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## Reduced-Order Sliding Mode Flux Observer and Nonlinear Control of an Induction Motor

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**Abstract:** This study describes an innovative strategy to the problem of non-linear estimation of states for electrical machine systems. This method allows the estimation of variables that are difficult to access or that are simply impossible to measure. Thus, as compared with a full-order sliding mode observer, in order to reduce the execution time of the estimation, a reduced or third-order discrete-time extended sliding mode observer is proposed for on-line estimation of rotor flux, rotor resistance and torque in an induction motor using a robust feedback linearization control. Simulations results on Matlab-Simulink environment for a 1.8 kW induction motor are presented to prove the effectiveness and high robustness of the proposed nonlinear control and observer against modeling uncertainty and measurement noise.

**Key words:** Nonlinear control, induction motor, reduced-order extended sliding mode observers, parameter estimation

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### INTRODUCTION

The Induction Motors (IM) become very popular for motion control applications due to its reasonable cost, simple and reliable construction. However, the control of IM is proved very difficult since the dynamic systems are non linear, the electric rotor variables are not usually measurable or the transducers are expensive (such as torque, flux transducers) and the physical parameters are often imprecisely known or variable. For instance, the rotor resistance drifts with the temperature of the rotor current frequency.

This naturally structure of non-linear and multivariable state of IM models induces the use of the non-linear control methods and in particular the robust feedback linearization strategy (Isidori, 1995; Yazdanpanah *et al.*, 2008; Mohanty *et al.*, 2002) to permit a decoupling, assure a good dynamic performance and stability of the IM.

However, a variation of the rotor resistance can induce a state-space coupling which can induce a degradation of the system. In order to achieve better dynamic performance, an on-line estimation of rotor fluxes and rotor resistance is necessary. An approach proposed by Derdiyok (2005), Jingchuan *et al.* (2005), Benchaib *et al.* (1999) and Shraim *et al.* (2007) to estimate with success the state variables in an IM is the use of the full-order Sliding Mode Observer (SMO). This latter, built from the dynamic model of the IM by adding corrector gains with switching terms, is used to provide not only the unmeasurable state variable estimation (rotor fluxes and Torque) but also the estimation of the measurable parameters (stator currents and speed). However the determination of the measurable parameters estimation imposes some estimation algorithms very long and usually sophisticated

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with an increase of the computational volume. Therefore, in order to reduce the accuracy and the computation rate of the estimation algorithms, the measured parameters estimation is not necessary.

Thus a Reduced-order Discrete-time Extended Sliding Mode Observer (RDESMO) for the IM is presented in this study to solve only and specially the problem of the unmeasurable parameters estimation (rotor fluxes and rotor time constant).

## MATERIALS AND METHODS

### Robust Feedback Control of an Induction Motor Model

By assuming that the saturation of the magnetic parts and the hysteresis phenomenon are neglected, the classical dynamic model of the IM in a (d, q) synchronous reference frame can be described by De Fornel and Louis (2007) and Mendes *et al.* (2002):

$$\begin{cases} V_{ds} = R_s I_{ds} + \frac{d\Phi_{ds}}{dt} - \omega_s \Phi_{qs} \\ V_{qs} = R_s I_{qs} + \frac{d\Phi_{qs}}{dt} + \omega_s \Phi_{ds} \end{cases}; \begin{cases} \Phi_{ds} = \frac{L_m}{L_r} \Phi_{dr} + \sigma L_s I_{ds} \\ \Phi_{qs} = \frac{L_m}{L_r} \Phi_{qr} + \sigma L_s I_{qs} \end{cases} \quad (1a)$$

$$\begin{bmatrix} \Phi_{ds} \\ \Phi_{dr} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix} \begin{bmatrix} I_{ds} \\ I_{dr} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} \Phi_{qs} \\ \Phi_{qr} \end{bmatrix} = \begin{bmatrix} L_s & L_m \\ L_m & L_r \end{bmatrix} \begin{bmatrix} I_{qs} \\ I_{qr} \end{bmatrix} \quad (1b)$$

The load mechanical equation is:

$$\frac{J}{p} \frac{d\omega_r}{dt} + \frac{f}{J} \omega_r = C_{em} - C_r \quad (1c)$$

