can be arbitrarily assigned by proper selection of the estimator gain matrix L, provided that (A, C) is observable. Generally, the estimator dynamics should be faster than the controller dynamics (eigenvalues of A-BK).

Validation of State Variables Determination and Limit Cycle Oscillation (LCO) Existence

Wing rock manifests itself as diverging oscillation which becomes periodic. The motion is characterized by large change in roll angle (Liebst and Nolan, 1997). The verification of the state variables and the LCO existence consists of the following steps:

- The initial condition for integration was based on lateral-directional Dutch roll mode shape (Eigen vector). Limit cycle oscillation existence was verified by varying the initial conditions from start up on both interior and exterior of limit cycle trajectory. The initial condition determination for both aircraft models is presented in Appendix A.
- The numerical results show a good agreement with the published work (Schmidt and Wright, 1991) and (Schmidt, 1998) in roll angle determination at different values of Dutch roll damping. The comparative results are shown in Fig. 4. In present study, the values of directional stability derivative and dihedral effects derivative are obtained from the Trigger parameter analysis (Liebst and Nolan, 1992). These values are altered during the process until the condition of the onset of wing rock is met.

Predication Procedure

The aircraft stability derivatives, inertia data and physical characteristics of selected models are obtained from previous indicated references. The ratio of short period to Dutch roll frequencies is 2.0 (minimum limit cycle oscillations) for A-4D aircraft and 2.4 for Mig-21 aircraft. The following procedures are used in current study:

- The first step in this process is to construct the lateral-directional and longitudinal plant matrices of the baseline selected aircraft.
- The couple of stability derivatives ($N_{\dot{\beta}}$ and $L_{\dot{\beta}}$) are modified in plant matrix and the Dutch roll instability, roll and spiral modes, are altered according to modified stability derivatives to demonstrate the behavior of limit cycle wing rock motion. These modifications have been done to satisfy the condition of lightly damp system from dynamic point of view.
- There are many ways to numerically alter Dutch roll dynamic instability, roll and spiral modes. The method considered here is the pole placement technique to form the appropriate plant matrix basis of the wing rock analysis.
- The complete set of nonlinear motion equations is solved to characterize the state variables including this couple of effects.

RESULTS AND DISCUSSION

Two baseline aircraft, an A-4D with $M = 0.4$ at sea level and a Mig-21 with $M = 0.6$ at $H = 10$ Km, were taken into consideration in this analysis.

A representative roll angle to a limit oscillation for A-4D aircraft is shown in Fig. 1. The elliptical trajectory behavior of β vs. ϕ explains the oscillatory behavior of the map at constant amplitude. This behavior is shown in Fig. 2. It was assumed reasonable to use linear aerodynamics for a weak Dutch-roll instability when the inertial coupling resulted in small angle perturbations (Schmidt and Wright, 1991). The dynamic response of nonlinear oscillator was presented in the more complex situation of aircraft wing rock. It is apparent from Fig. 3 that the limit cycle, when $\zeta_{D-R} < 0$ corresponds

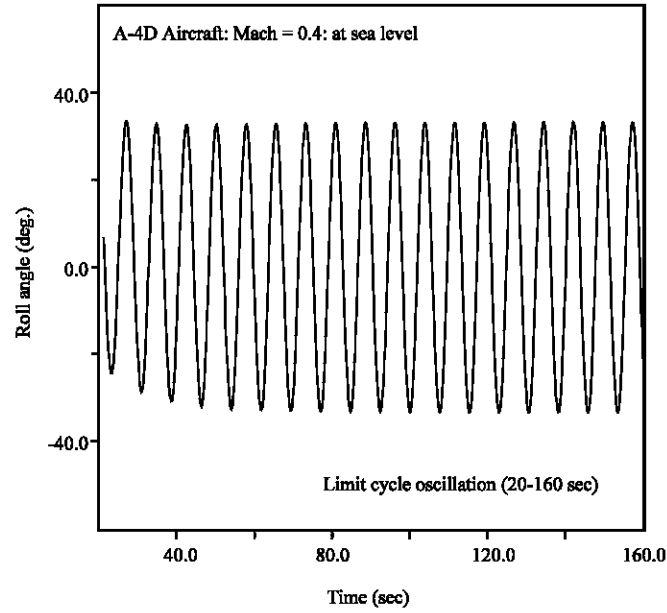


Fig. 1: Roll limit-cycle at $\zeta_{D-R} = -0.02$ $N_{\beta} = 0.8 \text{ sec}^{-2}$; $L_{\beta} = 11.5 \text{ sec}^{-2}$

