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A Multiple Items Inventory Model under Uncertain External Inflationary Conditions

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ABSTRACT

The inflation rate in the most of the previous researches has been considered constant and well-known over the time horizon, although the future rate of inflation is inherently uncertain and unstable and is difficult to predict it accurately. Therefore, assumption of the constant inflation rates is not valid. This study presents an inventory model with stochastic inflation rate for multiple items with budget constraint. The inventory systems costs are divided to the internal and external costs which will be affected with the internal and external rates over a finite time horizon. The discounted value method is used for mathematical analysis. The numerical examples have also been given to illustrate the developed models.

Key words: Inventory, stochastic, inflation, budget, discounted cost, chi-square test

INTRODUCTION

Since 1975 a series of related studies appeared that considered the effects of inflation on the inventory system. Before the 1990s, the earlier efforts have been studied simple situations. Buzacott (1975) proposed an Economic Order Quantity (EOQ) model with inflation subject to different types of pricing policies. Misra (1979) developed a discounted-cost model for various costs associated with an inventory system.

There are some researches to have founded in the deteriorating inventory area in recent years. Products such as vegetables, fish, medicine, blood, gasoline and radioactive chemicals have finite shelf life and start to deteriorate once they are produced. In a few of these works, deterioration rate is not constant. For instance, Chen (1998) proposed an inflationary model with time-proportional demand and weibull distribution for deteriorating items using dynamic programming. Balkhi (2004a, b) presented a production lot size inventory model that the production, demand and deterioration rates are known, continuous and differentiable functions of time. Shortages are allowed, but only a fraction of the stock out is backordered and the rest is lost. Lo *et al.* (2007) developed an integrated production-inventory model with assumptions of varying rate of deterioration, partial backordering, inflation, imperfect production processes and multiple deliveries. The most of the inventory systems for deteriorating items are considered a constant deterioration rate which will state in continuance. Another effort on inflationary inventory systems for deteriorating items has been made by Hsieh and Dye (2010) with considering pricing and lot-sizing policies.

Some research in inflationary inventory systems assumed time-varying demand rate. Datta and Pal (1991) investigated a finite-time-horizon inventory model with linear time-dependent demand rate when shortages are allowed. Yang *et al.* (2001) extended the inventory lot-size models to allow for inflation and fluctuating demand, which is more general than constant, increasing, decreasing and log-concave demand patterns. Chern *et al.* (2008) proposed partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation. Other works are performed by Chen (1998) and Balkhi (2004b). There are some items whose value or utility or quantity increase with time and those items can be termed as ameliorating item. There are a few models for ameliorating items. Moon *et al.* (2005) develop models for ameliorating/deteriorating items with time varying demand pattern over a finite planning horizon, taking into account the effects of inflation and time value of money.

Several authors have considered finite replenishment rate for inflationary inventory systems. Wee and Law (1999) derived a deteriorating inventory model under inflationary conditions for determining economic production lot size when, the demand rate is a linear decreasing function of the selling price. Sarker and Pan (1994) surveyed the effects of inflation and the time value of money on order quantity with finite replenishment rate. Balkhi (2004a) proposed two flexible production lot size inventory models for deteriorating items in which the production rate at any instant depends on the demand and the on-hand inventory level at that instant. Another research is performed by Lo *et al.* (2007). Maity (2010) presents a one machine multiple-product problem with production-inventory system under Fuzzy inequality constraint.

The stock-dependent demand rate models are prepared with some researchers. Vrat and Padmanabhan (1990) determined optimal ordering quantity for stock-dependent consumption rate items and showed that as the inflation rate increases, ordering quantity and the total system cost increase. Liao and Chen (2003) surveyed a retailer's inventory control system for the optimal delay in payment time for initial stock-dependent consumption rate when, a wholesaler permits delay in payment. The effect of inflation rate, deterioration rate, initial stock-dependent consumption rate and a wholesaler's permissible delay in payment is discussed. Hou and Lin (2006) present a deterministic Economic Order Quantity (EOQ) inventory model taking into account inflation and time value of money developed for deteriorating items with price- and stock-dependent selling rates. An efficient solution procedure is presented to determine the optimal number of replenishment, the cycle time and selling price. Hou (2006) prepared an inventory model for deteriorating items with stock-dependent consumption rate. Maiti *et al.* (2006) proposed an inventory model with stock-dependent demand rate and two storage facilities under inflation and time value of money where, the time horizon is stochastic in nature and follows exponential distribution with a known mean. An inventory model under inflation proposed by Yang *et al.* (2009) for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. Sana (2010) developed a multi-item EOQ model of deteriorating and ameliorating item for demand influenced by enterprises' initiatives.

