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Synchronization and Adaptive Synchronization of Hyperchaotic Lü Dynamical System

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Abstract: This study addresses the synchronization and adaptive synchronization problem of a hyperchaotic dynamical system with unknown system parameter. This technique is applied to achieve synchronization for hyperchaotic Lü system. Lyapunove direct method of stability is used to prove the asymptotic stability of solutions for the error dynamical system. Numerical simulations results are used to demonstrate the effectiveness of the proposed control strategy.

Key words: Hyperchaotic Lü system, synchronization, adaptive synchronization, nonlinear control function, lyapunove function, numerical simulation

INTRODUCTION

In mathematics and physics, chaos theory describes the behavior of certain nonlinear dynamical systems that under specific conditions exhibit dynamics that are sensitive to initial conditions (popularly referred to as the butterfly effect). As a result of this sensitivity, the behavior of chaotic systems appears to be random, because of an exponential growth of errors in the initial conditions. This happens even though these systems are deterministic in the sense that their future dynamics are well defined by their initial conditions and there are no random elements involved. This behavior is known as deterministic chaos, or simply chaos.

Chaotic behavior has been observed in the laboratory in a variety of systems including electrical circuits, lasers, oscillating chemical reactions, fluid dynamics and mechanical and magneto-mechanical devices. Observations of chaotic behaviour in nature include the dynamics of satellites in the solar system, the time evolution of the magnetic field of celestial bodies, population growth in ecology, the dynamics of the action potentials in neurons and molecular vibrations. Everyday examples of chaotic systems include weather and climate (Sneyers, 1998). There is some controversy over the existence of chaotic dynamics in the plate tectonics and in economics (Serletis and Gogas, 1997, 1999, 2000).

In recent years, researches on chaos control and synchronization have attracted increasing attention due to its potential applications to physics, chemical reactors, control theories, biological networks, artificial neural networks and secure communication (Ott *et al*., 1990; Pyragas, 1992; Tao *et al*., 2005; Wang and Tian, 2004).

Chaos synchronization has been observed in various fields. Fujisaka and Yamada (1983) showed criterion of chaos synchronization using Lyapunov exponents. Since Pecora and Carroll (1990) proposed a synthesis method for synchronized chaotic systems, many methods have been proposed and its applications in chaos communication provide very fascinating studies (Pecora and Carroll, 1990).

Synchronization of chaos is a phenomenon that may occur when two, or more, chaotic oscillators are coupled, or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly

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effect, which causes the exponential divergence of the trajectories of two identical chaotic system started with nearly the same initial conditions, having two chaotic system evolving in synchrony might appear quite surprising. However, synchronization of coupled or driven chaotic oscillators is a phenomenon well established experimentally and reasonably understood theoretically.

It has been found that chaos synchronization is quite a rich phenomenon that may present a variety of forms. When two chaotic oscillators are considered, these include: identical synchronization, generalized synchronization, phase synchronization, anticipated and lag synchronization and amplitude envelope synchronization. All these forms of synchronization share the property of asymptotic stability. This means that once the synchronized state has been reached, the effect of a small perturbation that destroys synchronization is rapidly damped and synchronization is recovered again. Mathematically, asymptotic stability is characterized by a positive Lyapunov exponent of the system composed of the two oscillators, which becomes negative when chaotic synchronization is achieved.

Hyperchaotic systems have received much attention in recent years, particularly the hyperchaotic Rossler attractors and its variation, which are obtained by introducing a quadratic term to a linear system (Rossler, 1979a; Liao and Huang, 1999), or by using piecewise-linear systems (Matsumat *et al*., 1986; Tsubone and Saito, 1998). Owing to their strong resistance to dynamics reconstruction, hyperchaotic systems are more suitable for some special engineering applications such as chaos-based encryption and secure communication.

Hyperchaotic systems is usually classified as a chaotic system with more than one positive Lyapunov exponent, indicating that the chaotic dynamics of the system are expanded in more than one direction giving rise to a more complex attractor. In recent years, hyperchaos has been studied with increasing interests, in the fields of secure communication (Udaltsov *et al*., 2003), multimode lasers (Shahverdiey *et al*., 2004), nonlinear circuits (Barbara and Silvano, 2002), biological networks (Neiman *et al*., 1999), coupled map lattices (Zhan and Yang, 2000) and so on.

Since the discovery of the hyperchaotic Rossler (1979b) system, many hyperchaotic systems have been developed such as the hyperchaotic MCK circuit (Matsumot *et al*., 1986), the hyperchaotic Chen system (Li *et al*., 2005; Yan, 2005), hyperchaotic Lü system (Elabbasy *et al*., 2006), etc.