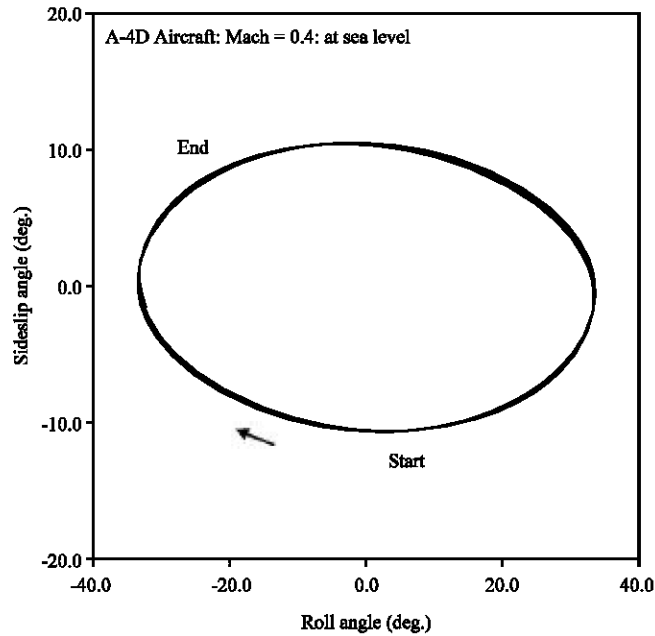


Fig. 2: Limit-cycle phase-plane trajectory: $\zeta_{D-R} = -0.02$ $N_{\beta} = 0.8 \text{ sec}^{-1}$; $L_{\beta} = -1.5 \text{ sec}^{-2}$

to a pitchfork type of Hopf bifurcation because the static equilibrium does not exist. The increase in the amplitude of roll angle is directly related to the rise in the level of initial Dutch roll instability at modified values of N_{β} and L_{β} (Fig. 4).

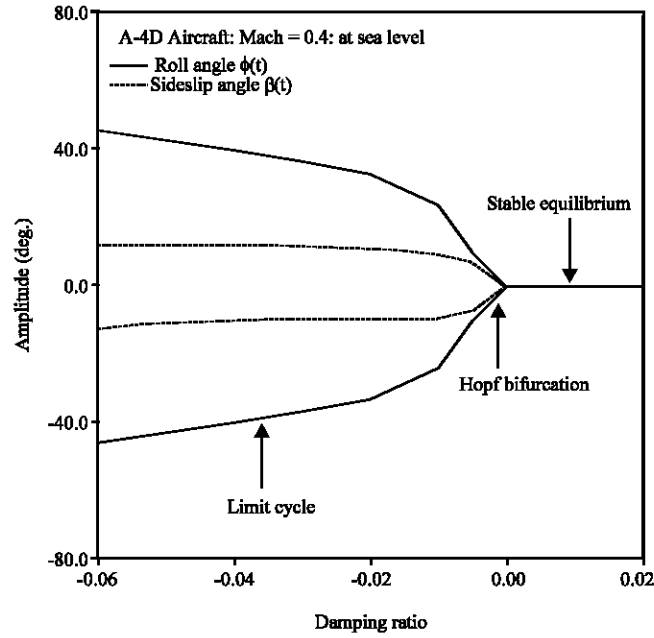


Fig. 3: Effect of damping ratio ζ_{D-R} upon limit cycle amplitude: $L_{\beta} = -11.5 \text{ sec}^{-2}$; $N_{\beta} = 0.8 \text{ sec}^{-1}$