Where:

$$C_{em} = p \cdot \frac{L_m}{L_r} (\Phi_{dr} I_{qs} - \Phi_{qr} I_{ds})$$

The application of Eq. 1a-c returns a system of fifth-order non-linear differential equation, with as state variables the stator currents ( $I_{ds}$ ,  $I_{qs}$ ), the rotor fluxes ( $\Phi_{dr}$ ,  $\Phi_{qr}$ ) and the rotor pulsation ( $\omega_r$ ):

$$\dot{x}_c = f_c(x_c) + g_c \cdot u \quad (2)$$

Where:

$$\begin{aligned} x_c &= [ I_{ds} \ I_{qs} \ \Phi_{dr} \ \Phi_{qr} \ \omega_r ]^T \\ u &= [ V_{ds} \ V_{qs} ]^T \end{aligned}$$

$$f_c(x_c) = \begin{bmatrix} -\lambda I_{ds} + \omega_s I_{qs} + \sigma_r \beta \Phi_{dr} + \beta \omega_r \Phi_{qr} \\ -\omega_s I_{ds} - \lambda I_{qs} - \beta \omega_r \Phi_{dr} + \beta \sigma_r \Phi_{qr} \\ \sigma_r L_m I_{ds} - \sigma_r \Phi_{dr} + \omega_{sl} \Phi_{qr} \\ \sigma_r L_m I_{qs} - \omega_{sl} \Phi_{dr} - \sigma_r \Phi_{qr} \\ p^2 \frac{L_m}{L_r J} (\Phi_{dr} I_{qs} - \Phi_{qr} I_{ds}) - \frac{p}{J} C_r - \frac{f}{J} \omega_r \end{bmatrix}; \quad g_c = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Where:

$$\sigma_r = \frac{1}{T_r}$$

$$\lambda = \lambda(\sigma_r) = \frac{1}{\sigma} \left( \frac{1}{T_s} + (1 - \sigma) \cdot \sigma_r \right)$$

$$\beta = \frac{1 - \sigma}{\sigma L_m}$$

$$\sigma = 1 - \frac{L_m^2}{L_s L_r}$$

Moreover, by choosing a rotating reference frame (d, q), so that the direction of axe d is always coincident with the direction of the rotor flux representative vector (field orientation), it is well known that this rotor field orientation in a rotating synchronous reference frame realizes:

$$\Phi_{dr} = \Phi_r = \text{Constant and } \Phi_{qr} = 0 \tag{3}$$

Thus the dynamic model of the IM, completed with the output equation, can be rewritten as:

$$\dot{x} = f(x) + g \cdot u ; y = [h_1(x) \ h_2(x)]^T = [\Phi_r \ \omega_r]^T \text{ with } x = [I_{ds} \ I_{qs} \ \Phi_r \ \omega_r]^T, u = [V_{ds} \ V_{qs}]^T$$

$$f(x) = \begin{bmatrix} -\lambda \cdot I_{ds} + \omega_s \cdot I_{qs} + \sigma_r \cdot \beta \cdot \Phi_r \\ -\omega_s \cdot I_{ds} - \lambda \cdot I_{qs} - \beta \cdot \omega_r \cdot \Phi_r \\ \sigma_r \cdot L_m \cdot I_{ds} - \sigma_r \cdot \Phi_r \\ p^2 \cdot \frac{L_m}{L_r \cdot J} \cdot \Phi_r \cdot I_{qs} - \frac{p}{J} \cdot C_r - \frac{f}{J} \cdot \omega_r \end{bmatrix} ; g = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 \\ 0 & \frac{1}{\sigma L_s} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = [g_1 \ g_2] \tag{4}$$

From the expressions Eq. 2 and 3, one can write:

$$\begin{cases} \frac{dI_{mr}}{dt} = \sigma_r \cdot I_{ds} - \sigma_r \cdot I_{mr} \\ \sigma_r = \omega_{sl} \cdot \frac{I_{mr}}{I_{qs}} \\ C_{em} = \frac{p \cdot L_m}{L_r} \cdot \Phi_r \cdot I_{qs} \end{cases} \tag{5}$$

Where:

$$I_{mr} = \frac{\Phi_r}{L_m}$$

This relation Eq. 5 shows that the dynamic model of the IM can be represented as a non-linear function of the rotor time constant. A variation of this parameter can induce, for the IM, a lack of field orientation, performance and stability. Thus, to preserve the reliability, robustness performance and stability of the system under parameters variation (in particular the rotor time constant variations) and disturbances, we can use a robust feedback linearization strategy to regulate the motor states.

