Trends in Applied Sciences Research

ISSN 1819-3579

Academic Journals Inc.

www.academicjournals.com

Trends in Applied Sciences Research 6 (10): 1185-1196, 2011
ISSN 1819-3579 / DOI: 10.3923/tasr.2011.1185.1196

An Efficient and Limited Feedback Opportunistic Beamforming Method

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ABSTRACT

In the current decade, limited feedback methods for data communication in multi-antenna broadcast channels have attracted a lot of interest. The opportunistic beamforming through multiple orthonormal random beamformers (ORBF method) is one of such remarkable methods. The main benefit of this method is obtaining the optimal scaling law of sum-capacity when the number of system users tends to infinity. But, reducing the required feedback amount of opportunistic data transmission is another important factor for practical multi-user systems. In this study, a new and efficient method is proposed which reduces the feedback amount of ORBF method to just $[\text{lod}_2 \text{ m}]$ bits per user using a predetermined threshold, when M is the number of transmitter antennas. The proposed method consists of two scenarios for user selection. Computer simulations of the sum-rate throughput and the scaling factor of each scenario, show that our method outperforms on the one existent opportunistic beamforming method with the same amount of feedback while it is a lot near to the throughput of ORBF.

Key words: Opportunistic communication, orthonormal random beamforming

INTRODUCTION

Sum-capacity is the desired performance in data transmission through multi-antenna broadcast channels and it is known that this performance can be achieved by Dirty Paper Coding (DPC) techniques (Weingarten *et al.*, 2004; Caire and Shamai, 2003; Liejun and Shengwu, 2011). It is shown that for fixed Signal to Noise Ratio (SNR) and M number of antenna elements in the transmitter, the sum-capacity of the system using DPC techniques scales with n, when the number of system users showed with n increases without restriction (i.e., $n \gg 1$). This scaling factor is called as optimal scaling law of sum-capacity. Although the DPC techniques are optimal but they are computationally complex and require the exact Channel State Information (CSI) of all users in the transmitting system. A simpler method named Zero-Forcing (ZF) beamforming is also proposed and has been shown to be optimal in terms of sum-capacity growth for large number of system users (Yoo and Goldsmith, 2006). However, this method needs exact CSI of all users in the transmitter too.

Opportunistic beamforming methods which benefit from their multi-user diversity are new solutions for data transmission in multi-antenna broadcast channels with band limited feedback channels. The idea of such transmission was first proposed by Viswanath *et al.* (2002), where, by use of a random beamformer, data was transmitted to the user with maximum received SNR. Opportunistic Antenna Selection (Opp. Ant. Sel.) (Sanayei and Nosratinia, 2007) is also another type of opportunistic beamforming methods which uses a threshold to determine the eligible users for data transmission. An eligible user in this method is the one whose maximum received SNR is greater than a predetermined threshold. The index of the transmitting antenna element corresponding to this maximum value is sent back to the transmitter, using $[\log_2 M]$ bits of feedback. Then, the transmitter sends the data to a randomly selected eligible user through the related best antenna element. Using this method results in greater sum-rate throughput in comparison with Viswanath *et al.* (2002) Orthonormal Random Beamforming (ORBF) is another method by Sharif and Hassibi (2005) which employs a set of orthonormal random beamformers in the transmitter, simultaneously. By use of this method, data can be transmitted to M users out of n users, simultaneously. In this method, each user evaluates its M received Signal to Interference plus Noise Ratios (SINR) for each of the beamformers, by assuming one beamformer as the desired one and others as interfering signals. Then, each user sends back the value of its maximum received SINR ($SINR_{max}$) and the index of the corresponding beamformer to the transmitter. After receiving the reports of all users, the transmitter assigns the best user to each of the random beamformers. It is shown that the sum-rate of ORBF method will increase with optimal scaling law of sum-capacity by increasing the number of system users.