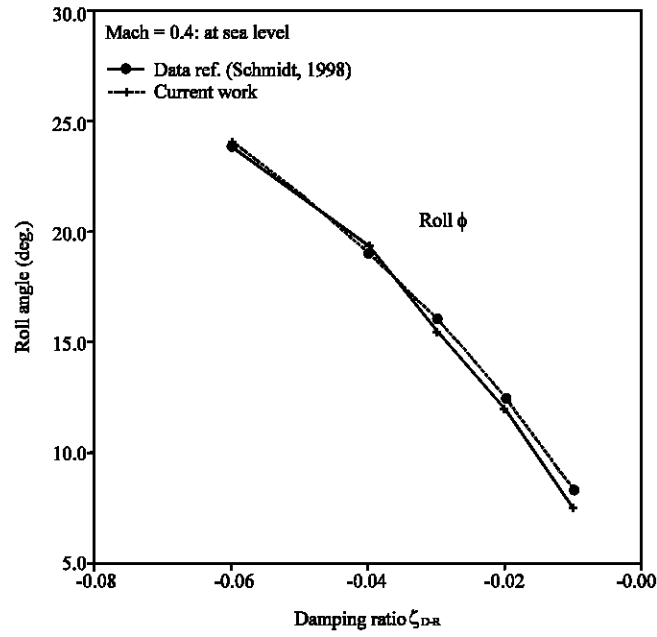


Fig. 4: Comparison results of the limit cycle amplitude (roll angle) as a function of damping ratio. A-4D aircraft

The effects of modified L_{β} on limit cycle amplitude (roll and sideslip angles) at $\zeta_{D-R} = -0.02$ and $N_{\beta} = 0.8 \text{ sec}^{-2}$ are shown in Fig. 5. A sharp increase in both roll and sideslip angles is obtained when

Table 1: Desired roots of Dutch roll instability, roll and spiral modes for A-4D aircraft

N_β (sec ⁻²)	ζ_{D-R}	L_β (sec ⁻²)	λ_{D-R}	λ_{roll}	λ_{spiral}	ϕ (deg.)
0.8	-0.02	-11.5	0.0167±0.8389i	-1.7621	0.0187	33.4
		-13.5	=	-1.7597	0.0153	34.7
		-14.5	=	-1.7585	0.0136	46.9
		-15.0	=	-1.7579	0.0128	67.4
0.7	-0.03	-11.5	0.0245±0.8169i	-1.7621	0.0187	36.9
		-13.5	=	-1.7597	0.0153	38.9
		-14.5	=	-1.7585	0.0136	58.4
		-15.0	=	-1.7579	0.0128	82.1
0.6	-0.04	-11.5	0.03185±0.7963i	-1.7621	0.0187	40.3
		-13.5	=	-1.7597	0.0153	45.9
		-14.5	=	-1.7585	0.0136	70.3
		-15.0	=	-1.7579	0.0128	95.4
0.5	-0.06	-11.5	0.0466±0.7766i	-1.7621	0.0187	46.2
		-13.5	=	-1.7597	0.0153	59.9
		-14.5	=	-1.7585	0.0136	91.4
		-14.8	=	-1.7581	0.0131	106.8

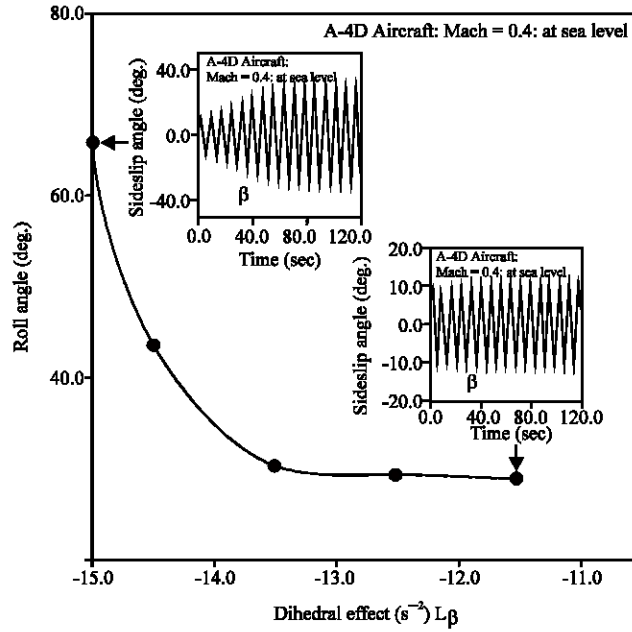


Fig. 5: Effects of modified L_β on limit cycle amplitude (roll and sideslip angles): $\zeta_{D-R} = -0.02$; $N_\beta = 0.8 \text{ sec}^{-2}$

the level of dihedral derivative increases in negative sign. Table 1 presents the detailed results with the desired roots of Dutch roll instability, spiral and roll modes. The augmentation of roll angle is directly proportional to dihedral effect values. This influence is shown in Fig. 6.