As a matter of fact, we can see that the system Eq. 4 has relative degree  $r_1 = r_2 = 2$  and can be transformed into a linear and controllable system by chosen:

- A suitable change of coordinates given by:

$$z_1 = h_1(x); z_2 = L_r h_1(x); z_3 = h_2(x); z_4 = L_r h_2(x)$$

- The feedback linearization control having the following form:

$$u = \begin{bmatrix} L_{g1}L_f h_1(x) & L_{g2}L_f h_1(x) \\ L_{g1}L_f h_2(x) & L_{g2}L_f h_2(x) \end{bmatrix}^{-1} \cdot \begin{bmatrix} v_1 - L_f^2 h_1(x) \\ v_2 - L_f^2 h_2(x) \end{bmatrix} = \begin{bmatrix} \frac{\sigma_r L_m}{\sigma L_s} & 0 \\ 0 & \frac{p^2 L_m \Phi_r}{\sigma L_s L_r J} \end{bmatrix}^{-1} \cdot \begin{bmatrix} v_1 - L_f^2 h_1(x) \\ v_2 - L_f^2 h_2(x) \end{bmatrix}$$

where,  $\Phi_r \neq 0$  and  $[v_1, v_2]^T$  are the new input vector of the obtained decoupled systems.

- Two robust controllers  $C(s)$  to provide a good regulation and convergence of the rotor flux ( $\Phi_r$ ) and speed ( $\omega_r$ ). On the other hand, in order to impose after a closed loop a second order dynamic behaviour defined by  $H(s)$ , the controller  $C(s)$  can be chosen by Doyle *et al.* (1992):

$$C(s) = \frac{R(s).H(s)^{-1}}{1 - R(s)} \quad (6)$$

Where:

$$R(s) = \frac{1}{(1 + t_0 s)^2}$$

$$H(s) = \frac{1}{1 + \frac{2\xi\omega_0}{\omega_0} s + \frac{1}{\omega_0^2} s^2}$$

where, the real  $t_0$  is an adjusting positive parameter.

The block diagram structure for the control of ( $\Phi_r, \omega_r$ ) is shown in Fig. 1.

Furthermore, as the control of an IM generally required the knowledge of the instantaneous flux of the rotor that is not measurable, a full-order SMO built from the model Eq. 2 by adding corrector gains with switching terms is widely used (Asseu *et al.*, 2008; Tursini *et al.*, 2000) with success for on-line estimation at one and the same time of rotor time constant, fluxes, currents, speed or torque. The equivalent value of the switching function depends on the current errors given by the difference between the estimated currents to their real or measured values. However, as the currents and speed are already measurable, their estimated values are not therefore necessary.

Here, in order to respect to the rotor time constant variations and reduce the execution time of the observation, a reduced-order extended sliding mode observer is proposed to provide only the unmeasurable parameters estimation (rotor fluxes and rotor time constant). And the switching term of this reduced observer will be only function of the measurable parameters (voltage, currents and speed).

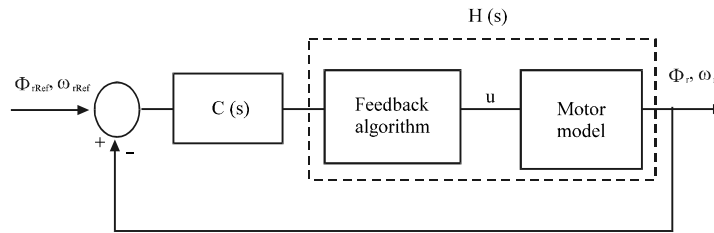


Fig. 1: The block diagram structure for the control of ( $\Phi_r, \omega_r$ )

### Reduced Order Discrete-Time Extended Sliding Mode Observer

Let us consider the dynamic model of the IM given by the system Eq. 2. Assume that among the state variable, the stator currents ( $I_{ds}$ ,  $I_{qs}$ ) and the rotor speed ( $\omega_r$ ) are measurable, therefore their on-line estimation is not necessary. Moreover, as previously underline, a variation of the rotor resistance can induce performance degradation of the system. Thus from the expression Eq. 2, in order to estimate the rotor flux and rotor time constant, a two-dimensional state vector extended to the rotor time constant defined by  $X_r = [\Phi_{dr} \ \Phi_{qr} \ \sigma_r]^T = [x_1 \ x_2 \ x_3]^T$  can be introduced with  $\sigma_r = R_r/L_r$ . The corresponding reduced order extended state space equation become: where,  $v(t) = [I_{ds}, I_{qs}]^T$  is the new input vector. We can write:

$$\begin{cases} \dot{x}_1 = -x_3 \cdot x_1 + \omega_{sl} \cdot x_2 + x_3 \cdot L_m \cdot I_{ds} \\ \dot{x}_2 = -\omega_{sl} \cdot x_1 - x_3 \cdot x_2 + x_3 \cdot L_m \cdot I_{qs} \\ \dot{x}_3 = \epsilon \end{cases} \quad (7)$$

where,  $\epsilon$  presents the slow variation of  $\sigma_r$ . The fact that the state vector only consists of the rotor flux and resistance offers an advantage namely the reduction of the computational volume and complexity. Thus the rotor flux and resistance can be more easily and rapidly estimated.