First we need to recall some concepts and terms from synchronization theory.

Consider the systems of differential equation:

$$\dot{x} = f(x) \tag{1}$$

and

$$\dot{y} = g(y, x) \tag{2}$$

where, $x \in R^n$, $y \in R^n$, f, g: $R^n \rightarrow R^n$ are assumed to be analytic function

Let $x(t, x_0)$ and $y(t, y_0)$ be solutions to (1) and (2), respectively. The solutions $x(t, x_0)$ and $y(t, y_0)$ are said that are synchronized if

$$\lim_{t\to\infty} \|x(t, x_0) - y(t, y_0)\| = 0$$

SYSTEM DESCRIPTION

In this study we study the synchronization of the hyperchaotic Lü system (Elabbasy *et al*., 2006)

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= -xz + cy + w \\ \dot{z} &= xy - bz \\ \dot{w} &= z - rw \end{aligned} \tag{3}$$

where, a, b, c and r are four unknown uncertain parameters. This new system exhibits a chaotic attractor at the parameter values a = 15, b = 5, c = 10 and r = 1 (Fig. 1).

The divergence of the flow Eq. 3 is given by:

$$\nabla.F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} + \frac{\partial F_4}{\partial w} = -a + c - b - r < 0.$$

where, $F = (F_1, F_2, F_3, F_4) = (a(y - x),\ -xz + cy + w,\ xy - bz,\ z - rw)$

Hence the system is dissipative when: c<a+b+c

The system has three equilibrium points:

$$E_0 = (0,0,0),\ E_+ = (\sigma_1, \sigma_1, \frac{\sigma_1^2}{b}, \frac{\sigma_1^2}{br}),\ E_- = (\sigma_2, \sigma_2, \frac{\sigma_2^2}{b}, \frac{\sigma_2^2}{br})$$

where, $s_1 = \frac{1+\sqrt{1+4bcr^2}}{2d}$ and $s_2 = \frac{1-\sqrt{1+4bcr^2}}{2d}$.

To study the stability of E_0 the associated Jacobian J_0 is

$$J_0 = \begin{bmatrix} -a & a & 0 & 0 \\ -z & c & -x & 1 \\ y & x & -b & 0 \\ 0 & 0 & 1 & -r \end{bmatrix}$$

The characteristic polynomial of the matrix J_0 is given by

$$(\lambda + a)(\lambda - c)(\lambda + b)(\lambda + r) = 0 \tag{4}$$

The eigenvalues are $\lambda_1 = -a$, $\lambda_2 = c$, $\lambda_3 = -b$ and $\lambda_4 = -r$. Then the equilibrium point E_0 is stable if $c < 0$ other with the equilibrium is unstable.

To study the stability of E_+ the associated Jacobian J_+ is

$$J_+ = \begin{bmatrix} -a & a & 0 & 0 \\ -\frac{2cbr^2+1+\sqrt{1+4cbr^2}}{2br^2} & c & -\frac{1+\sqrt{1+4cbr^2}}{2r} & 1 \\ \frac{1+\sqrt{1+4cbr^2}}{2r} & \frac{1+\sqrt{1+4cbr^2}}{2r} & -b & 0 \\ 0 & 0 & 1 & -r \end{bmatrix}$$

The characteristic polynomial of the matrix J_+ is given by

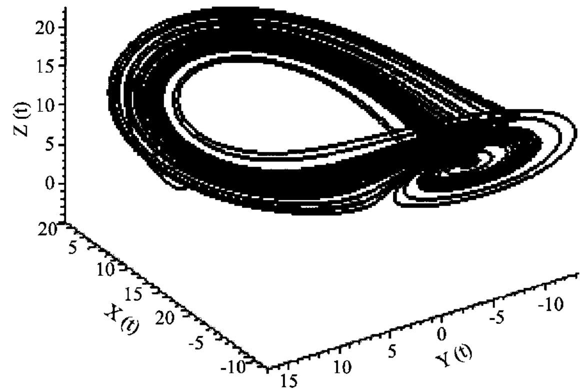

Fig. 1a: Chaotic attractor of hyperchaotic Lü system at a = 15, b = 5, c = 10 and r = 1 in x, y, z subspace