Other efforts in inventory systems under inflationary conditions are performed under the assumption of the permissible delay in payments. Chang (2004) proposed an EOQ model for deteriorating items under inflation when, the supplier offers a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. Shah (2006) derived an inventory model by assuming constant rate of deterioration of units in an inventory, time value of money under the conditions of permissible delay in payments. Other models are prepared by Liao and Chen (2003) and Hsu *et al.* (2008).

The inflationary inventory models with two warehouses are proposed previously. Yang (2004) discussed the two-warehouse inventory problem for deteriorating items with a constant demand rate and shortages. Yang (2006) extended the models introduced in Yang (2004) to incorporate partial backlogging and then compare the two two-warehouse inventory models based on the minimum cost approach. Huang and Lai (2006) developed an optimal lot-sizing model under two warehouses and two-level delay permitted using algebraic method. A Two storage inventory problem with dynamic demand and interval valued lead-time over finite time horizon under inflation and time-value of money considered by Dey *et al.* (2008).

The above mentioned studies have considered a constant and well-known inflation rate over the time horizon. But, many economic factors may affect on the future changes in the inflation rate; such as changes in the world inflation rate, rate of investment, demand level, labor costs, cost of raw materials, rates of exchange, rate of unemployment, productivity level, tax, liquidity, etc. Therefore, assumption of the constant inflation rates is not valid. Mirzazadeh (2010) assumed the inflation is time-dependent and demand rate is assumed to be inflation-proportional. Horowitz (2000) discussed a simple EOQ model with a Normal distribution for the inflation rate and the firm's cost of capital. He showed the importance of taking into account the inflation rate and time discounting, especially when the former is relatively high or when there is considerable uncertainty as to either the inflation rate or the marginal cost of capital. Ameli *et al.* (2011) presented an economic order quantity model with imperfect items under fuzzy inflationary conditions.

This study considers internal and external inflation rates. Since, the internal costs are controllable, it is assumed that the Internal Inflation Rate (IIR) is constant. But, the External Inflation Rate (EIR) is stochastic. However, we can develop easily the proposed mathematical model for both IIR and EIR stochastic conditions. The world inflation rates since 1970 have been surveyed using the Chi-square test. It has been concluded that the Uniform distribution is suitable for the inflation rate.

THE BASIC ASSUMPTIONS AND NOTATIONS

The following assumptions have been considered:

- A multiple items inventory system has been considered
- The available budget for purchasing is constrained and will increase through the External Inflation Rate (EIR)
- The demand rates are constant
- The inventory system costs are known at the beginning of time horizon and will increase through the inflation rates
- Replenishment is instantaneous, i.e., the replenishment rate is infinite
- The stock level at the beginning of time horizon and at the end of time horizon is zero
- The time horizon is constant and finite

The following notations are used in the model:

n	= The number of items
Q_i	= The order quantity for i-th item
D_i	= The annual demand rate for i-th item
S_i	= The ordering cost for i-th item at the beginning of time horizon

- C_i = The purchasing cost per unit for i-th item at time zero
 h_{ki} = The internal (for $k = 1$) and the external (for $k = 2$) holding cost per unit per unit time for i-th item at time zero
 f_k = The internal inflation rate (IIR) for $k = 1$ and the External Inflation Rate (EIR) for $k = 2$
 R_k = The discount rate net of inflation: $R_k = r - f_k$ where r is the discount rate
 B = The maximum available budget at the beginning of time horizon
 H = The time horizon
 N_i = The number of replenishment cycles over time horizon: $N_i = HD_i/Q_i$
 $E[PWC_{ij}]$ = The expected present value of the total costs for i-th item for j-th cycle where, $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, N_i$