Denote  $\hat{x}_1$ ,  $\hat{x}_2$  and  $\hat{x}_3$  the estimates of the fluxes and rotor time constant. The proposed reduced order ESMO is a copy of the model Eq. 7 by adding corrector gains with switching terms:

$$\begin{cases} \dot{\hat{x}}_1 = -\hat{x}_3 \cdot \hat{x}_1 + \omega_{sl} \cdot \hat{x}_2 + \hat{x}_3 \cdot L_m \cdot I_{ds} + \Gamma_1 \cdot I_s \\ \dot{\hat{x}}_2 = -\omega_{sl} \cdot \hat{x}_1 - \hat{x}_3 \cdot \hat{x}_2 + \hat{x}_3 \cdot L_m \cdot I_{qs} + \Gamma_2 \cdot I_s \\ \dot{\hat{x}}_3 = \epsilon + \Gamma_3 \cdot I_s \end{cases} \quad (8)$$

where,  $\Gamma_1$  and  $\Gamma_2$  are the observer gains. The switching  $I_s$  is defined as:

$$I_s = \begin{bmatrix} \text{sign}(s_1) \\ \text{sign}(s_2) \end{bmatrix} \quad \text{with } S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = M \cdot \tilde{Z}_r \quad \text{and } M = \begin{bmatrix} \beta \cdot \sigma_r & \beta \cdot \omega_r \\ -\beta \cdot \omega_r & \beta \cdot \sigma_r \end{bmatrix}^{-1} \quad (9)$$

where,  $\tilde{Z}_r$  is a function depending on the stator currents, voltages and speed measurements.

Setting  $\tilde{x}_i = x_i - \hat{x}_i$  with  $i = 1, 2, 3$ . Firstly to determine  $\Gamma_1$  and  $\Gamma_2$ , let us assume that the estimation error of the rotor constant time converge to zero ( $\tilde{x}_3 = x_3 - \hat{x}_3 \cong 0$ ). The estimation error dynamics of the fluxes is given by:

$$\begin{cases} \dot{\tilde{x}}_1 = -\sigma_r \cdot \tilde{x}_1 + \omega_{sl} \cdot \tilde{x}_2 - \Gamma_1 \cdot I_s \\ \dot{\tilde{x}}_2 = -\omega_{sl} \cdot \tilde{x}_1 - \sigma_r \cdot \tilde{x}_2 - \Gamma_2 \cdot I_s \end{cases}$$

With the following observer gain matrices:

$$\begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} = \begin{bmatrix} q - \sigma_r & \omega_{sl} \\ -\omega_{sl} & q - \sigma_r \end{bmatrix} \Delta; \quad \Delta = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix} \quad (10)$$

where,  $q$  and  $n$  are positive constants, the estimation  $[\tilde{x}_1 \ \tilde{x}_2]$  error converge to zero.

Secondly, to determine observer gain  $\Gamma_3$ , it can be supposed that the observation errors of the fluxes converge to zero. The estimation errors of the fluxes  $\tilde{x}_i = x_i - \hat{x}_i = 0$  ( $i = 1, 2$ ) are then given by:

$$\begin{aligned} 0 &= -\tilde{x}_3 \cdot \hat{x}_1 + \hat{x}_3 \cdot \tilde{x}_1 + \omega_{sl} \cdot \tilde{x}_2 + L_m \cdot I_{ds} \cdot \tilde{x}_3 - \Gamma_1 \cdot I_s \\ 0 &= -\omega_{sl} \cdot \tilde{x}_1 - \tilde{x}_3 \cdot \hat{x}_2 + \hat{x}_3 \cdot \tilde{x}_2 + L_m \cdot I_{qs} \cdot \tilde{x}_3 - \Gamma_2 \cdot I_s \end{aligned}$$

By replacing the expressions of  $\Gamma_1$  and  $\Gamma_2$ , we obtain:

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = \frac{1}{q} \begin{bmatrix} L_m I_{ds} - \hat{x}_1 \\ L_m I_{qs} - \hat{x}_2 \end{bmatrix} \tilde{x}_3$$

The estimation error dynamics of the rotor time constant is given by:

$$\tilde{x}_3 = -\Gamma_3 I_s = -\Gamma_3 \Delta^{-1} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} = -\Gamma_3 \Delta^{-1} \frac{1}{q} \begin{bmatrix} L_m I_{ds} - \hat{x}_1 \\ L_m I_{qs} - \hat{x}_2 \end{bmatrix} \tilde{x}_3$$

We can see that this error dynamics is locally and exponentially stable by chosen:

$$\Gamma_3 = m \cdot q \begin{bmatrix} L_m I_{ds} - \hat{x}_1 \\ L_m I_{qs} - \hat{x}_2 \end{bmatrix}^T \Delta \text{ with } m > 0 \quad (11)$$

The parameter  $m$  is adjusted with respect to rotor time constant estimation.

