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## State Variable Model of a Solar Power System

<sup>2</sup>A.B. Chattopadhyay, <sup>1</sup>A. Choudhury and <sup>2</sup>A. Nargund

<sup>1</sup>Department of Electronics and Instrumentation Engineering, Birla Institute of Technology and Science, Pilani-Dubai, UAE

<sup>2</sup>Department of Electrical and Electronics Engineering, Birla Institute of Technology and Science, Pilani-Dubai, UAE

*Corresponding Author: A.B. Chattopadhyay, Department of Electrical and Electronics Engineering, Birla Institute of Technology and Science, Pilani-Dubai, P.O. Box 345055, Plot No. UG 06, Dubai International Academic City, Dubai, UAE Tel: (009714) 4200700/225 Fax: (009714) 4200555*

### ABSTRACT

This study develops a mathematical technique for the solution of a non-linear state variable model of a solar array power system. Significance of the technique lies in the fact that experimental complexities can be avoided to reach a desired conclusion regarding the design of the controller associated with a solar array power system. An iterative method has been used in which the initiating assumption has been made to consider the system to depend entirely upon its initial values at zero time instant and taking the forcing function to be zero. In the next step of the analysis the obtained time response of the state variable again has been used iteratively to reach the final solution with the forcing function considered. The non-linearity appearing in the state variable formulation is due to the fact that the forcing function is a function of the state variable itself. In the intermediate stage of analysis Maclaurin Series has been applied to find the Laplace Transform of certain mathematical function containing singularity at zero time instant. The time response expression resulting from the analysis has been used to obtain various plots. The results of simulation in MATLAB establish the fact that there is a difference in the energy stored between the inductor and capacitor. This difference is attributed to the natural resistance value associated with the inductor coil and the fictitious resistance (responsible for power loss) associated with the capacitor and any incremental resistance in the controller circuit.

**Key words:** Equivalent circuit, non-linear, iteration, controller, load, characteristics

### INTRODUCTION

Growing energy demands and limited fossil fuel reserves have led to an increased awareness to incorporate renewable sources of energy. Solar Arrays were initially employed to power satellites and spacecraft (Bae *et al.*, 2008), but over the years their application scope has broadened and they are now also being used in terrestrial applications. In light of these recent developments, they are bound to play a promising role in future.

A solar array power system consists of a solar array, solar array regulator (Mourra *et al.*, 2010; Cho and Cho, 2001) and a battery that is charged together with the load. Numerous research works have been performed on the modeling of such systems (Jensen *et al.*, 2010; Wang *et al.*, 2010; Xiaolei *et al.*, 2010; Ramaprabha *et al.*, 2009; Bouchafaa *et al.*, 2010). One of the models helps in the prediction of power output (Jensen *et al.*, 2010). The system possesses a non-linear current

voltage characteristic and the companion research paper (Wang *et al.*, 2010; Xiaolei *et al.*, 2010) explains a method to calculate the current voltage curve of a complex solar array based on basic parameters.

It is evident from the research work (Bouchafaa *et al.*, 2010) that application of fuzzy logic makes it easier to tackle non-linear systems. Many researchers (Huynh and Cho, 1996; Hua and Shen, 1998; Siri and Conner, 2002) have also focused their attention on maximum power tracking and as a result many different tracking techniques have been proposed to optimize the operation of solar arrays. Apart from tackling non-linear systems, neural networks have also provided improved methods in maximum peak power tracking (Ramaprabha *et al.*, 2009). In general, both fuzzy logic (Radaideh, 2003) and neural networks (Zayandehroodi *et al.*, 2010) have provided valuable results in the area of power systems.

Issues of stability have been addressed by researchers based on analytical modeling in terms of small perturbation model (Siri, 2000a, b). On the other hand, in large signal model, researchers have focused on analyzing the system stability by classifying the effective load characteristics of different control methods such as constant power load, variable power load, constant voltage load, constant current load and constant resistive load (Bae *et al.*, 2008). The inherent non-linearity in the system results in multiple equilibrium points. To evaluate the stability of these multiple equilibrium points, the authors of research paper (Bae *et al.*, 2008) have performed an analysis using the state plane technique.