THE MATHEMATICAL MODEL AND ANALYSIS

The many studies of inventory management systems have been reported. Analysis of the inventory systems in the literature is carried out using two procedures. The first procedure determines the optimal values of the control variables by minimizing the average annual cost and the alternative (and in theory more correct) procedure determines the optimal ordering policy by minimizing the discounted value of all future costs. Hadley in 1964, showed, by detailed computations in the simplest case, corresponding to the familiar deterministic lot size model, the order quantities computed by minimizing the average annual cost and by minimizing the discounted cost do not differ significantly over a wide range of the values of the pertinent parameters and certainly not for the range values normally encountered in inventory problems. It is shown, however, that in extreme circumstances sizable differences can be obtained. Therefore, the discounted value method is used in this study. The present value of the inventory system costs (including purchasing, ordering and carrying costs) for the i-th item for the j-th cycle is as follows:

$$PWC_{ij} = S_i e^{-(r-f_k)(j-1)Q_i/D_i} + \sum_{k=1}^2 \left[\frac{h_{ki} Q_i}{r-f_k} - \frac{h_{ki} D_i}{(r-f_k)^2} \left(1 - e^{-(r-f_k)Q_i/D_i} \right) e^{-(r-f_k)(j-1)Q_i/D_i} \right] + Q_i C_i e^{-(r-f_k)(j-1)Q_i/D_i} \quad (1)$$

The external inflation rate has uniform distribution function as follows:

$$f(f_2) = \frac{1}{b-a} I_{[a,b]}(f_2) \quad (2)$$

The objective is minimization of the expected present value of the total costs for the total items for the total cycles over the time horizon:

$$\text{Min}Z = \sum_{i=1}^n \sum_{j=1}^{N_i} E[PWC_{ij}] \quad (3)$$

Where:

$$E[PWC_{ij}] = \int_a^b PWC_{ij} f(f_2) df_2 = S_i e^{-(r-f_1)(j-1)Q_i/D_i} + \left[\frac{h_{1i} Q_i}{r-f_1} - \frac{h_{1i} D_i}{(r-f_1)^2} (1 - e^{-(r-f_1)Q_i/D_i}) \right] e^{-(r-f_1)(j-1)Q_i/D_i} + \frac{h_{2i} Q_i}{b-a} I_1 - \frac{h_{2i} D_i}{(b-a)} (I_2 - I_3) + \frac{Q_i C_i}{b-a} I_4 \quad (4)$$

I_1, I_2, I_3 and I_4 are as follows:

$$I_1 = \int_a^b \frac{e^{-(j-1)(r-f_2)Q_i/D_i}}{r-f_2} df_2 \quad (5)$$

$$I_2 = \int_a^b \frac{e^{-(j-1)(r-f_2)Q_i/D_i}}{(r-f_2)^2} df_2 \quad (6)$$

$$I_3 = \int_a^b \frac{e^{-(j-1)(r-f_2)Q_i/D_i}}{(r-f_2)^2} df_2 \quad (7)$$

$$I_4 = \int_a^b e^{-(j-1)(r-f_2)Q_i/D_i} df_2 \quad (8)$$

The solution of I_1 after expansion the series is:

$$I_1 = - \left[\text{Ln}|r-f_2| + \sum_{m=1}^{\infty} \frac{(-1)^m (j-1)^m (r-f_2)^m (Q_i/D_i)^m}{m.m!} \right]_a^b \quad (9)$$

The third term and following terms of Σ in the above Equation are negligible. Therefore, I_1 equals to:

$$I_1 = \text{Ln} \left| \frac{r-a}{r-b} \right| + \frac{(j-1)(a-b)Q_i}{D_i} - \frac{(j-1)^2 Q_i^2}{4D_i^2} [(r-b)^2 - (r-a)^2] \quad (10)$$