New behavior of limit cycle is obtained when the value of L_β is changed from 15.45 to 15.5 sec⁻² (Fig. 7). A four point attractor is predicted with sharp change in the values of roll angle. This influence shows the significant role of the dihedral derivative when it is coupled with directional stability derivative (Fig. 8).

To confirm the importance of this coupled stability effects, the baseline model of Mig-21 attack aircraft was taken in consideration in this study. The phase plan trajectory (periodic orbit) shows the infinite oscillation of constant amplitude (Fig. 9). The effect of damping ratio upon a limit cycle is shown in Fig. 10.

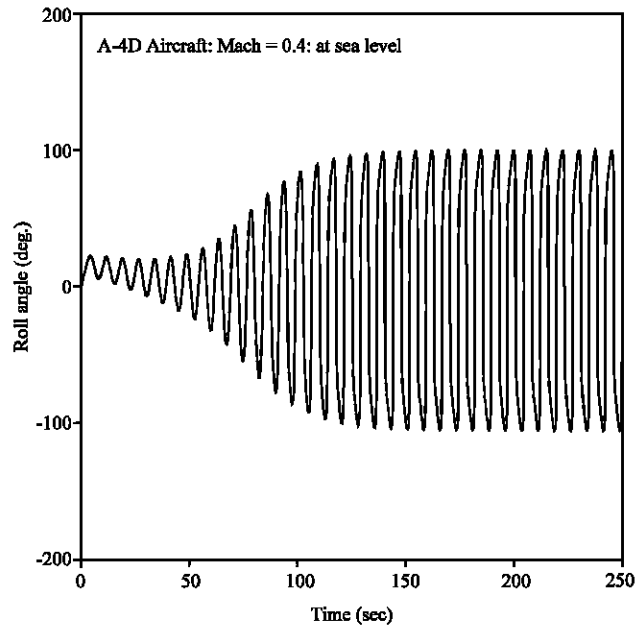


Fig. 6: Roll limit cycle build up, $\zeta_{D-R} = -0.02$ $N_{\beta} = 0.8 \text{ sec}^{-2}$; $L_{\beta} = 15.45 \text{ sec}^{-2}$; two point attractor

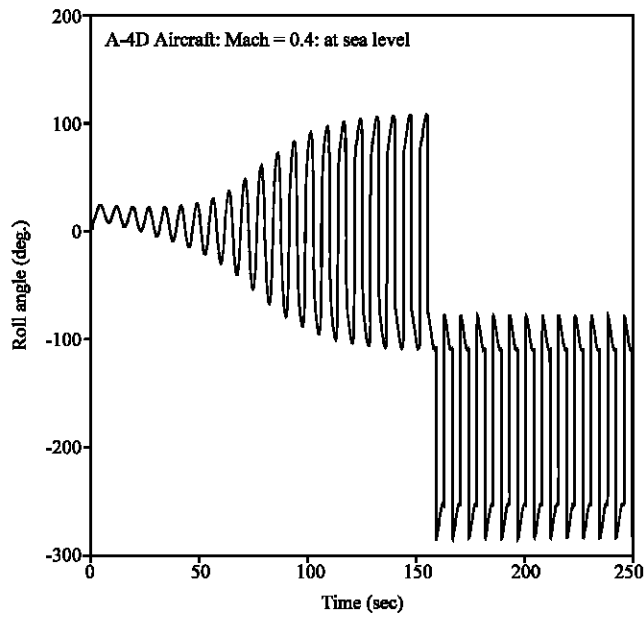


Fig. 7: Roll limit cycle build up, $\zeta_{D-R} = -0.02$ $N_{\beta} = 0.8 \text{ sec}^{-2}$; $L_{\beta} = 15.5 \text{ sec}^{-2}$

The increase in roll angle is directly related to the rise in the value of dihedral derivative. This behavior is shown in Fig. 11 and 12. Table 2 introduces the couple of stability derivatives effects on roll angle in lateral dynamics of this model.