Even though some analysis has been covered in literature, the authors of the present paper feel that the state variable model containing some non-linearity can be dealt analytically using some special mathematical technique to have a better prediction on the current through the inductor and voltage across the capacitor in connection with the controller circuit of the solar array power system. Such work is based on the strategy stated in research paper (Bae *et al.*, 2008), but the exact way of tackling the nonlinearity mathematically has not been presented in that paper. Such detailed aspects are presented in the next section of the present paper. In the research paper (Bae *et al.*, 2008), the state equations have been defined and an experimental approach has been used to analyze the stability. In the present study, the state variable model of the solar array power system has been developed and the explicit expressions for the time responses of the state variables have been derived using a mathematical technique. Having the explicit expressions of the state variables associated with the controller circuit in hand basically helps any designer to predict any abnormal rise in capacitor voltage or inductor current. Such aspect seems to be very crucial so far the design of the controller circuit is concerned. One of the advantages of state variable modeling is that it is easily amenable to computer simulation. This advantage is due to the fact the state space representation basically indicates a set of first order differential equations. State variable modeling has also been applied in other areas such as DC-DC boost converters by one of the authors of the present paper (Chattopadhyay *et al.*, 2009, 2005). As any formulation needs a simulation, proposed methodology in the present study also follows simulation results presented in the appropriate section. Ghorab *et al.* (2007) have used MATLAB to perform the simulation of a turbo generator. The results of the present study maintain similarity with the physical facts involved in a controller circuit consisting of the energy storing elements such as inductor and capacitor. The study ends up with suitable concluding remarks.

## **PROBLEM FORMULATION**

The analysis of the present paper is based on the configuration of the solar array power system presented in Fig. 1 (Bae *et al.*, 2008). Furthermore to start the analysis, the equivalent circuit

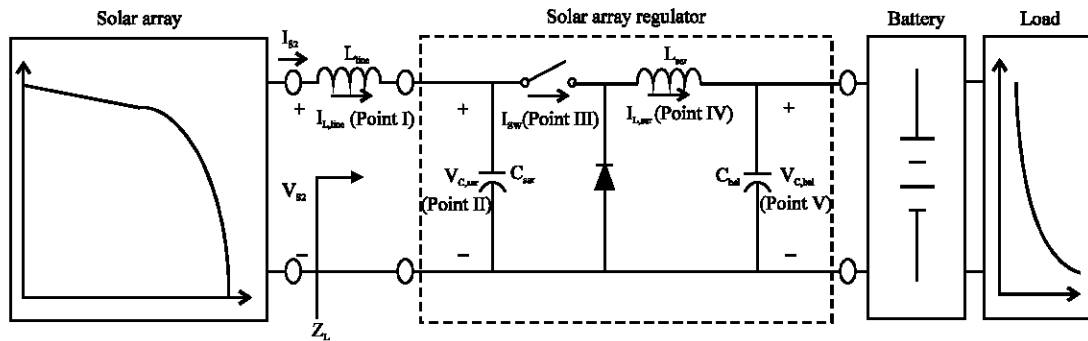


Fig. 1: Series configuration of a solar array regulator system (Bae *et al.*, 2008)

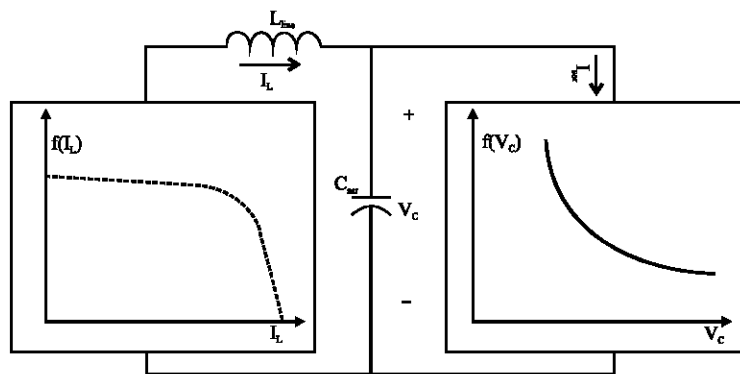


Fig. 2: Equivalent circuit model of constant power load (Bae *et al.*, 2008)

model for any load with a specific nature (e.g., constant voltage, constant power type etc.) is needed at this stage. Such nature is shown in Fig. 2 (Bae *et al.*, 2008) and it is a constant power type load. It may be noted that the array output and the load impedance have clearly non-linear characteristics.

The mathematical technique has been summarized in Fig. 3 and can be treated as a generalized approach to non-linear state variable analysis. The state equations are given as follows:

$$\frac{di_L}{dt} = \frac{1}{L}f(i_L) - \frac{v_c}{L} \tag{1}$$

$$\frac{dv_c}{dt} = \frac{i_L}{C} - \frac{1}{C}f(v_c) \tag{2}$$

With the help of these state equations, the iterative technique has been applied on the state variable model developed in the next section. This project work was conducted in the Department of Electrical and Electronics Engineering, BITS, Pilani-Dubai, Dubai International Academic City, U.A.E during the period from 01-02-2010 to 15-09-2010.