Similarly, I_2 accordingly is:

$$I_2 = \frac{e^{-(j-1)(r-b)Q_i/D_i}}{r-b} - \frac{e^{-(j-1)(r-a)Q_i/D_i}}{r-a} - \frac{(j-1)Q_i}{D_i} \text{Ln} \left| \frac{r-a}{r-b} \right| - \frac{(j-1)^2 Q_i^2 (a-b)}{D_i^2} - \frac{(j-1)^3 Q_i^3}{4D_i^3} [(r-b)^2 - (r-a)^2] \quad (11)$$

I_3 can be calculated analogously. Finally, after calculation the last item, I_4 , we have:

$$I_4 = \begin{cases} b-a & \text{for } j=1 \\ \frac{e^{-(j-1)(r-b)Q_i/D_i}}{(j-1)Q_i/D_i} - \frac{e^{-(j-1)(r-a)Q_i/D_i}}{(j-1)Q_i/D_i} & \text{for } j=1 \end{cases} \quad (12)$$

By substituting the above results in Eq. 4 and simplifying, the expected present value for i-th item for j-th cycle will be obtained. Thus, the total costs of i-th item over the time horizon is:

$$\sum_{j=1}^{N_i} E[PWC_{ij}] \quad (13)$$

The following series will be used for calculation of Eq. 13:

$$\sum_{j=1}^{N_i} j = \frac{D_i(Q_i + D_i)}{2Q_i^2} \quad (14)$$

$$\sum_{j=1}^{N_i} j(2j-1) = \frac{D_i(Q_i + D_i)(4D_i - Q_i)}{6Q_i^3} \quad (15)$$

$$\sum_{j=1}^{N_i} [e^{-(j-1)(r-b)Q_i/D_i} - e^{-j(r-b)Q_i/D_i}] = 1 - e^{-r+b} \quad (16)$$

$$\sum_{j=1}^{N_i} [e^{-(j-1)(r-a)Q_i/D_i} - e^{-j(r-a)Q_i/D_i}] = 1 - e^{-r+a} \quad (17)$$

$$\sum_{j=1}^{N_i} e^{-(j-1)(r-f_1)Q_i/D_i} = \frac{1 - e^{-r+f_1}}{1 - e^{-(r-f_1)Q_i/D_i}} \quad (18)$$

Thus, the objective function can be shown in Eq. 19:

$$\text{MinZ} = \sum_{i=1}^n \left[Q_i C_i + \frac{D_i C_i}{b-a} \sum_{j=2}^{N_i} \frac{e^{-(j-1)(r-b)Q_i/D_i} - e^{-(j-1)(r-a)Q_i/D_i}}{j-1} + \frac{h_{2i}(D_i + Q_i)}{2} \right. \\ \left. + \frac{h_{2i}(4D_i^2 + 3D_i Q_i - Q_i^2)}{24D_i(b-a)} [(r-b)^2 - (r-a)^2] - \frac{h_{2i} D_i}{(b-a)} \left(\frac{1 - e^{-r+b}}{r-b} - \frac{1 - e^{-r+a}}{r-a} \right) \right. \\ \left. + \left[S_i + \frac{h_{1i} Q_i}{r-f_1} \left(1 - \frac{1 - e^{-(r-f_1)Q_i/D_i}}{r-f_1} \right) \right] \frac{1 - e^{-(r-f_1)}}{1 - e^{-(r-f_1)Q_i/D_i}} \right] \quad (19)$$

THE BUDGET CONSTRAINT

The available budget at time zero is B which increases by EIR. Also, the unit price, C_i , increases by EIR. Therefore, the budget constraint at time t is:

$$\sum_{i=1}^n Q_i C_i e^{f_2 t} \leq B e^{f_2 t} \quad \text{for } t \in [0 - H] \quad (20)$$