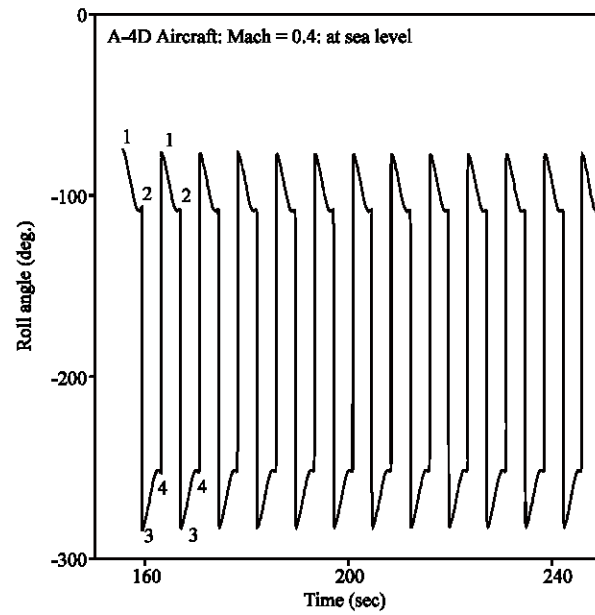


Fig. 8: Limit cycle oscillation of roll angle at time (155-250) sec. $\zeta_{D-R} = -0.02$ $N_{\beta} = 0.8 \text{ sec}^{-2}$: $L_{\beta} = 15.5 \text{ sec}^{-2}$: four point attractor

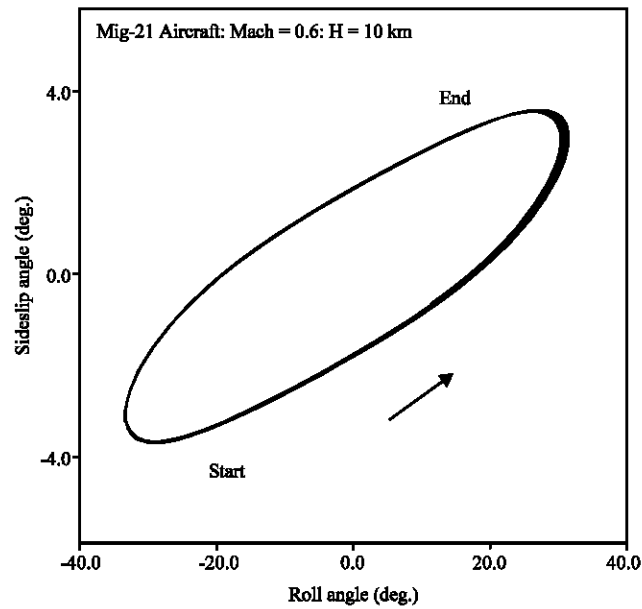
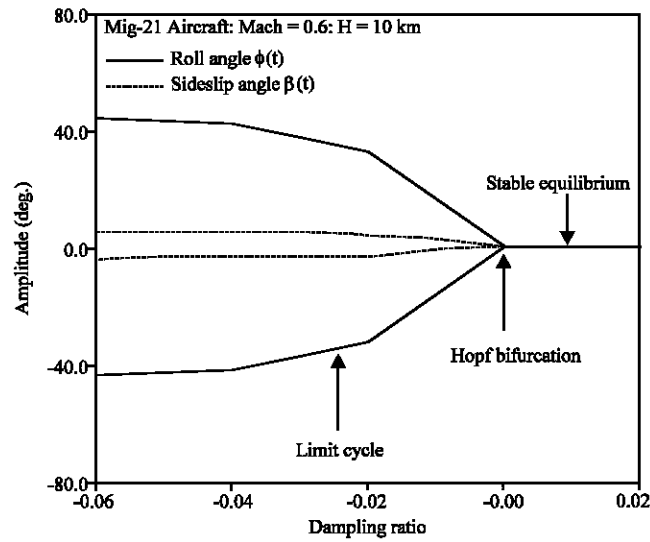
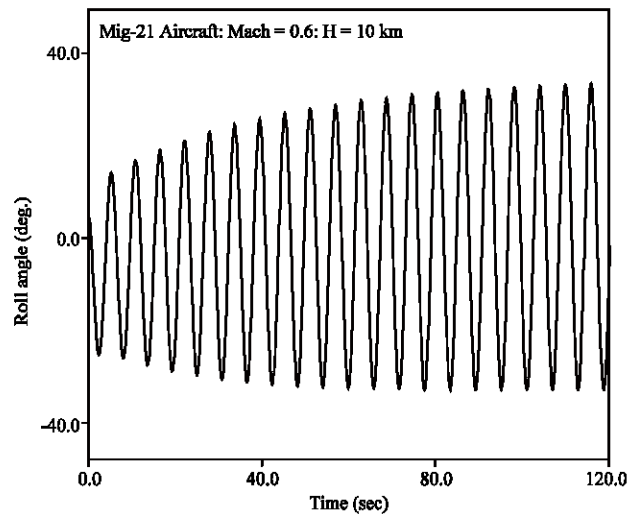