## DEVELOPMENT OF NONLINEAR STATE VARIABLE MODEL AND METHOD OF SOLUTION

For the formulation of the state variable matrix, let the state variable function be:

$$\frac{dx}{dt} = Ax + Bu \tag{3}$$

where 'x' is the state variable and 'u' is the forcing function.

In case of a solar array power system,

$$\begin{aligned} u &= f(x) \\ \Rightarrow \frac{dx}{dt} &= Ax + Bf(x) \end{aligned} \tag{4}$$

Equation 4 clearly indicates that the control input is not independent rather it depends on the state variable itself. Hence the traditional mathematical technique of solving a linear state variable model will not hold good in the present case. The proposed iterative method for tackling such non-linear system needs a flowchart which is presented in Fig. 3.

The flowchart aids in the analysis of a non-linear state variable model of any system and the technique has been elucidated in this paper with the solar array power system. The state equations are given as follows:

$$\frac{di_L}{dt} = \frac{1}{L}f(i_L) - \frac{v_c}{L} \tag{5}$$

$$\frac{dv_c}{dt} = \frac{i_L}{C} - \frac{1}{C}f(v_c) \tag{6}$$

Let  $i_L = x_1$  and  $v_c = x_2$

Therefore,

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/L & 0 \\ 0 & -1/C \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \end{bmatrix} \tag{7}$$

Let  $f(x_1) = u_1$  and  $f(x_2) = u_2$

$$\begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} = A \text{ and } \begin{bmatrix} 1/L & 0 \\ 0 & -1/C \end{bmatrix} = B$$

In the first iteration both  $u_1$  and  $u_2$  are taken as zero and the system is treated as autonomous, depending only on the initial value. Therefore,

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{8}$$

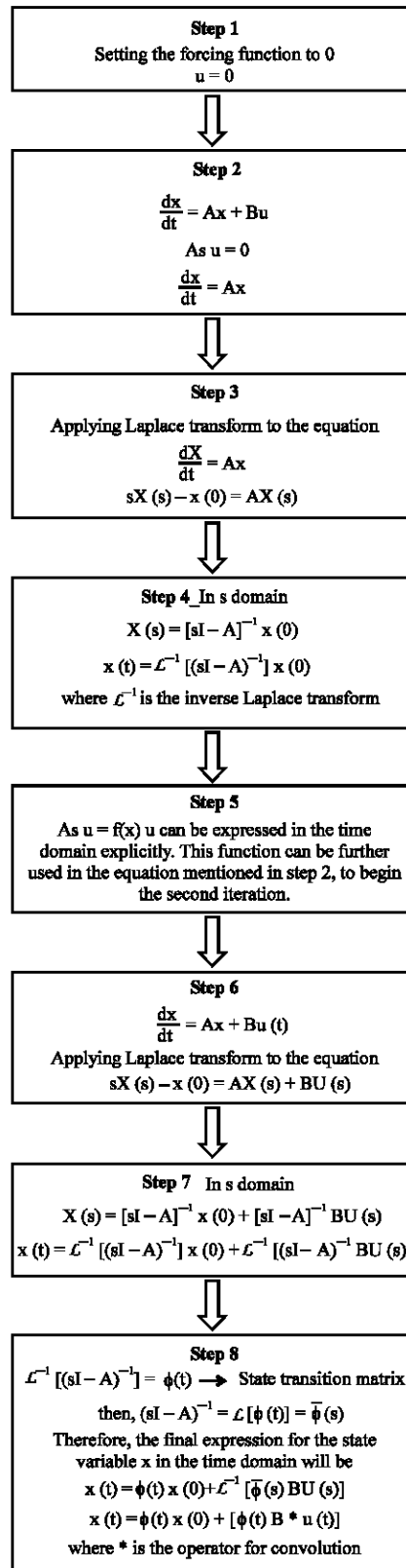


Fig. 3: Flowchart illustrating the technique for analysis of non-linear state variable model

The above matrix is analogous to:

$$\frac{dx}{dt} = Ax$$

Obeying the algorithm, we arrive at the following equation:

$$X(s) = [sI - A]^{-1}x(0) \tag{9}$$

$$[sI - A]^{-1} = \left( s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \right)^{-1} \tag{10}$$

$$= \frac{1}{s^2 + 1/LC} \begin{bmatrix} s & -1/L \\ 1/C & s \end{bmatrix} \tag{11}$$