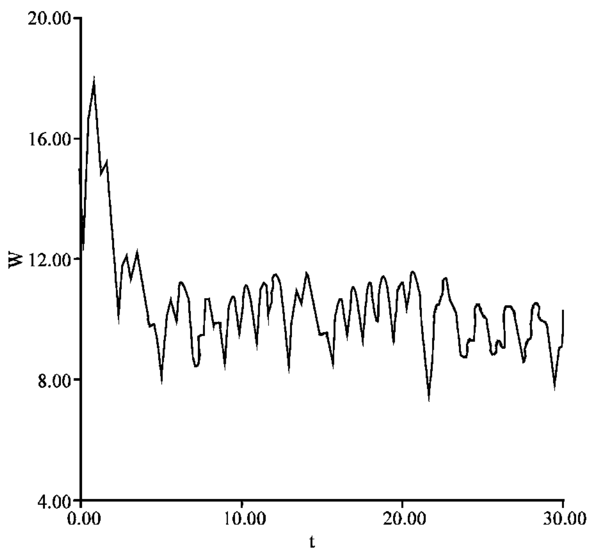

Fig. 1b: Time responses for the variable w(t) of the hyperchaotic Lü system

$$\lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4 = 0 \tag{5}$$

where:

$$c_1 = r + b - c + a$$

$$c_2 = \frac{a + b + 2b^2r^3 + (a + b)\sqrt{1 + 4cbr^2} - 2br^3c + 2abr^3 + 2ab^2r^3}{2br^2}$$

$$c_3 = \frac{3ab + ar + 2ab^2r^3 + (ar + 3ab)\sqrt{1 + 4cbr^2} + 4acb^2r^2}{2br^2}$$

$$c_4 = \frac{a + 4abcr^2 + a\sqrt{1 + 4cbr^2}}{2r}$$

A set of necessary and sufficient conditions for all the roots of Eq. 5 to have negative real parts is given by the well-known Routh-Hurwitz criterion in the following form

$$c_1>0,\ c_1c_2-c_3>0,\ c_1(c_2c_3-c_1c_4)-c_3^2>0 \text{ and } c_1c_4(c_2c_3-c_1c_4)-c_4c_3^2>0$$

i.e.,

$$c_1>0,\ \ c_4>0,\ c_1c_2-c_3>0 \text{ and } c_1(c_2c_3-c_1c_4)-c_3^2>0$$

However, the above values of c_1, c_4 and c_3 guaranteed that $c_1c_2-c_3<0$. Hence the equilibrium point E_+ is unstable.

To study the stability of E_ the associated Jacobian J_ is

$$J_- = \begin{bmatrix} -a & a & 0 & 0 \\ -\dfrac{2cbr^2+1-\sqrt{1+4cbr^2}}{2br^2} & c & -\dfrac{1-\sqrt{1+4cbr^2}}{2r} & 1 \\ \dfrac{1-\sqrt{1+4cbr^2}}{2r} & \dfrac{1-\sqrt{1+4cbr^2}}{2r} & -b & 0 \\ 0 & 0 & 1 & -r \end{bmatrix}$$

The characteristic polynomial of the matrix J_ is given by

$$\lambda^4+c_1\lambda^3+c_2\lambda^2+c_3\lambda+c_4=0 \tag{6}$$

where:

$$c_1=r+b-c+a$$

$$c_2=\frac{a+b+2b^2r^3-(a+b)\sqrt{1+4cbr^2}-2br^3c+2abr^3+2ab^2r^3}{2br^2}$$

$$c_3=\frac{3ab+ar+2ab^2r^3-(ar+3ab)\sqrt{1+4cbr^2}+4acb^2r^2}{2br^2}$$

$$c_4=\frac{a+4abcr^2-a\sqrt{1+4cbr^2}}{2r}$$

As above, one can see that E_ is also unstable since $c_1c_2-c_3$ will be negative.

SYNCHRONIZATION OF HYPERCHAOTIC Lü SYSTEM

We study the synchronization problem of the familiar hyperchaotic Lü system using the method proposed by Pecora and Carroll (1990) and Carrol and Pecora (1991), where, a stable subsystem of a chaotic system is synchronized with a separate chaotic subsystem under suitable conditions. This method has been further extended to cascading chaos synchronization with multiple stable subsystem.