Fig. 9: Limit cycle phase-plane trajectory, $\zeta_{D-R} = -0.02$ $N_{\beta} = 1.25 \text{ sec}^{-2}$: $L_{\beta} = 11.5 \text{ sec}^{-2}$

Finally, the numerical results indicated that the situation of wing rock dynamics becomes more and more critical (divergent influence of roll and sideslip angles) as dihedral effect derivative increases and directional stability derivative decreases.

Table 2: Desired roots of Dutch roll instability, roll and spiral modes for Mig-21 aircraft

N_β (sec ⁻²)	ζ_{D-R}	L_β (sec ⁻²)	λ_{D-R}	λ_{roll}	λ_{spiral}	ϕ (deg.)
1.25	-0.02	-11.5	$0.0241 \pm 1.205i$	-0.7861	0.0064	32.80
		-13.5	=	-0.8255	0.0045	40.01
		-15.0	=	-0.8536	0.0033	46.00
0.7	-0.04	-11.5	$0.0396 \pm 0.9915i$	-0.7861	0.0064	36.19
		-13.5	=	-0.8255	0.0045	42.65
		-15.0	=	-0.8536	0.0033	49.78
0.6	-0.06	-11.5	$0.057 \pm 0.95i$	-0.7861	0.0064	42.50
		-13.5	=	-0.8255	0.0045	45.90
		-15.0	=	-0.8536	0.0033	56.48
0.5	-0.07	-11.5	$0.0635 \pm 0.907i$	-0.7861	0.0064	44.35
		-13.5	=	-0.8255	0.0045	51.30
		-15.0	=	-0.8536	0.0033	58.83

Fig. 10: Effect of damping ratio ζ_{D-R} upon limit cycle amplitude: $L_\beta = -11.5 \text{ sec}^{-2}$ Fig. 11: Roll limit cycle build up, $\zeta_{D-R} = -0.02$ $N_\beta = 1.25 \text{ sec}^{-2}$; $L_\beta = -11.5 \text{ sec}^{-2}$

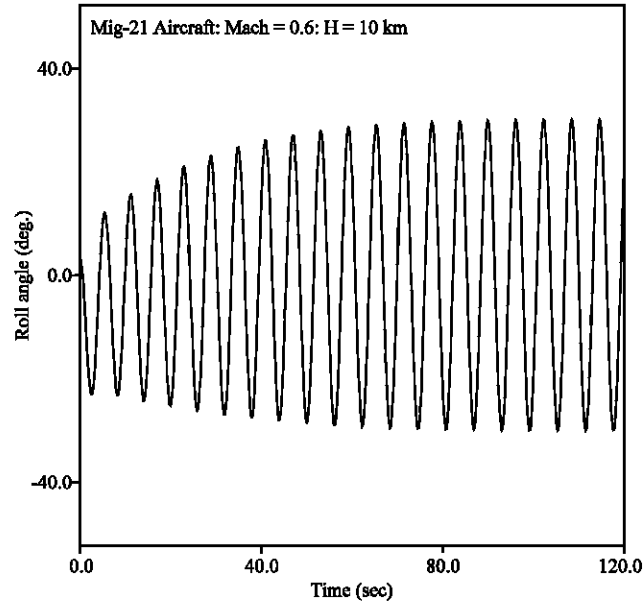


Fig. 12: Roll limit cycle build up, $\zeta_{D-R} = -0.02$ $N_{\beta} = 1.25 \text{ sec}^{-2}$; $L_{\beta} = -13.5 \text{ sec}^{-2}$