As it is obvious, for proper decision in ORBF method each user must send back one integer and one real valued number to the transmitter. Diaz *et al.* (2006) reduced the total required feedback to just one bit per user by dividing users in M groups and using a threshold. Although the required feedback was well decreased with Diaz method but the system throughput remarkably fell down. A more efficient method is proposed by Adlband *et al.* (2008) which outperforms Diaz method with the same feedback amount. This method employs the multi-user diversity more efficiently by enlarging the user groups which test each beamformer.

The current study proposed with the nearest output to the optimal method i.e., ORBF.

In this study a more efficient and low rate feedback method which has the nearest output to the optimal method(ORBF) is proposed.

In this method that can be considered as the generalization of Adlband *et al.* (2008) each eligible user (who has $SINR_{max}$ greater than an optimized threshold) informs the transmitter about the index of chosen transmitter antenna by use of $[\log_2 M]$ bits of feedback.

In fact all users are allowed to measure their M received SINRs and evaluate the maximum received SINR due to the threshold. This result to better benefitting of multi-user diversity and spatial multiplexing inherent in the system. Thus, the proposed method will be more efficient comparing with similar opportunistic methods.

In the sequel, $\mathbb{C}^M$ denotes the set of all M-dimensional complex vectors which are shown using boldface lowercase letters. $(.)^T$ and $(.)^H$ show the transposition and hermitian of a vector, respectively. Also, $E\{\cdot\}$ indicates the expected value of a random variable and $\Pr\{A\}$ denotes the probability of the event A.

SYSTEM MODEL

We assume a network of n number of single antenna users. The transmitter is equipped with an array of M antennas and it can use a set of M orthonormal random beamformers , $U = \{u_1,...,u_M\}$ for data transmission to the users. These random beamformers $u_i \epsilon \mathbb{C}^M$ are generated from an isotropic distribution (Hassibi and Marzetta, 2002). By use of these M vectors, data can be transmitted to M independent users, simultaneously. Thus, the transmit signal vector from the base station is:

$$x = \sum_{m=1}^{M} u_m s_m \quad (1)$$

where, s_m is the unit power data transmitted through the mth beamformer. It is assumed that s_m is independent from s_k for $k \neq m$.

Here, we show the channel vector between the transmitter and the ith user by $h_i \in \mathbb{C}^M$. It is usually assumed that the components of this channel vector are i.i.d., zero-mean unity variance complex Gaussian random variables. Additionally, it is considered that h_i is independent from h_j when $j \neq i$ (i.e., the channel vectors of different users are independent from each other). In this case, the received signal by the ith user can be expressed as:

$$y_i = \sqrt{\rho_i}\, h_i^T x + w_i \quad (2)$$

where, ρ_i and w_i are the ith user channel power and additive white Gaussian noise, respectively. In general, users may have different channel powers. Here, we consider a homogenous network in which $\rho_i = \rho$ for I = 1,...,n (Diaz *et al*., 2006).

Assuming the symbol s_m in the transmitted signal vector as the desired signal for the ith user, Eq. 2 can be rewritten as:

$$y_i = \underbrace{\sqrt{\rho}\, h_i^T u_m s_m}_{\text{desired signal}} + \underbrace{\sqrt{\rho} \sum_{k=1, k \neq m}^{M} h_i^T u_k s_k}_{\text{interference}} + w_i \quad (3)$$

Thus, the received SINR for the ith user will be:

$$\mathrm{SINR}_{i,m} = \frac{\rho\, |h_i^T u_m|^2}{1 + \rho \sum_{k \neq m} \left| h_i^T u_k \right|^2} \quad (4)$$

Due to the independency of different users channel vectors and also the orthogonality of random beamformers u_i, it can be shown that the random variables $\mathrm{SINR}_{i,m}$ are i.i.d for I = 1,...,n and m = 1,...,M. Also, each received $\mathrm{SINR}_{i,m}$ has the following cumulative density function:

$$F(x) = \Pr\{\mathrm{SINR}_{i,m} < x\} = 1 - \frac{e^{-x/\rho}}{(1+x)^{M-1}} \quad (5)$$

PROPOSED METHOD

In our proposed method, each user measures its M received SINRs, such as the ORBF method. For this aim, each user needs neither estimating its corresponding channel vector nor estimatong M transmit beamformers. This measurement can be done for example in the ith user by estimating the values of $|h^T_i\, u_m|$ for m = 1,... ,M, using channel training sequences transmitted from the central transmitter and then using Eq. 4.