The drive system is:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) \\ \dot{y}_1 &= -x_1 z_1 + c y_1 + w_1 \\ \dot{z}_1 &= x_1 y_1 - b z_1 \\ \dot{w}_1 &= z_1 - r w_1 \end{aligned} \tag{7}$$

Here, since the (x_1, z_1, w_1) subsystem is stable for all values of a, b and r, in which the conditional lyapunov exponents are negative. Then we will use y_1 to be drive the (x_2, y_2, z_2) subsystem of the response system:

$$\begin{aligned} \dot{x}_2 &= a(y_1 - x_2) \\ \dot{z}_2 &= x_2 y_1 - b z_2 \\ \dot{w}_2 &= z_2 - r w_2 \end{aligned} \tag{8}$$

and the difference system for:

$$e_x = x_2 - x_1, \quad e_z = z_2 - z_1 \text{ and } e_w = w_2 - w_1 \tag{9}$$

then the error dynamical system is given by:

$$\begin{aligned} \dot{e}_x &= -a e_x \\ \dot{e}_2 &= y_1 e_x - b e_z \\ \dot{e}_w &= e_z - r e_w \end{aligned} \tag{10}$$

The solution of system Eq. 10 is given by:

$$\begin{aligned} e_x &= \exp(-at + a_1) \\ e_z &= \frac{y_1}{b - a}\exp(-at + a_1) + a_2 \exp(-bt) \\ e_w &= \frac{y_1}{(b - a)(r - a)}\exp(-at + a_1) + \frac{a_2}{r - b}\exp(-bt) + a_3 \exp(-rt) \end{aligned} \tag{11}$$

where, α_1, α_2 and α_3 are constants of integration.

Then

$$\lim_{t\to\infty} e_x = 0, \quad \lim_{t\to\infty} e_z = 0 \quad \text{and} \quad \lim_{t\to\infty} e_w = 0 \tag{12}$$

and then the response system with y-derive configuration does synchronize.

Numerical Results

We have verified that when applying the synchronization method of Pecora and Carroll (1990) of the hyperchaotic Lü system using only y(t) as the drive the stability condition can be satisfied while a = 15, b = 5, c = 10 and r = 1. By using Fourth-order Runge-Kutta method with time step size 0.001. The initial states of the drive system are $x_1(0) = -20$, $y_1(0) = 5$, $z_1(0) = 0$ and $w_1(0) = 15$ and of the response system are $x_2(0) = 10$, $z_2(0) = 5$ and $w_2(0) = 10$. Then $e_x(0) = 30$, $e_z(0) = 5$ and $e_w(0) = -5$ are chosen in all simulations. Figure 2a displays the trajectories x_1 and x_2, (b) displays the trajectories z_1 and z_2, © displays the trajectories w_1 and w_2 and (d) shows that the trajectories of $e_x(t)$, $e_z(t)$ and $e_w(t)$ of the error system tended to zero.

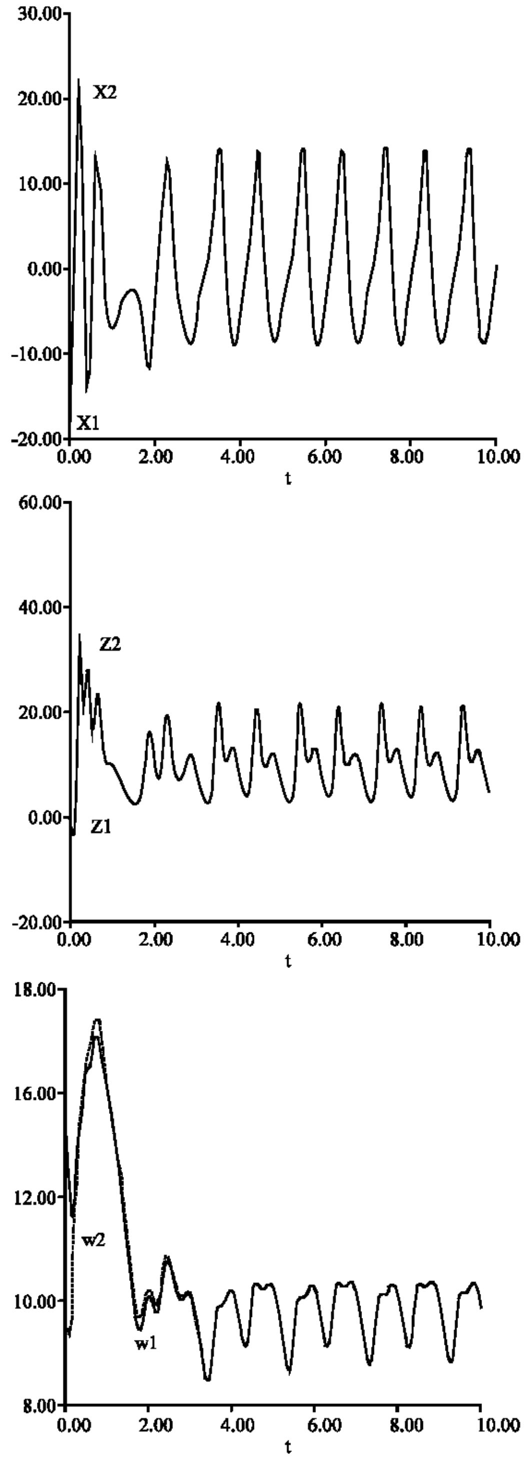

Fig. 2: Solutions of the drive and response systems with Pecora and Carroll method, (a) signals x_1 and x_2, (b) signals z_1 and z_2 and © signals w_1 and w_2