Finally, from the expressions Eq. 10 and 11, it can be seen that there are three positive adjusting gains:  $q$ ,  $n$  and  $m$  which play a critical role in the stability and the velocity of the observer convergence. These three adjusting parameters must be chosen so that the reduced observer satisfies robustness properties, global or local stability, good accuracy and considerable rapidity.

In order to implement the reduced-order ESMO algorithm in a DSP for real-time applications, the corresponding three-dimension state space equation defined in Eq. 7 must be discretized using Euler's approximation (2nd order). Thus the new discrete-time varying model represented by a function depending on the stator current is given by:

$$\begin{cases} x_i(k+1) = x_i(k) + T_e L_1(x_i(k)) + \frac{T_e^2}{2!} L_2^i(x_i(k)) \\ \quad = x_i(k) + T_e J_1(x_i(k), v(k)) + \frac{T_e^2}{2!} J_2(x_i(k), v(k)) \\ y_i(k) = h(x_i(k)) \end{cases} \quad (12)$$

Where:

$$x_i(k) = [\Phi_{dr}(k) \quad \Phi_{qr}(k) \quad \sigma_r(k)]^T$$

$$v(k) = [I_{ds}(k) \quad I_{qs}(k)]^T$$

$$J_1(x_i(k), v(k)) = \begin{bmatrix} -\sigma_r(k) \cdot \Phi_{dr}(k) + \omega_{sr}(k) \cdot \Phi_{qr}(k) + \sigma_r(k) \cdot L_m I_{ds}(k) \\ -\omega_{sr}(k) \cdot \Phi_{dr}(k) - \sigma_r(k) \cdot \Phi_{qr}(k) + \sigma_r(k) \cdot L_m I_{qs}(k) \\ 0 \end{bmatrix};$$

$$J_2(x_i(k), v(k)) = \begin{bmatrix} (\sigma_r^2(k) - \omega_{sr}^2(k)) \cdot \Phi_{dr}(k) - 2 \cdot \omega_{sr}(k) \cdot \sigma_r(k) \cdot \Phi_{qr}(k) \\ \quad - \sigma_r^2(k) \cdot L_m I_{ds}(k) + \sigma_r(k) \cdot L_m \cdot \omega_{sr}(k) \cdot I_{qs}(k) \\ 2 \cdot \omega_{sr}(k) \cdot \sigma_r(k) \cdot \Phi_{dr}(k) + (\sigma_r^2(k) - \omega_{sr}^2(k)) \cdot \Phi_{qr}(k) \\ \quad - \sigma_r(k) \cdot L_m \cdot \omega_{sr}(k) \cdot I_{ds}(k) - \sigma_r^2(k) \cdot L_m \cdot I_{qs}(k) \\ 0 \end{bmatrix}$$

where,  $k$  means the  $k$ th sampling time, i.e.,  $t = k \cdot T_e$  with  $T_e$  the adequate sampling period chosen without failing the stability and the accuracy of the discrete-time model.

The proposed RDESMO can be defined by the following equation:

$$\hat{x}_r(k+1) = \hat{x}_r(k) + T_e \cdot J_1(\hat{x}_r(k), v(k)) + \frac{T_e^2}{2!} \cdot J_2(\hat{x}_r(k), v(k)) + G(k) \cdot I_s(k) \quad (13)$$

This Eq. 13 is composed of:

- The correction:  $\theta = G(k) \cdot I_s(k)$
- The prediction vector:  $\hat{x}_r(k+1) = \hat{x}_r(k) + T_e \cdot J_1(\hat{x}_r(k), v(k)) + \frac{T_e^2}{2!} \cdot J_2(\hat{x}_r(k), v(k))$   
with  $\hat{x}_r(k) = [\check{\Phi}_\alpha(k) \quad \check{\Phi}_\varphi(k) \quad \check{\sigma}_r(k)]^T$

The switching vector  $I_s(k)$ , deduced from the continuous case given by Eq. 9, can be written as:

$$I_s(k) = \begin{bmatrix} \text{sign}(s_1(k)) \\ \text{sign}(s_2(k)) \end{bmatrix} \quad \text{with } S = \begin{bmatrix} s_1(k) \\ s_2(k) \end{bmatrix} = T_e \cdot M(k) \cdot \tilde{z}(k+1) \quad (14)$$