After this measurement, the index of the beamformer causing the $SINR_{max}$ of each user will be sent back to the transmitter if the value of $SINR_{max}$ is greater than a predetermined threshold. Otherwise, there is no feedback information. In fact, in this method the value of $SINR_{max}$ isn't sent back in contrast with ORBF method and also the index of corresponding beamformer is transmitted back just when the condition of $SINR_{max}>\alpha$ satisfies. Here, such is named as eligible user.

Then, after receiving the reports of all users in central transmitter, it has to choose one appropriate user for each beamformer. The used decision rule for scheduling is illustrated in two different scenarios:

- **Scenario 1:** One eligible user among those with feedback index i, is randomly peaked, for data transmission through the ith beamformer. For each beamformer which is not requested with any user, one user is peaked randomly from the rest of users

- **Scenario 2:** Scheduling is done just when each beamformer of set U is requested at least by one user. Thus, while all of u_is are not requested, a new set will be generated in the transmitter and be tested by all of users. After finding the proper set U, the ith beamformer will be allocated to one randomly peaked eligible user with feedback index i

As it is obvious from the description of two scenarios, we expect to gain greater values of sum-rate throughput in the second scenario compared to the first one. The reason is that all peaked users in the second scenario have $SINR_{max}>\alpha$. Also, it should be mentioned that finding the proper set of beamformers in second scenario requires a question and answering process between the transmitter and users and also impose larger amount of information feedback, while this is not required in the first scenario. Thus, the best scenario selection is a tradeoff between the sum-rate throughput and the feedback amount.

SUM-RATE THROUGHPUT

Here, the sum-rate throughput of the proposed method will be investigated in details, for the two scenarios.

First Scenario: Similar to the first theorem of Diaz *et al.* (2006), it can easily be shown that the sum-rate throughput of this system, R is lower bounded as:

$$R \geq \sum_{i=1}^{M} P_i E\{\log(1+SINR_{max,i}) \mid SINR_{max,i} > \alpha\} \tag{6}$$

where, P_i is the probability of the ith beamformer to be requested by at least one user. Also, $SINR_{max,i}$ indicates the received $SINR_{max}$ of the user which is scheduled for data transmission via the ith beamformer.

As the $SINR_{max,i}$s have the same distribution for i = 1,...,M, the lower bound of sum-rate can be written as:

$$R_{lb} = \left(\sum_{i=1}^{M} P_i\right) C_{SINR_{max}>\alpha} \tag{7}$$

where:

$$C_{SINR_{max}>\alpha} = E\left\{\log(1+SINR_{max}) \middle| SINR_{max} > \alpha\right\}$$

Here, for finding the R_{lb} we have to evaluate the value of P_i for i = 1,...,M to. Since all beamformers have the same condition for being requested by at least one user, it can easily be shown that:

$$P_1 = \cdots = P_M = P$$

Now, considering p as the probability of event that one user does not request a specific beamformer after measuring its maximum received SINR, the probability of a beamformer not to be requested by any user is equal to: p^n and the probability that one beamformer being requested by at least one user is:

$$P = 1 - p^n \tag{8}$$

As a result, evaluating the probability P is sufficient to find probabaility p. For this we consider the tth user and its decisions after evaluating the values of $SINR_{t,m}$ (m = 1,... ,M). As it can be implemented from the description of the proposed method, the tth user functions like the flowchart in Fig. 1.