Therefore Eq. 9 in matrix form is as follows:

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \frac{1}{s^2 + 1/LC} \begin{bmatrix} s & -1/L \\ 1/C & s \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} \tag{12}$$

$$= \begin{bmatrix} \frac{s}{s^2 + 1/LC} & -1/L(s^2 + 1/LC) \\ 1/C(s^2 + 1/LC) & \frac{s}{s^2 + 1/LC} \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} \tag{13}$$

Taking  $\frac{1}{LC} = \omega_0^2$

$$= \begin{bmatrix} \frac{s}{s^2 + \omega_0^2} & -1/L(s^2 + \omega_0^2) \\ 1/C(s^2 + \omega_0^2) & \frac{s}{s^2 + \omega_0^2} \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathcal{L}^{-1} \begin{bmatrix} \frac{s}{s^2 + \omega_0^2} & -1/L(s^2 + \omega_0^2) \\ 1/C(s^2 + \omega_0^2) & \frac{s}{s^2 + \omega_0^2} \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} \tag{15}$$

$$\mathcal{L}^{-1} \left[ \frac{s}{s^2 + \omega_0^2} \right] = \cos(\omega_0 t)$$

$$\mathcal{L}^{-1}\left[\frac{-1}{L(s^2 + \omega_0^2)}\right] = \mathcal{L}^{-1}\left[\frac{-1}{\omega_0 L} \times \frac{\omega_0}{(s^2 + \omega_0^2)}\right] = \frac{-1}{\omega_0 L} \sin(\omega_0 t)$$

$$\mathcal{L}^{-1}\left[\frac{1}{C(s^2 + \omega_0^2)}\right] = \mathcal{L}^{-1}\left[\frac{1}{\omega_0 C} \times \frac{\omega_0}{(s^2 + \omega_0^2)}\right] = \frac{1}{\omega_0 C} \sin(\omega_0 t)$$

Therefore,

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \cos(\omega_0 t) & \frac{-1}{\omega_0 L} \sin(\omega_0 t) \\ \frac{1}{\omega_0 C} \sin(\omega_0 t) & \cos(\omega_0 t) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} \quad (16)$$

$$x_1(t) = x_1(0) \cos(\omega_0 t) - \frac{x_2(0)}{\omega_0 L} \sin(\omega_0 t) \quad (17)$$

$$x_2(t) = \frac{x_1(0)}{\omega_0 C} \sin(\omega_0 t) + x_2(0) \cos(\omega_0 t) \quad (18)$$

Once the functions  $x_1(t)$  and  $x_2(t)$  are obtained at the force free condition of the system, at this stage it is necessary to find out the actual state variable response of the system due to the actual control inputs  $u_1$  and  $u_2$ . Earlier it was assumed that  $u_1 = f(x_1)$  and  $u_2 = f(x_2)$  but mere such assumptions will not serve the actual purpose. More clearly it is to be decided that  $u_1$  is which type of function of  $x_1$  and  $u_2$  is which type of function of  $x_2$ . To decide the strategy, certain practical or physical criteria are needed. Such criteria are based on the plots shown in Fig. 4a and b (Bae *et al.*, 2008). For simplicity of calculations, the solar array output is considered to be a constant. Figure 4a has been obtained based on an approximation. Similar approach has been used by other researchers (Bondar *et al.*, 2008) in connection with the non-linear analysis of power amplifiers.

We can define the following functions for  $u_1$  and  $u_2$

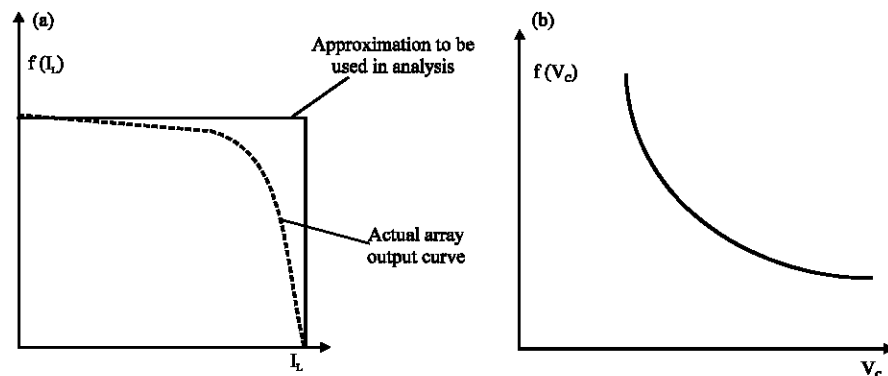


Fig. 4: (a) Array output curve (Bae *et al.*, 2008) and (b) Load line (Bae *et al.*, 2008)