Where:

$$M(k) = \begin{pmatrix} -\hat{\sigma}_r(k) + \frac{T_e}{2}(\hat{\sigma}_r^2(k) - \omega_\alpha^2(k)) & \omega_\alpha(k) - T_e \cdot \hat{\sigma}_r(k) \cdot \omega_\alpha(k) \\ -\omega_\alpha(k) + T_e \cdot \hat{\sigma}_r(k) \cdot \omega_\alpha(k) & \hat{\sigma}_r(k) + \frac{T_e}{2}(\hat{\sigma}_r^2(k) - \omega_\alpha^2(k)) \end{pmatrix}$$

$$\tilde{z}(k+1) = \begin{pmatrix} z_{rd}(k+1) - \hat{z}_{rd}(k+1) \\ z_{rq}(k+1) - \hat{z}_{rq}(k+1) \end{pmatrix}$$

Thus the output vector becomes  $h(x_r(k)) = [h_1(k), h_2(k)]^T$  where,  $h_1$  and  $h_2$  are given by:

$$h_1 = z_{rd}(k+1) - \Phi_\alpha(k+1) - \Phi_\alpha(k) - T_e \cdot \omega_\alpha(k) \cdot \Phi_\varphi(k), \quad h_2 = z_{rq}(k+1) - \Phi_\varphi(k+1) - \Phi_\varphi(k) + T_e \cdot \omega_\alpha(k) \cdot \Phi_\alpha(k)$$

From the electrical Eq. 1a of the IM, an approximate (1st order) discrete-time relation of the fluxes is given by:

$$\begin{cases} z_{rd}(k+1) = \frac{T_e \cdot L_r}{L_m} [V_{ds}(k) - R_s \cdot I_{ds}(k)] - \frac{\sigma L_s \cdot L_r}{L_m} [I_{ds}(k+1) - I_{ds}(k) - T_e \cdot \omega_s(k) \cdot I_{qs}(k)] \\ z_{rq}(k+1) = \frac{T_e \cdot L_r}{L_m} [V_{qs}(k) - R_s \cdot I_{qs}(k)] - \frac{\sigma L_s \cdot L_r}{L_m} [I_{qs}(k+1) - I_{qs}(k) + T_e \cdot \omega_s(k) \cdot I_{ds}(k)] \end{cases}$$

and

$$\begin{pmatrix} \hat{z}_{rd}(k+1) \\ \hat{z}_{rq}(k+1) \end{pmatrix} = \begin{pmatrix} \check{\Phi}_{dr}(k+1) - \hat{\Phi}_\alpha(k) - T_e \cdot \omega_s(k) \cdot \hat{\Phi}_\varphi(k) \\ \check{\Phi}_{qr}(k+1) - \hat{\Phi}_\varphi(k) + T_e \cdot \omega_s(k) \cdot \hat{\Phi}_\alpha(k) \end{pmatrix} \quad (15)$$

The proposed gain matrix representation  $G(k)$ , deduced from the continuous case given by Eq. 10 and 11, can be defined as follows (discrete-time approach):

$$G_r(k) = T_e \cdot \begin{pmatrix} \Gamma_1(k) \\ \Gamma_2(k) \\ \Gamma_3(k) \end{pmatrix} = \begin{pmatrix} q - T_e \cdot \hat{\sigma}_r(k) & T_e \cdot \omega_\alpha(k) \\ -T_e \cdot \omega_\alpha(k) & q - T_e \cdot \hat{\sigma}_r(k) \\ T_e \cdot (m \cdot L_m \cdot I_{ds}(k) - \hat{\Phi}_{dr}(k)) & T_e \cdot (m \cdot L_m \cdot I_{qs}(k) - \hat{\Phi}_{qr}(k)) \end{pmatrix} \Delta \quad (16)$$

Once the fluxes are estimated, it is easy to deduce the estimated torque defined by:

$$\hat{C}_{em}(k) = p \cdot \frac{L_m}{L_r} (\hat{\Phi}_\alpha(k) \cdot I_{qs}(k) - \hat{\Phi}_\varphi(k) \cdot I_{ds}(k)) \quad (17)$$



## RESULTS AND DISCUSSION

In order to verify the feasibility of the proposed RDESMO, the simulation on SIMULINK from Mathwork has been carried out for a 1.8 kW induction motor controlled with a robust linearization via feedback algorithm (Fig. 2). The nominal parameters of the induction motor are shown in the Table 1.

The RDESMO is implanted in a S\_function using C language. In order to evaluate its performances and effectiveness, the comparisons between the observed state variables and the simulated ones have been realized for several operating conditions with the presence of about 15% noise on the simulated currents ( $I_{ds}$ ,  $I_{qs}$ ) or speed. Thus, using a sampling period  $T_e = 1$  m sec, the simulations are realized at first in the nominal case with the nominal parameters of the induction motor (Table 1) and then, in the second case, with 50% variation of the nominal rotor time constant ( $\sigma_r = 1.5 \sigma_{rn}$ ) in order to verify the rotor time constant tracking and flux estimation.