According to Fig. 1, there are M+1 number of events which may happen for the tth user decision. Event A happens when all received SINRs are less than the threshold,

$$A = \{\{SINR_{t,1} < \alpha\} \cap \cdots \cap \{SINR_{t,M} < \alpha\}\} \tag{9}$$

and event E_i happens when $SINR_{t,i}$ is greater than the others. This event can be explained with,

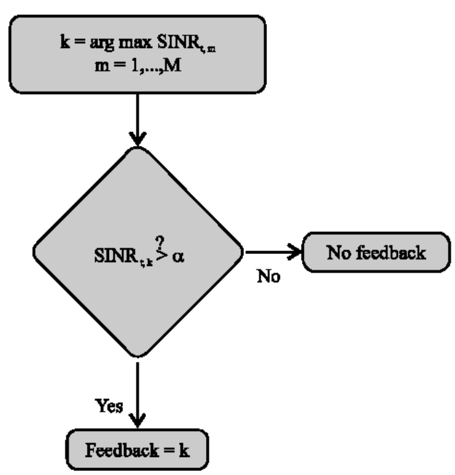

Fig. 1: Flowchart of the tth user decision

$$E_i = \{\{SINR_{t,i} > \alpha\} \cap \{SINR_{t,i} > SINR_{t,j}\}\} \quad (10)$$

for j = 1,...,M and J ≠ i. It should be recalled that there won't be any feedback information to the transmitter if A happens and when E_i takes place the value i must to be sent back. Using Eq. 10 it can be concluded that: $E_i \cap E_j = \Phi$ for j ≠ i. Therefore, the probability of event A is:

$$\Pr\{A\} = 1 - \sum_{i=1}^{M} \Pr\{E_i\} \quad (11)$$

Also, since $SINR_{t,i}$s are i.i.d. random variables, the probability of one beamformer to be requested is equal to the another one. In other words, we have: Pr $\{E_1\} = \cdots =$ Pr $\{E_M\}$. Thus Eq.11 will be:

$$\Pr\{A\} = 1 - M\Pr\{E_1\} \quad (12)$$

In fact, the Pr $\{E_i\}$is less than 1/M.

Also from the definition of event A we have:

$$\Pr\{A\} = F^M(\alpha) \quad (13)$$

where, F(α) is substituted from Eq. 5. Using Eq. 12 and 13, we can write:

$$\Pr\{E_i\} = \frac{1}{M}\left(1 - F^M(\alpha)\right) \qquad i = 1, \ldots, M$$

therefore, it can be concluded that:

$$p = 1 - \Pr\{E_i\} = 1 - \frac{1 - F^M(\alpha)}{M} \quad (14)$$

and

$$P = 1 - \left(1 - \frac{1 - F^M(\alpha)}{M}\right)^n \quad (15)$$

Then, the lower bound will be:

$$R_{lb} = M\left(1 - \left(1 - \frac{1 - F^M(\alpha)}{M}\right)^n\right) C_{SINR_{max} > \alpha} \quad (16)$$

Which is a function of M, n and α. This sum-rate scales with M log log (n) for large values of n, such as ORBF method while requiring lower feedback amount.

To show this behavior, consider the smallest value of throughput when all peaked users have the same received $SINR_{max}$ equals to α. In this condition we have:

$$R_{1b,\alpha} = M\Big(1-\big(1-\frac{1-F^M(\alpha)}{M}\big)^n\Big)\log(1+\alpha) \tag{17}$$

Assuming the threshold value as:

$$\alpha = \rho\log(n) - \rho M\log\log(n) \tag{18}$$

we have:

$$\frac{e^{-\alpha/\rho}}{(1+\alpha)^{M-1}} = \frac{e^{-\log(n)+M\log\log(n)}}{(1+\rho\log(n)-\rho M\log\log(n))^{M-1}} = \frac{\frac{1}{n}(\log(n))^M}{(1+\rho\log(n)-\rho M\log\log(n))^{M-1}} = \frac{\log(n)}{n}\Big(\frac{\log(n)}{1+\rho\log(n)-\rho M\log\log(n)}\Big)^{M-1}$$

For large values of n the second term is almost equal to $1/\rho^{M-1}$. Thus,

$$F(\alpha) \approx 1-\frac{\log(n)}{n\rho^{M-1}} \tag{19}$$

as n goes to infinity.