$$u_1 = f(x_1) = \text{constant} = k \tag{19}$$

$$u_2 = f(x_2) = \frac{C_1}{x_2(t)} \tag{20}$$

$$= \frac{C_1}{\frac{x_1(0)}{\omega_0 C} \sin(\omega_0 t) + x_2(0) \cos(\omega_0 t)}$$

$$= \frac{C_1}{P \sin(\omega_0 t) + Q \cos(\omega_0 t)} \tag{21}$$

Where:

$$P = \frac{x_1(0)}{\omega_0 C} \text{ and } Q = x_2(0)$$

$$P \sin(\omega_0 t) + Q \cos(\omega_0 t) = r \sin(\omega_0 t + \theta)$$

$$r = \sqrt{P^2 + Q^2} \text{ and } \theta = \tan^{-1} \frac{Q}{P}$$

Therefore,

$$u_2 = f(x_2) = \frac{C_1}{r \sin(\omega_0 t + \theta)} = \frac{C_1}{r} \operatorname{cosec}(\omega_0 t + \theta) \tag{22}$$

Incorporating the values of  $u_1$  and  $u_2$  in the following equation:

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/L & 0 \\ 0 & -1/C \end{bmatrix} \begin{bmatrix} f(x_1) \\ f(x_2) \end{bmatrix} \tag{23}$$

Applying Laplace transform to the above equation we arrive at the following equation:

$$\begin{bmatrix} sX_1(s) - x_1(0) \\ sX_2(s) - x_2(0) \end{bmatrix} = A \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} + B \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix} \tag{24}$$

$$U_1(s) = \frac{k}{s} \tag{25}$$

$$U_2(s) = \mathcal{L} \left[ \frac{C_1}{r} \operatorname{cosec}(\omega_0 t + \theta) \right]$$

The Laplace transform of  $\text{cosec}(\omega_0 t + \theta)$  is not possible, even after expanding it in terms of its polynomial series due to the presence of singularity.

Therefore, in order to acquire the Laplace expression, instead of using the conventional technique, we expand  $\text{cosec}(\omega_0 t + \theta)$  by using Maclaurin series at zero time instant.

$$\begin{aligned} \text{cosec}(\omega_0 t + \theta) &= \text{cosec}(\omega_0 t + \theta)|_{t=0} + t \times \text{cosec}'(\omega_0 t + \theta)|_{t=0} + \frac{t^2}{2!} \times \text{cosec}''(\omega_0 t + \theta)|_{t=0} \\ &= \text{cosec}\theta - \omega_0 t \text{cosec}\theta \cot\theta + \frac{\omega_0^2 t^2}{2!} [\text{cosec}\theta (\text{cosec}^2\theta + \cot^2\theta)] \\ \mathcal{L}[\text{cosec}(\omega_0 t + \theta)] &= \frac{\text{cosec}\theta}{s} - \frac{\omega_0}{s^2} \text{cosec}\theta \cot\theta + \frac{\omega_0^2}{s^3} [\text{cosec}\theta (\text{cosec}^2\theta + \cot^2\theta)] \\ \mathcal{L}[\text{cosec}(\omega_0 t + \theta)] &= \frac{\text{cosec}\theta}{s} - \frac{\omega_0}{s^2} \text{cosec}\theta \cot\theta + \frac{\omega_0^2}{s^3} [\text{cosec}\theta (\text{cosec}^2\theta + \cot^2\theta)] \\ &= \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \end{aligned} \tag{26}$$

Where,

$$X' = \text{cosec}\theta, Y = \text{cosec}\theta \cot\theta \text{ and } Z = \text{cosec}\theta (\text{cosec}^2\theta + \cot^2\theta)$$

Therefore,

$$U_2(s) = \frac{C_1}{r} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \tag{27}$$