CONCLUSIONS

Based on Hopf bifurcation, the behavior of limit cycle wing rock motion resulting from new couple of effects, specifically dihedral derivative and directional stability derivative, is presented in this study. The complete set of nonlinear equations of motion is solved. The following conclusions have been obtained in the present study:

- The numerical results indicated that the dihedral effect derivative is the most significant stability derivative in the determination of wing rock.
- The situation of wing rock dynamics becomes more and more critical (divergent influence of roll and sideslip angles) as dihedral effect derivative increases and directional stability derivative decreases.
- Aircraft roll damping exerted a weak influence upon the behavior of limit cycle when the directional stability derivative was coupled with the dihedral derivative.

APPENDIX A

Initial Condition Calculation:

For Mig-21 aircraft model:

$$[A_{\text{Lat-Dir}}] = \begin{bmatrix} -0.026 & 0.0 & 0.055 & -1.0 \\ -11.5 & -0.479 & 0.0 & 0.248 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 1.25 & 0.0069 & 0.0 & -0.0153 \end{bmatrix}$$

The state vector of the system is:

$$X = [\beta \ p \ \phi \ r \ \alpha \ q \ \theta]^T$$

The initial condition for the integration was based on lateral-directional Dutch roll mode shape (Eigen vector).

$$X_o = \begin{Bmatrix} \beta \\ p \\ \phi \\ r \end{Bmatrix} = \begin{Bmatrix} 0.0904 \\ 0.7657 \\ 0.6303 \\ -0.0909 \end{Bmatrix}$$

Normalized with respect to β , as:

$$X_o = \begin{Bmatrix} \beta \\ p \\ \phi \\ r \end{Bmatrix} = \begin{Bmatrix} 1.0 \\ 8.47 \\ 6.972 \\ -1.005 \end{Bmatrix}; \quad \alpha = \theta = 0.0 \text{ and } q = 0.0$$

The initial condition (i.c) is $\beta = 2 \text{ deg (0.0349 rad)} \times \text{mode shape}$ then

$$X_o = [0.0349 \ 0.29565 \ 0.24336 \ -0.03508 \ 0.0 \ 0.0 \ 0.0]^T$$

In the same manner, the initial condition of integration for A-4D aircraft is

$$X_o = [0.0349 \ 0.2239 \ 0.0605 \ -0.136 \ 0.0 \ 0.0 \ 0.0]^T$$

REFERENCES

- Balakrishna *et al.*, 1986. Stability and control characteristics of fighter aircraft. Special Publication, 8602, (SED), India.
- Emad, N., 2003. Numerical investigation of roll inertia cross coupling and its effects on stability and response of supersonic aircraft. Ph.D Thesis, University of Technology, Baghdad.
- Giovanni and F. Giulietti, 2004. Unified algebraic approach to approximation of lateral-directional modes and departure criteria. J. Guid. Control and Dynamics, 27: 724-728.
- Liebst and Nolan, 1992. Method for the prediction of the onset of wing rock. J. Aircraft, 31: 1419-1421.
- McRuer *et al.*, 1973. Aircraft Dynamic and Automatic Control. Princeton University Press.
- Nelson and A. Pelletier, 2003. The Unsteady aerodynamics of slender wings and aircraft undergoing large amplitude maneuvers. Prog. Aerospace J., 39: 185-248.
- Phillips, W.F., 2002. Analytical solution of wing dihedral effect. J. Aircraft, 39: 514-516.
- Saad, A.A., 2000. Simulation and analysis of wing rock physics for generic fighter model with three degrees of freedom. Ph.D Thesis, AFIT/DS/ENY/00-01, Graduate School of Engineering and Management, Air Force Institute of Technology, Wright-Patterson AFB, OH.
- Schmidt and Wright, 1991. Aircraft wing rock by inertia coupling. AIAA Paper, 2885, pp: 367-372.
- Schmidt, 1998. Introduction to aircraft flight dynamics. Naval Postgraduate School, AIAA Education Series, Monterey, California.