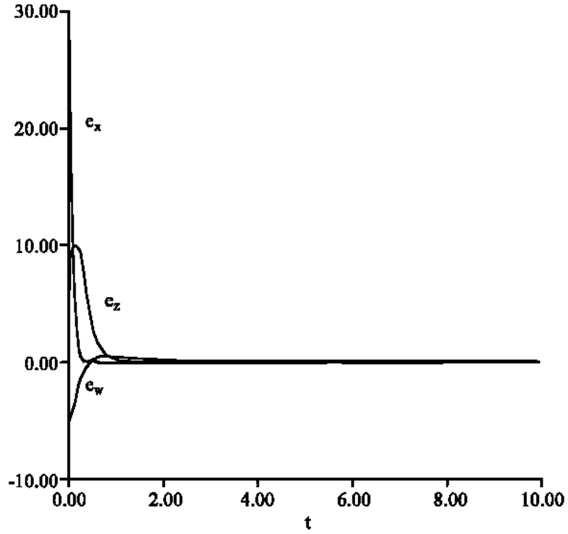

Fig. 2d: Behaviour of the trajectories e_x, e_y and e_z of the error system tends to zero as t tends to 3

ADAPTIVE IDENTICAL SYNCHRONIZATION

In order to observe the adaptive synchronization behaviour in hyperchaotic Lü system, we have two identical hyperchaotic Lü systems where the drive system with four state variables denoted by the subscript 1 drives the response system having identical equations denoted by the subscript 2. However, the initial condition of the drive system is different from that of the response system, therefore two hyperchaotic Lü systems are described, respectively, by the following equations:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) \\ \dot{y}_1 &= -x_1 z_1 + c y_1 + w_1 \\ \dot{z}_1 &= x_1 y_1 - b z_1 \\ \dot{w}_1 &= z_1 - r w_1 \end{aligned} \tag{13}$$

and

$$\begin{aligned} \dot{x}_2 &= a(y_2 - x_2) + u_1(t) \\ \dot{y}_2 &= -x_2 z_2 + c y_2 + w_2 + u_2(t) \\ \dot{z}_2 &= x_2 y_2 - b z_2 + + u_3(t) \\ \dot{w}_2 &= z_2 - r w_2 + u_4(t) \end{aligned} \tag{14}$$

We have introduced four control inputs, $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$ in Eq. 14, $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$, are to be determined for the purpose of synchronizing the two identical hyperchaotic Lü systems with the same but unknown parameters a, b, c and r in spite of the differences in initial conditions.

Remark 1: The hyperchaotic Lü system is dissipative system and has a bounded, zero volume, globally attracting set. Therefore, the state trajectories $x_1(t)$, $y_1(t)$, $z_1(t)$ and $w_1(t)$ are globally bounded for all t = 0 and continuously differentiable with respect to time. Consequently, there exist three positive constants s_1, s_2, s_3 and s_4 such that:

$|x_1(t)| \le s_1 < \infty$, $|y_1(t)| \le s_2 < \infty$, $|z_1(t)| \le s_3 < \infty$ and $|w_1(t)| \le s_4 < \infty$ hold for all $t = 0$.

Let us define the state errors between the response system that is to be controlled and the controlling drive system as:

$$e_x = x_2 - x_1,\ e_y = y_2 - y_1,\ e_z = z_2 - z_1 \text{ and } e_w = w_2 - w_1$$

Then the error dynamical system can be written as:

$$\begin{aligned} \dot{e}_x &= a(e_y - e_x) + u_1 \\ \dot{e}_y &= ce_y - x_1 e_z - z_2 e_x + e_w + u_2 \\ \dot{e}_2 &= y_2 e_x + x_1 e_y - be_z + u_3 \\ \dot{e}_w &= e_z - re_w + u_4 \end{aligned} \tag{16}$$

Then the synchronization problem is now replaced by the equivalent problem of stabilizing the system Eq. 16 using a suitable choice of the control laws $u_1(t)$, $u_2(t)$, $u_3(t)$ and $u_4(t)$. Let us now discuss the following one case of control input $u_2(t)$.