In the second iteration, the function for  $X(s)$  is given as follows:

$$X(s) = [sI - A]^{-1} x(0) + [sI - A]^{-1} B U(s) \tag{28}$$

$$\begin{aligned} [sI - A]^{-1} B &= \frac{1}{s^2 + 1/LC} \begin{bmatrix} s & -1/L \\ 1/C & s \end{bmatrix} \times \begin{bmatrix} 1/L & 0 \\ 0 & -1/C \end{bmatrix} \\ &= \frac{1}{s^2 + 1/LC} \begin{bmatrix} s/L & 1/LC \\ 1/LC & -s/C \end{bmatrix} \end{aligned} \tag{29}$$

$$\begin{aligned} [sI - A]^{-1} B \times U(s) &= \frac{1}{s^2 + 1/LC} \begin{bmatrix} s/L & 1/LC \\ 1/LC & -s/C \end{bmatrix} \times \begin{bmatrix} \frac{k}{s} \\ \frac{C_1}{r} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \end{bmatrix} \\ &= \frac{1}{s^2 + 1/LC} \begin{bmatrix} \frac{k}{L} + \frac{C_1}{rLC} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \\ \frac{k}{sLC} - \frac{sC_1}{rC} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \end{bmatrix} \end{aligned} \tag{30}$$

Therefore,

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \frac{1}{s^2 + 1/LC} \begin{bmatrix} x_1(0)s - \frac{x_2(0)}{L} \\ \frac{x_1(0)}{C} + x_2(0)s \end{bmatrix} + \frac{1}{s^2 + 1/LC} \begin{bmatrix} \frac{k}{L} + \frac{C_1}{rLC} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \\ \frac{k}{sLC} - \frac{sC_1}{rC} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \end{bmatrix} \quad (31)$$

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \frac{1}{s^2 + 1/LC} \begin{bmatrix} x_1(0)s - \frac{x_2(0)}{L} + \frac{k}{L} + \frac{C_1}{rLC} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \\ \frac{x_1(0)}{C} + x_2(0)s + \frac{k}{sLC} - \frac{sC_1}{rC} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 1/LC} \begin{bmatrix} x_1(0)s - \frac{x_2(0)}{L} + \frac{k}{L} + \frac{C_1}{rLC} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \\ \frac{x_1(0)}{C} + x_2(0)s + \frac{k}{sLC} - \frac{sC_1}{rC} \left[ \frac{X'}{s} - \frac{\omega_0 Y}{s^2} + \frac{\omega_0^2 Z}{s^3} \right] \end{bmatrix} \right\} \quad (33)$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \frac{k}{\omega_0 L} \sin(\omega_0 t) + \frac{C_1}{r} \left\{ X'(1 - \cos(\omega_0 t)) - Y(\omega_0 t - \sin(\omega_0 t)) + Z \left( \frac{\omega_0^2 t^2}{2!} - 1 + \cos(\omega_0 t) \right) \right\} + x_1(0) \cos(\omega_0 t) - \frac{x_2(0)}{\omega_0 L} \sin(\omega_0 t) \\ k(1 - \cos(\omega_0 t)) - \frac{C_1}{\omega_0 r C} \left( X' \sin(\omega_0 t) - Y(1 - \cos(\omega_0 t)) + Z(\omega_0 t - \sin(\omega_0 t)) \right) + \frac{x_1(0)}{\omega_0 C} \sin(\omega_0 t) + x_2(0) \cos(\omega_0 t) \end{bmatrix} \quad (34)$$

The constants in the following step have been chosen in a manner such that the number of terms can be reduced to simplify calculations. The following values have been chosen after several numerical approximations.

$$k = 4, C_1 = 4, L = 100 \mu\text{H}, C = 200 \mu\text{F}, \theta = \frac{3\pi}{2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 7071.067812 \text{ Hz}$$

$$\theta = \tan^{-1} \frac{Q}{P} = \frac{3\pi}{2}$$

$$\Rightarrow \frac{Q}{P} = \tan \frac{3\pi}{2}$$

$$\Rightarrow \frac{Q}{P} = \infty$$

As  $Q = x_2(0) \neq \infty$

Therefore,

$$P = \frac{x_1(0)}{\omega_0 C} = 0$$

Therefore,  $x_1(0) = 0$  and assuming  $x_2(0) = 1$  V, we reduce Eq. 34 to the following:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 4.2426 \sin(\omega_0 t) - 2\omega_0^2 t^2 \\ 4 - 3 \cos(\omega_0 t) + (20000)t \end{bmatrix} \quad (35)$$

From Eq. 35, the expression for current through the capacitor is as follows:

$$i_c(t) = C \left( \frac{dx_2(t)}{dt} \right) = (4 + (0.0006\omega_0 \sin(\omega_0 t))) \quad (36)$$

Similarly using Eq. 35, the expression for voltage across inductor is as follows:

$$v_L(t) = L \left( \frac{dx_1(t)}{dt} \right) = (3\cos(\omega_0 t) - (20000t)) \quad (37)$$

Once the state variables and their time derivatives are obtained as suitable functions of time, it is needed to have an estimate of the energy storage phenomenon of the elements in the controller circuit. Such investigation needs suitable data based on numerical results and these data are presented in Table 1 and 2.