By use of this result we have:

$$\frac{M-1}{M} < 1-\frac{1-F^M(\alpha)}{M} < 1 \tag{20}$$

According to the above inequality and Eq. 15 and it can be concluded that P tends to the value 1 as n increases unlimitedly. Then this results in:

$$R_{1b,\alpha} \approx M\log\,(1+\rho\log\,(n)-\rho M\log\log\,(n)) \tag{21}$$

which scales with Mloglog (n).

Second scenario: In this scenario the system sum-rate throughput can be found more accurately compared with the first scenario, since here we are sure that all chosen users have $SINR_{max}>\alpha$.

Firstly, suppose that the ith beamformer is requested with K_i users (I = 1,...,M) Since each of beamformers must be requested at least by one user, the value of K_i is bounded such that:

$$1 \le K_i \le n-M+1 \tag{22}$$

Let us show the total number of users having feedback by K_s,

$$K_s = K_1 + \cdots + K_M$$

According to Eq. 22, K_s is bounded as:

$$M \le K_s \le n$$

The lower bound of K_s happens when each beamformer is requested by just one user. Also, K_s = M indicates the case in which all users have feedback information to the transmitter.

It should be mentioned that the data transmission rate through one beamformer depends on the $SINR_{max}$ of the chosen user for it. Therefore for investigating the system sum-rate, the statistics of $SINR_{max}$ must be studied. In fact, according to the definition, in the ith user we have:

$$SINR_{max} = \max_{m=1,\ldots,M} SINR_{t,m}$$

Thus, $SINR_{max}$ is the maximum of M i.i.d. random variables with CDF in (5) and we have:

$$\begin{aligned} F_{SINR_{max}}(x) &= \Pr\left\{\{SINR_{t,1} \le x\} \cap \ldots \cap \{SINR_{t,M} \le x\}\right\} \\ &= \left(\Pr\{SINR_{t,1} \le x\}\right)^M = F^M(x) \end{aligned} \tag{23}$$

This CDF of the $SINR_{max}$ is satisfied for all users.
Here, for a distinct discussion from the previous scenario (where we defined $SINR_{max,\,i}$), here, we denote the maximum received SINR of the ith user by η_t. Since the η_t is a random variable with the value between zero and infinity, the system sum-rate will have different values among these two quantities. Now, let us order the set of maximum received SINRs of all users (η_1, η_2,...,η_n) descending manner. The new random variables are named (X_1, X_2,...,X_n) as in (Sanayei and Nosratinia, 2007) and we have:

$$X_n \le \ldots \le X_{j+1} \le X_j \le X_{j-1} \le \cdots \le X_1$$

In fact, X_j indicates the maximum received SINR of the jth best user of all users. The CDF of X_j is shown by Arnold *et al.* (1992) as:

$$F_i(x) = \sum_{l=0}^{i-1} \frac{n!}{(n-l)!\,l!} (F(x))^{n-l} (1-F(x))^l \tag{24}$$

when the channel gains are i.i.d..

In such a system the data transmission rate through the ith beamformer is:

$$C_j = \int_0^\infty \log(1+x)\, dF_j(x) \tag{25}$$

when this beamformer is selected for the jth best user. We know that this event happens just when the jth best user is in the ith user group and also the scheduler selects it for u_i. The probability of this event is named as p_j^i and we have:

$$P_j^i = \Pr\{C \cap D\} \tag{26}$$

where, C is the event of jth best user being one of K_i users in ith user group and D shows the event of jth best user being chosen with the scheduler. Since scheduling to one user is done without any priority between users, we can write:

$$\Pr\{D\} = \frac{1}{K_i} \tag{27}$$

On the other hand probability of C depends on the distribution of users requesting u_i among all eligible users.