The state variable y_1 of the drive system is coupled to the second equation of the response system and an external control with the state y_2 as the feedback variable is also introduced into the second Eq. in 16. Therefore, the feedback control law is described as:

$$u_2 = -\tilde{k}_1 e_y,\ u_1 = 0,\ u_3 = 0 \text{ and } u_4 = 0 \tag{17}$$

where, $\tilde{k}_1$ is an estimated feedback gain updated according to the following adaptation algorithm

$$\dot{\tilde{k}}_1 = \gamma\, e_y^2, \qquad \tilde{k}_1(0) = 0 \tag{18}$$

Then the resulting error dynamical system can be expressed by:

$$\begin{aligned} \dot{e}_x &= a(e_y - e_x) \\ \dot{e}_y &= ce_y - x_1 e_z - z_2 e_x + e_w - \tilde{k}_1 e_y \\ \dot{e}_2 &= y_2 e_x + x_1 e_y - be_z + u_3 \\ \dot{e}_w &= e_z - re_w \\ \dot{\tilde{k}}_1 &= \gamma e_y^2, \qquad \tilde{k}_1(0) = 0 \end{aligned} \tag{19}$$

Consider a Lyapunov function as follows:

$$V = \frac{1}{2}(e_x^2 + e_y^2 + e_z^2 + e_w^2 + \frac{1}{\gamma}(\tilde{k}_1 - k_1^{**})^2) \tag{20}$$

where, k_1^{**} is a positive constant which will be defined later. Taking the time derivative of Eq. 20, then we get:

$$\begin{aligned}
\dot{V} &= e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z + e_w\dot{e}_w + \frac{2}{\gamma}(\tilde{k}_1 - k_1^{**})\dot{\tilde{k}}_1 \\
&= -ae_x^2 + ae_xe_y + (c-\tilde{k}_1)e_y^2 - z_2e_xe_y - x_1e_ze_y + e_we_y + y_2e_xe_z \\
&\quad + x_1e_ye_z - be_z^2 + e_ze_w - re_w^2 + (\tilde{k}_1 - k_1^{**})e_y^2 \\
&= -ae_x^2 + (c-k_1^{**})e_y^2 - be_z^2 - re_w^2 + (a-z_2)e_xe_y + y_2e_xe_z + e_ye_w + e_ze_w \\
&\le -ae_x^2 + (c-k_1^{**})e_y^2 - be_z^2 - re_w^2 + (a+s_3)e_xe_y + s_2e_xe_z + e_ye_w + e_ze_w \\
&= -[ae_x^2 + (k_1^{**}-c)e_y^2 + be_z^2 + re_w^2 - (a+s_3)e_xe_y - s_2e_xe_z - e_ye_w - e_ze_w] \\
&= -\begin{bmatrix} |e_x| & |e_y| & |e_z| & |e_w| \end{bmatrix}
\begin{bmatrix} a & -\frac{s_3+a}{2} & -\frac{s_2}{2} & 0 \\ -\frac{s_3+a}{2} & k_1^{**}-c & 0 & -\frac{1}{2} \\ -\frac{s_2}{2} & 0 & b & -\frac{1}{2} \\ 0 & -\frac{1}{2} & -\frac{1}{2} & r \end{bmatrix}
\begin{bmatrix} |e_x| \\ |e_y| \\ |e_z| \\ |e_w| \end{bmatrix} \\
&= -\begin{bmatrix} |e_x| & |e_y| & |e_z| & |e_w| \end{bmatrix} \Psi(d_1^*, d_2^*) \begin{bmatrix} |e_x| & |e_y| & |e_z| & |e_w| \end{bmatrix}^T
\end{aligned} \tag{21}$$

$$\text{(i) For } k_1^{**} > c + \frac{1}{a}\left(\frac{a+s_3}{2}\right)^2 = d_1$$

$$\text{(ii) For } k_1^{**} > \frac{4abc}{4ab-s_2^2} + \frac{b(s_3^2+a^2)}{4ab-s_2^2} + \frac{2s_3ab}{4ab-s_2^2} - \frac{s_2^2c}{4ab-s_2^2} = d_2$$

$$\text{(iii) For } k_1^{**} > \frac{4ab(4cr + ar + 2rs_3 + 1) + 2s_2(s_3 + 2s_2br + a)}{4(4abr - s_2^2r - a)} - \frac{4ac + 4s_2^2rc + 2as_3 + a^2 + s_2^2 + s_3^2}{4(4abr - s_2^2r - a)} = d_3$$

If we choose $k_1^{**} = \max(d_1, d_2, d_3)$ then the 4×4 matrix $\Psi(k_1^{**})$ is positive definite.