## RESULTS

Using Eq. 35, the plots of state variables shown in Fig. 5 and 6 were obtained with the help of MATLAB.

The plot for current through capacitor shown in Fig. 7 was obtained by using Eq. 36.

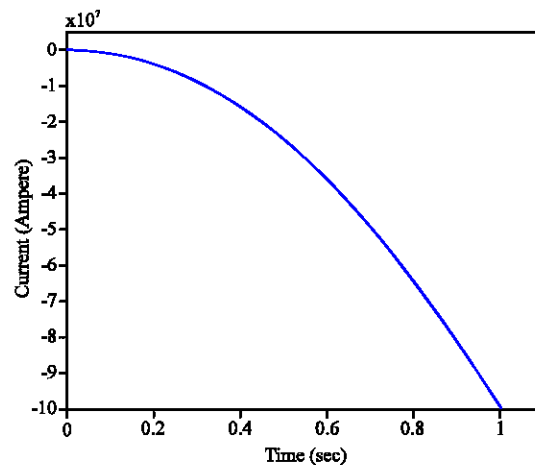


Fig. 5: Plot of  $x_1(t)$  versus time

Table 1: Values of  $f(t)$  at different instants of time

Time t (sec)	$v_L(t)$	$x_1(t)$	$f(t) = v_L(t) \times x(t)$
0	3	0	0
9.00E-10	3	2.70E-05	8.10E-05
1.90E-09	2.9999	5.70E-05	1.71E-04
2.90E-09	2.9999	8.70E-05	2.61E-04
3.90E-09	2.9999	0.000117	3.51E-04
4.90E-09	2.9999	0.000147	4.41E-04
5.90E-09	2.9999	0.00017699	5.31E-04
6.90E-09	2.9998	0.00020699	6.21E-04
7.90E-09	2.9998	0.00023699	7.11E-04
8.90E-09	2.9998	0.00026699	8.01E-04
9.90E-09	2.9998	0.00029699	8.91E-04
1.09E-08	2.9998	0.00032698	9.81E-04
1.19E-08	2.9997	0.00035698	1.07E-03
1.29E-08	2.9997	0.00038698	1.16E-03
1.39E-08	2.9997	0.00041698	1.25E-03
1.49E-08	2.9997	0.00044697	1.34E-03
1.59E-08	2.9997	0.00047697	1.43E-03
1.69E-08	2.9996	0.00050697	1.52E-03
1.79E-08	2.9996	0.00053696	1.61E-03
1.89E-08	2.9996	0.00056696	1.70E-03
1.99E-08	2.9996	0.00059695	1.79E-03
2.09E-08	2.9996	0.00062695	1.88E-03
2.19E-08	2.9995	0.00065695	1.97E-03
2.29E-08	2.9995	0.00068694	2.06E-03
2.39E-08	2.9995	0.00071694	2.15E-03
2.49E-08	2.9995	0.00074693	2.24E-03
2.50E-08	2.9995	0.00074993	2.25E-03

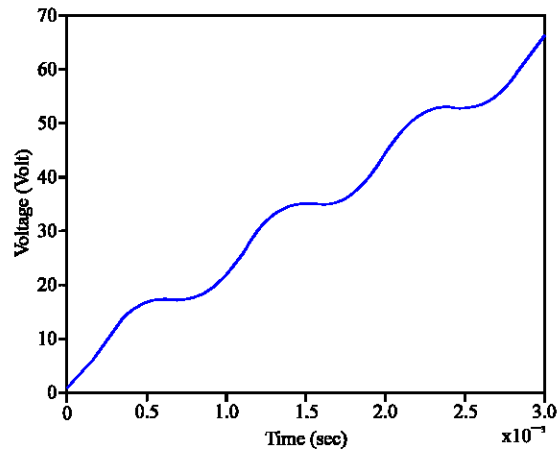


Fig. 6: Plot of  $x_2(t)$  versus time

The plot for voltage across inductor shown in Fig. 8 was obtained by using Eq. 37.

The plot of  $f(t)$  (Fig. 9) was created in MATLAB by using the values of  $f(t)$  computed in Table 1.