Figure 3 and 4 shows the simulation results for a step input of the rotor speed and flux. One can see that in both nominal (Fig. 3a, c) or non-nominal (Fig. 4a, c) cases, the estimated values of fluxes and torque converge very well to their simulated values.

The observed fluxes (Fig. 3a) indicate the good orientation ( $\Phi_{dr}$  is constant and  $\Phi_{qr}$  converges to zero) which is due to a favorable rotor time constant estimation (Fig. 3b, 4b). The estimated torque (Fig. 3c) is in good agreement with the simulated value.

Once the fluxes are estimated, we can deduce the algorithm of the feedback linearization control (Fig. 2). The waveforms show the good uncoupling between the rotor flux and the speed because a step variation in  $\Phi_{dr}$  (Fig. 4a) can not generate a speed  $\omega_r$  change (Fig. 4d). Thus the field orientation and the synthesis of robust linearization and decoupling control are well verified.

All those results show the satisfying tuning, the excellent performance of the robust decoupling control and RDESMO against rotor resistance variations and perturbations or noises.

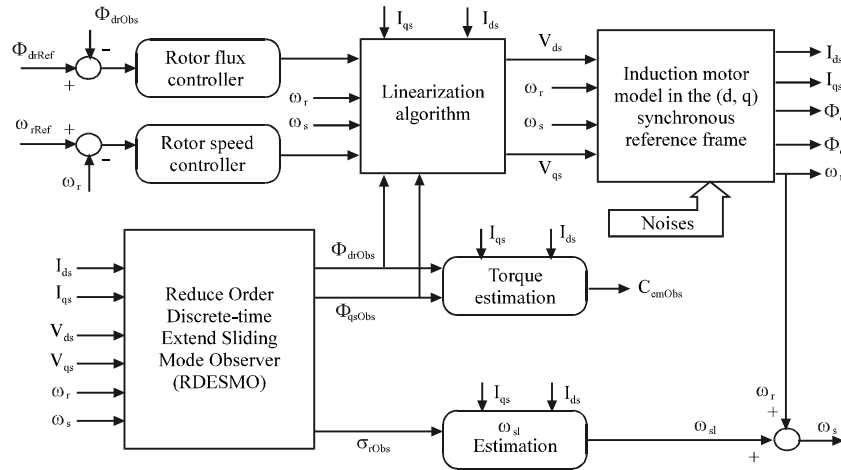


Fig. 2: Simulation scheme

Table 1: Nominal parameters of the Induction motor

$P_{mn} = 1.8$ kW	$U_n = 220/380$ V	$I_n = 20.8/12$ A	$p = 2$
$F_n = 50$ Hz	$\Omega_n = 1420$ rpm	$J_n = 0.15$ kg N m <sup>-2</sup>	$f_n = 0.05$ Nm sec rad <sup>-1</sup>
$R_n = 5.7$ $\Omega$	$R_m = 1.475$ $\Omega$	$L_{en} = 0.1766$ H	$L_m = 0.1262$ H
$L_n = 0.0504$ H	$L_{mn} = 0.1262$ H		

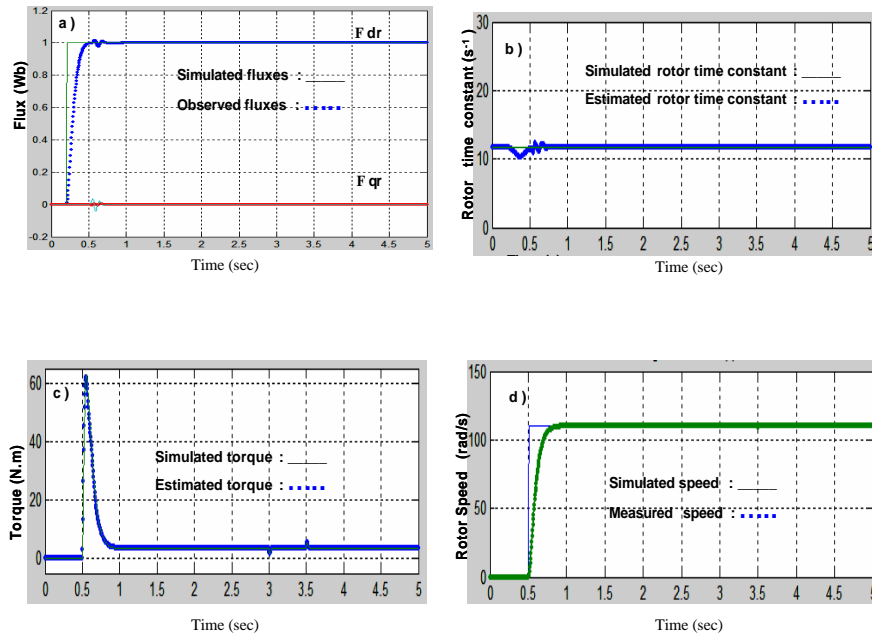


Fig. 3: Nominal case ( $R_r = R_m$ )