Where, s_2 and s_3 are defined in remark 1. If k_1^{**} is appropriately chosen such that the 4×4 matrix $\Psi(k_1^{**})$ in Eq. 21 is positive definite, then $\dot{V} \le 0$ holds. Since V is a positive and decreasing function and $\dot{V}$ is negative semidefinite (we choose $a(k_1^{**}-c) - (\frac{a+s_3}{2})^2 > 0$). It follows that the equilibrium point $(e_x = 0, e_y = 0, e_z = 0, e_w = 0, \tilde{k}_1 = k_1^{**})$ of the system (19) is uniformly stable, i.e., $e_x(t)$, $e_y(t)$, $e_z(t)$, $e_w(t) \in L_\infty$ and $\tilde{k}_1(t) \in L_\infty$. From Eq. 20 we can easily show that the squares of $e_x(t)$, $e_y(t)$, $e_z(t)$ and $e_w(t)$ are integrable with respect to time t, i.e., $e_x(t)$, $e_y(t)$, $e_z(t)$ and $e_w(t) \in L_2$. Next by Barbalat's Lemma Eq. 16 implies that $\dot{e}_x(t)$, $\dot{e}_y(t)$, $\dot{e}_z(t)$, $\dot{e}_w(t) \in L_\infty$, which in turn implies $e_x(t)\to 0$, $e_y(t)\to 0$, $e_z(t)\to 0$ and $e_w(t)\to 0$ as $t\to\infty$. Thus, in the closed-loop system $x_2(t)$, $\to x_1(t)$, $y_2(t)\to y_1(t)$, $z_2(t)\to z_1(t)$ and $w_2(t)\to w_1(t)$ as $t\to 8$. This implies that the two hyperchaotic Lü systems have been globally asymptotically synchronized under the control law Eq. 17 associated with Eq. 18.

Numerical Experiment

Fourth-order Runge-Kutta method is used to solve differential equations. A time step size 0.001 is employed. The three parameters are chosen as a = 15, b = 5, c = 10 and r = 1 in all simulations so that the hyperchaotic Lü system exhibits a chaotic behaviour if no control is applied. The initial states of the drive system are $x_1(0) = -20$, $y_1(0) = 5$, $z_1(0) = 0$ and $w_1(0) = 15$ and of the response system are

$x_2(0) = 10$, $y_2(0) = -5$, $z_2(0) = 5$ and $w_2(0) = 10$. Then $e_x(0) = 30$, $e_y(0) = -10$, $e_z(0) = 5$ and $e_w(0) = -5$. In this case, we assume that the drive system and the response system are two identical hyperchaotic Lü system with different initial conditions. The evolutions of state synchronization errors and the history of the estimated feedback gain using the feedback control law (17) associated with the adaptation algorithm (18). These numerical results demonstrate the systems have been asymptotically synchronized using the proposed adaptive schemes (Fig. 3).

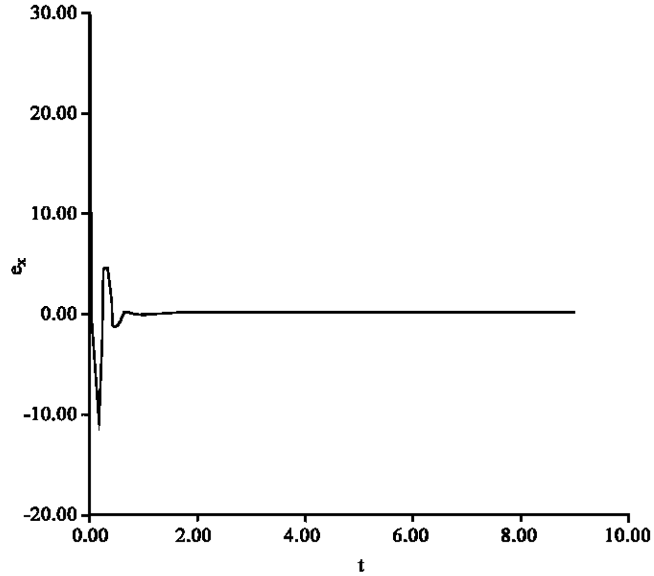

Fig. 3a: Behaviour of the trajectory e_x of the error system tends to zero as t tends to 2 when the parameter values are a = 15, b = 5, c = 10 and r = 1