It can easily be shown that the total number of all possible combinations for K_i users requesting u_i is equal to:

$$n_T = \prod_{p=0}^{K_i-1}(K_s - p) \tag{28}$$

Also, the number of possible combinations in which the ith best user be one of the assumed K_i users is:

$$\prod_{p=1}^{K_i-1}(K_s - p)$$

Thus, the total number of cases in which one of these K_i users is the X_i user equals to:

$$n_j = K_i \prod_{p=1}^{K_i-1}(K_s - p) \tag{29}$$

Using Eq. 28 and 29 we have:

$$\Pr\{C\} = \frac{n_j}{n_T} = \frac{K_i}{K_s} \tag{30}$$

and then,

$$P_j^i = \frac{1}{K_i} \times \frac{K_i}{K_s} = \frac{1}{K_s} \tag{31}$$

which just depends to the total number of users with feedback. Therefore, the probability of one arranged user to be selected for one beamformer is equal to another one.

Now, the data rate throughput via the ith beamformer will be:

$$R^i(K_s) = \sum_{j=1}^{K_s} P_j^i C_j = \frac{1}{K_s}\sum_{j=1}^{K_s} C_j$$

and the system sum-rate is,

$$R_{sum}(K_s) = \sum_{i=1}^{M} R^i(K_s) = \frac{M}{K_s}\sum_{j=1}^{K_s} C_j \quad (32)$$

It should be mentioned that this case of R_{sum} (K_s) happens just when there are K_s users having feedback. The probability of this happening can be denoted by p_{Ks} and it is equal to:

$$p_{K_s} = \frac{n!}{(n-K_s)!K_s!} p_\alpha^{K_s}(1-p_\alpha)^{n-K_s} \quad (33)$$

when $p_a = \Pr\{\eta_t > \alpha\}$.

Finally, using Eq. 32 and 33 the average of sum-rate throughput can be written as:

$$\bar{R}_{sum} = M\sum_{K_s=M}^{n}\left(\frac{p_{K_s}}{K_s}\sum_{j=1}^{K_s} C_j\right) \quad (34)$$

According to the fact that the sum-rate throughput of this scenario is greater than that of the first scenario, it can be concluded that this scenario can achieve the optimal scaling factor too. In other words it can be shown that for large values of n, the sum-rate throughput of system will grow with factor of M log log n.

FINDING THE OPTIMAL THRESHOLD

Since, the sum-rate throughput is the performance criterion of multi-user systems, an appropriate choice of threshold is the one which results in the maximum throughput.

In the second scenario, we have derived a formula for R_{sum}that depends on the threshold α. In this case for each value of M, n and ρ the threshold value must be chosen such that it maximizes the average sum-rate.

Also, in the first scenario we obtained a lower bound on the throughput. In this case by setting the threshold to maximize this lower bound, it is expected to attain greater sum-rates in the system. Thus, for each value of M, n and ρ in the system, the value of α is optimized to obtain the maximum R_{lb}.

However, there is no closed form formula for optimal threshold in both scenarios and the desired value can be evaluated through computer simulations.

COMPUTER SIMULATIONS

In this part, we demonstrate the performance of our proposed method scheme via simulation results and get a comparison with the throughput of other methods.