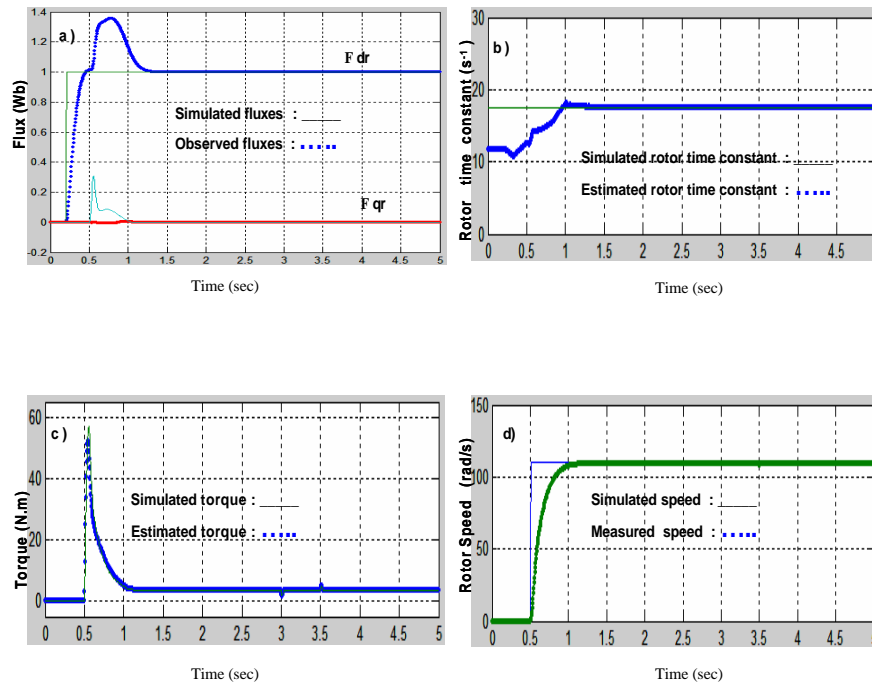


Fig. 4: Non nominal case ( $R_r = 1.5 R_m$ )

## CONCLUSION

In this study, a robust feedback linearization strategy and RDESMO are used to permit a regulation and observation for the Induction motor states in order to assure a good dynamic performance and stability of the global system. This RDESMO is based on SMO principle and extended for the reconstruction of the fluxes, the rotor time constant and the torque estimation.

The interesting simulation results obtained on the induction motor show the effectiveness, the convergence and the stability of this robust decoupling control and RDESMO against rotor resistance variations measured noise and load. Thus, in the industrial applications, one will appreciate very well the experimental implement of this robust estimator for the reconstitution of the fluxes and the torque as well as the rotor resistance.

## NOMENCLATURE

$C_{em}, C_l$	: Electromagnetic and load torques (Nm)
$I_{ds}, I_{qs}$	: Stationary frame (d, q)-axis stator currents (A)
$I_{dr}, I_{qr}, I_{mr}$	: Stationary frame (d, q)-axis rotor currents and rotor magnetizing current (A)
$p, J, f$	: $p$ : Pole pair No. $J$ : Inertia, $kg^2$ ; $f$ : Friction coefficient (Nm.s/rad)
$L_r, L_s, L_m, L_f$	: Rotor, stator, mutual and leakage inductances (H)
$R_s, R_r$	: Stator and rotor referred resistance ( $\Omega$ )
$T_e, T_r, T_s$	: Sampling period, rotor and stator time constant ( $T_r = L_r/R_r = 1/\sigma_r$ ; $T_s = L_s/R_s$ ), (s)
$V_{ds}, V_{qs}$	: Stationary frame d- and q-axis stator voltage (V)
$\Phi_{dr}, \Phi_{qr}, \Phi_{ds}, \Phi_{qs}$	: d-q components of rotor fluxes ( $\Phi_{dr}, \Phi_{qr}$ ) and stator fluxes ( $\Phi_{ds}, \Phi_{qs}$ ) (Wb)
$\omega_s, \omega_r, \omega_{sl}$	: Stator, rotor and slip pulsation (or speed) ( $rad\ sec^{-1}$ )

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