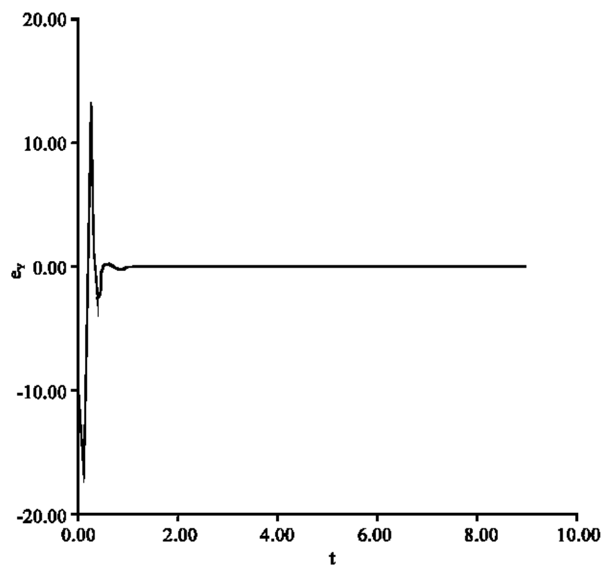

Fig. 3b: Behaviour of the trajectory e_z of the error system tends to zero as t tends to 2 when the parameter values are a = 15, b = 5, c = 10 and r = 1

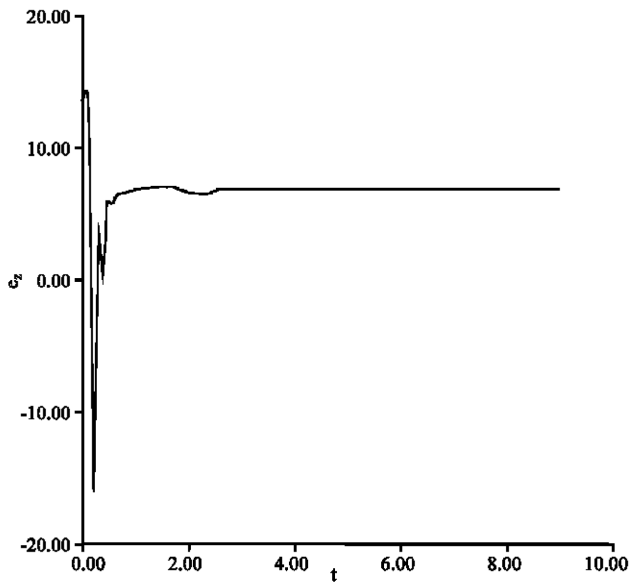

Fig. 3c: Behaviour of the trajectory e_z of the error system tends to zero as t tends to 2 when the parameter values are a = 15, b = 5, c = 10 and r = 1

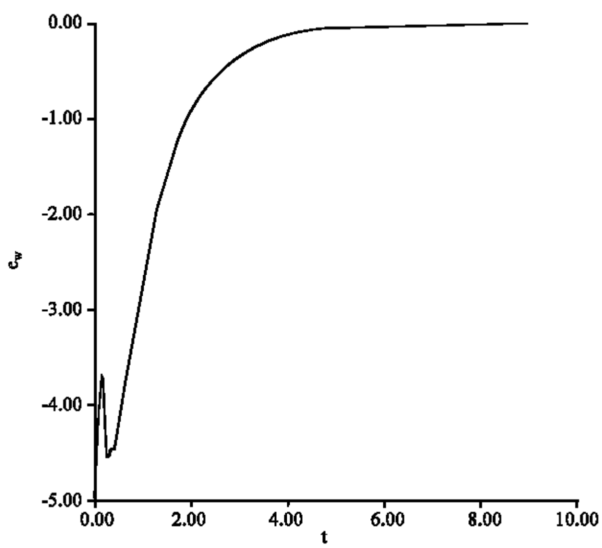

Fig. 3d: Behaviour of the trajectory e_w of the error system tends to zero as t tends to 8 when the parameter values are a = 15, b = 5, c = 10 and r = 1

CONCLUSION

In this study synchronization and adaptive synchronization using uncertain parameters of the hyperchaotic Lü system is demonstrated. The Pecora and Carroll method has been applied to achieve the synchronization of the hyperchaotic Lü system. All results are proved by using Lyapunov direct method. The proposed scheme is efficient in achieving simple synchronization in our example and can be applied to similar chaotic systems.

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