Figure 2 and 3 show the sum-rate throughput of our proposed method using both scenarios as well as the sum-rate throughput of ORBF method and Opp. Ant. Sel. method in a system

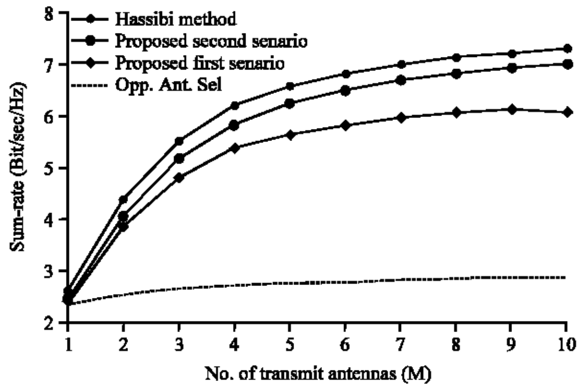

Fig. 2: System sum-rate throughput versus number of transmit antennas at the transmitter (M) for both proposed scenarios as well as ORBF and Opp. Ant. Sel. methods, for a system with n = 100 users and ρ = 0 dB

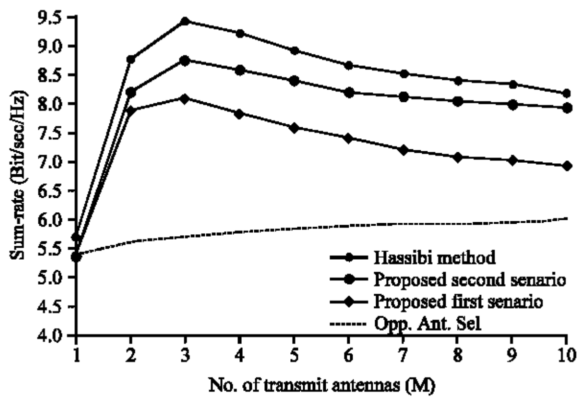

Fig. 3: System sum-rate throughput versus number of transmit antennas at the transmitter (M) for both our proposed scenarios as well as ORBF and Opp. Ant. Sel. methods, for a system with n = 100 users and ρ = 10 dB

with n = 100 number of users for ρ = 0 and ρ10 dB, respectively. This throughput values are resulted from averaging over 1000 number of simulation running.

In comparing the performance of the two scenarios, the second scenario outperforms the first one as it is expected. The reason is that all chosen users in the second scenario have maximum received SINR greater than the threshold, but it is not guaranteed for chosen users in the first scenario.

Also, it can be seen from the figure that the throughput of two scenarios are less than the sum-rate resulted from ORBF. This result can be explained due to the greater feedback amount of ORBF method compared with our method. Since the value of maximum SINRs must be sentback in addition to corresponding index.

It is clear from the curves that the difference between second scenario throughput and that of ORBF is less than 1 Bit/Sec/Hz in each point. This fact is true while the second scenario requires less feedback amount especially for small values of M and large number of system users.

Also, it is worthy to note the difference between the throughput of Opp. Ant. Sel. method and our proposed scenarios in spite of the fact that they have the same amount of feedback ($\lceil \log_2 M \rceil$ bits per user). In fact in Opp. Ant. Sel. method, although there are M antenna elements in the transmitter, but data is transmitted to just one selected user and there is no use of spatial multiplexing. Thus, our proposed scenarios with exploiting this inherent characteristic of system, outperforms this method which is the one existent method with the same feedback amount.

CONCLUSION

A simple and efficient method is proposed in this study to reduce the feedback amount required in opportunistic beamforming method of Sharif and Hassibi (2005) which uses a set of orthonormal random beamformers to serve M number of users (equal to the number of transmit antennas at the central transmitter), simultaneously. It is shown that the feedback amount of our proposed method which is explained in two scenarios is equal to $[\log_2 (M)]$ bits per user. By computer simulations we have shown that the sum-rate throughput of our proposed method is greater than the method by Sanayei and Nosratinia (2007) with the same amount of feedback. Also it is shown that the sum-rate throughput of our method in both scenarios scales with the optimal scaling law, i.e., M log log n as the number of users in the system goes to infinity.

ACKNOWLEDGMENT

This study has been supported by Islamic Azad university, Kazerun Branch.

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