By simplifying Eq. 20 we have:

$$\sum_{i=1}^n Q_i C_i \leq B \quad (21)$$

The stochastic inventory model with budget constraint using Eq. 19 and 21 is as follows:

$$\begin{aligned} \text{Min} Z &= \sum_{i=1}^n \sum_{j=1}^{N_j} E[PWC_{ij}] \\ &\sum_{i=1}^n Q_i C_i \leq B \end{aligned} \quad (22)$$

The lagrange method is used to optimize of the method:

$$L(Q_i, \lambda) = Z + \lambda \left[\sum_{i=1}^n Q_i C_i - B \right] \quad (23)$$

Minimization of the original model, Eq. 22, follows the minimization of the Lagrangian expression L. This can be done by taking the partial derivations of L (Q_i , λ) with respect to Q_i and λ and setting them equal to zero. Next section gives a numerical example to illustrate the model.

THE NUMERICAL EXAMPLE

The following numerical example is provided to clarify how the proposed model is applied. Let, $H = 2$ years, $B = 15000\$$, $r = 20\%$ and $n = 2$. The internal inflation rate, f_1 , is 8% and the external inflation rate has the Uniform distribution function with $a = 10\%$ and $b = 16\%$. The demand rates per unit time are: $D_1 = 2000$ units and $D_2 = 3500$ units. The ordering, purchasing and carrying costs at the beginning of time horizon are $S_1 = \$100/\text{order}$; $S_2 = \$110/\text{order}$; $C_1 = \$5/\text{unit}$; $C_2 = \$4/\text{unit}$; $h_{11} = \$2/\text{unit/year}$; $h_{12} = \$4/\text{unit/year}$; $h_{21} = \$4/\text{unit/year}$; $h_{22} = \$5/\text{unit/year}$.

The problem is to find the optimal ordering policy through minimizing the expected present value of the total inventory system costs. Considering the above information and using the numerical methods, the problem is solved and the optimum values of Q_1 , Q_2 and λ are obtained as follows:

$$Q_1^* = 390 \text{ units}; Q_2^* = 415 \text{ units}; \lambda^* = 0.23586$$

CONCLUSION

In the previous literature of the inventory system under inflationary conditions, usually, the inflation rates have been assumed constant over the time horizon. But, many economic factors may affect on the future changes in the inflation rate; such as changes in the world inflation rate, rate of investment, demand level, labor costs, cost of raw materials, rates of exchange, rate of unemployment, productivity level, tax, liquidity, etc. Therefore, assumption of the constant inflation

rates is not valid. The current developed model considers stochastic inflation rate with Uniform distribution function. The numerical example has been given to illustrate the theoretical results.

REFERENCES

- Ameli, A., A. Mirzazadeh and M.A. Shirazi, 2011. Economic order quantity model with imperfect items under fuzzy inflationary conditions. *Trends Applied Sci. Res.*, 6: 294-303.
- Balkhi, Z.T., 2004a. An optimal solution of a general lot size inventory model with deteriorated and imperfect products, taking into account inflation and time value of money. *Int. J. Syst. Sci.*, 35: 87-96.
- Balkhi, Z.T., 2004b. On the optimality of inventory models with deteriorating items for demand and on-hand inventory dependent production rate. *IMA J. Manage. Math.*, 15: 67-86.
- Buzacott, J.A., 1975. Economic order quantities with inflation. *Operat. Res. Q.*, 26: 553-558.
- Chang, C.T., 2004. An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. *Int. J. Prod. Econ.*, 88: 307-316.
- Chen, J.M., 1998. An inventory model for deteriorating items with time-proportional demand and shortages under inflation and time discounting. *Int. J. Prod. Econ.*, 55: 21-30.
- Chern, M.S., H.L. Yang, J.T. Teng and S. Papachristos, 2008. Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation. *Eur. J. Operat. Res.*, 191: 127-141.
- Datta, T.K. and A.K. Pal, 1991. Effects of inflation and time-value of money on an inventory model with linear time-dependent demand rate and shortages. *Eur. J. Operat. Res.*, 52: 326-333.
- Dey, J.K., S.K. Mondal and M. Maiti, 2008. Two storage inventory problem with dynamic demand and interval valued lead-time over finite time horizon under inflation and time-value of money. *Eur. J. Operat. Res.*, 185: 170-194.
- Horowitz, I., 2000. EOQ and inflation uncertainty. *Int. J. Prod. Econ.*, 65: 217-224.
- Hou, K.L., 2006. An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting. *Eur. J. Operat. Res.*, 168: 463-474.
- Hou, K.L. and L.C. Lin, 2006. An EOQ model for deteriorating items with price- and stock-dependent selling rates under inflation and time value of money. *Int. J. Syst. Sci.*, 37: 1131-1139.
- Hsieh, T.P. and C.Y. Dye, 2010. Pricing and lot-sizing policies for deteriorating items with partial backlogging under inflation. *Expert Syst. Appl. Int. J.*, 37: 7234-7242.
- Hsu, K.H., H. Hung-Fu, T. Yu-Cheng and H. Yung-Fu, 2008. Optimal inventory planning under permissible delay in payments when a larger order quantity. *J. Applied Sci.*, 8: 1049-1054.
- Huang, Y.F. and C.S. Lai, 2006. Optimal lot-sizing policy under two warehouses and two-level delay permitted using algebraic method. *J. Applied Sci.*, 6: 3060-3067.
- Liao, H.C. and Y.K. Chen, 2003. Optimal payment time for retailer's inventory system. *Inter. J. Syst. Sci.*, 34: 245-253.
- Lo, S.T., H.M. Wee and W.C. Huang, 2007. An integrated production-inventory model with imperfect production processes and Weibull distribution deterioration. *Int. J. Prod. Econ.*, 106: 248-260.
- Maiti, A.K., M.K. Maiti and M. Maiti, 2006. Two storage inventory model with random planning horizon. *Applied Math. Comput.*, 183: 1084-1097.
- Maity, A.K., 2010. One machine multiple-product problem with production-inventory system under fuzzy inequality constraint. *Applied Soft Comput.*, 10.1016/j.asoc.2009.12.029

- Mirzazadeh, A., 2010. Effects of variable inflationary conditions on an inventory model with inflation-proportional demand rate. *J. Applied Sci.*, 10: 551-557.
- Misra, R.B., 1979. A note on optimal inventory management under inflation. *Naval Res. Logist.*, 26: 161-165.
- Moon, I., B.C. Giri and B. Ko, 2005. Economic order quantity models for ameliorating/deteriorating items under inflation and time discounting. *Eur. J. Operat. Res.*, 162: 773-785.
- Sana, S.S., 2010. Demand influenced by enterprises initiatives-A multi-item EOQ model of deteriorating and ameliorating items. *Math. Comput. Modell.*, 52: 284-302.
- Sarker, B.R. and H. Pan, 1994. Effects of inflation and the time value of money on order quantity and allowable shortage. *Int. J. Prod. Econ.*, 34: 65-72.
- Shah, N.H., 2006. Inventory model for deteriorating items and time value of money for a finite time horizon under the permissible delay in payments. *Int. J. Syst. Sci.*, 37: 9-15.
- Vrat, P. and G. Padmanabhan, 1990. An inventory model under inflation for stock-dependent consumption rate items. *Eng. Costs Prod. Econ.*, 19: 379-383.
- Wee, H.M. and S.T. Law, 1999. Economic production lot size for deteriorating items taking account of the time-value of money. *Comput. Operat. Res.*, 26: 545-558.
- Yang, H.L., J.T. Teng and M.S. Chern, 2001. Deterministic inventory lot-size models under inflation with shortages and deterioration for fluctuating demand. *Naval Res. Logistics*, 48: 144-158.
- Yang, H.L., 2004. Two-warehouse inventory models for deteriorating items with shortages under inflation. *Eur. J. Operat. Res.*, 157: 344-356.
- Yang, H.L., 2006. Two-warehouse partial backlogging inventory models for deteriorating items under inflation. *Int. J. Prod. Econ.*, 103: 362-370.
- Yang, H.L., J.T. Teng and M.S. Chern, 2009. An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. *Int. J. Prod. Econ.*, 123: 8-19.