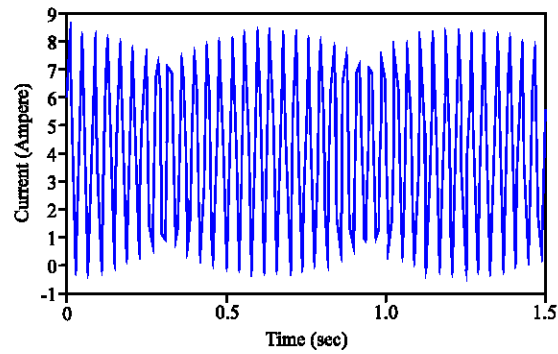


Fig. 7: Plot of  $i_c(t)$  versus time

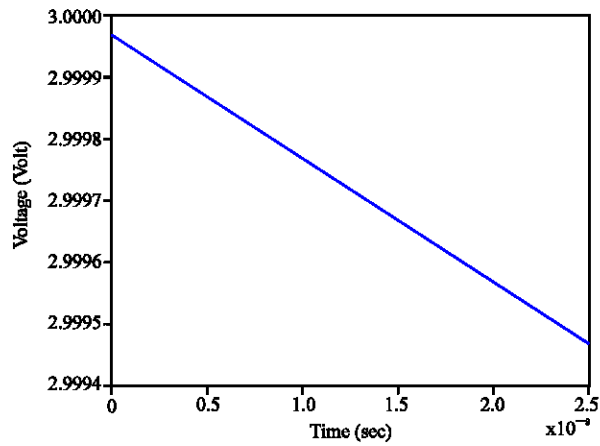


Fig. 8: Plot of  $v_L(t)$  versus time

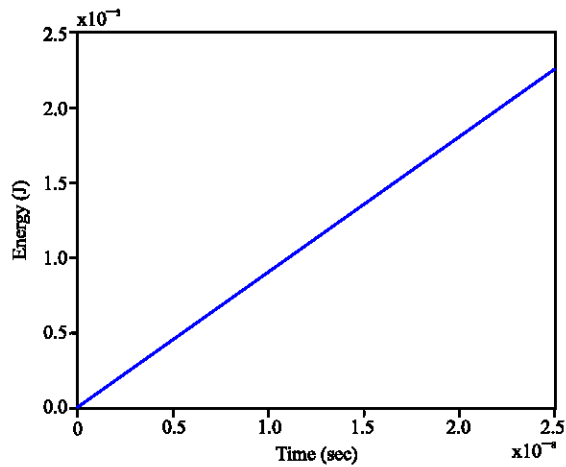


Fig. 9: Plot of  $f(t)$  versus time

Figure 9 was used for finding the area geometrically so as to have a quick look on the energy storage phenomenon.

Thus, the energy stored in the inductor was calculated as  $2.8111 \times 10^{-11}$  Joules.

Table 2: Values of  $g(t)$  at different instants of time

Time t (sec)	$i_C(t)$	$x_2(t)$	$g(t) = i_C(t) \times x_2(t)$
0	4	1	4
9.00E-10	4	1	4
1.90E-09	4.0001	1	4.0001
2.90E-09	4.0001	1.0001	4.00050001
3.90E-09	4.0001	1.0001	4.00050001
4.90E-09	4.0001	1.0001	4.00050001
5.90E-09	4.0002	1.0001	4.00060002
6.90E-09	4.0002	1.0001	4.00060002
7.90E-09	4.0002	1.0002	4.00100004
8.90E-09	4.0003	1.0002	4.00110006
9.90E-09	4.0003	1.0002	4.00110006
1.09E-08	4.0003	1.0002	4.00110006
1.19E-08	4.0004	1.0002	4.00120008
1.29E-08	4.0004	1.0003	4.00160012
1.39E-08	4.0004	1.0003	4.00160012
1.49E-08	4.0004	1.0003	4.00160012
1.59E-08	4.0005	1.0003	4.00170015
1.69E-08	4.0005	1.0003	4.00170015
1.79E-08	4.0005	1.0004	4.0021002
1.89E-08	4.0006	1.0004	4.00220024
1.99E-08	4.0006	1.0004	4.00220024
2.09E-08	4.0006	1.0004	4.00220024
2.19E-08	4.0007	1.0004	4.00230028
2.29E-08	4.0007	1.0005	4.00270035
2.39E-08	4.0007	1.0005	4.00270035
2.49E-08	4.0007	1.0005	4.00270035
2.50E-08	4.0007	1.0005	4.00270035

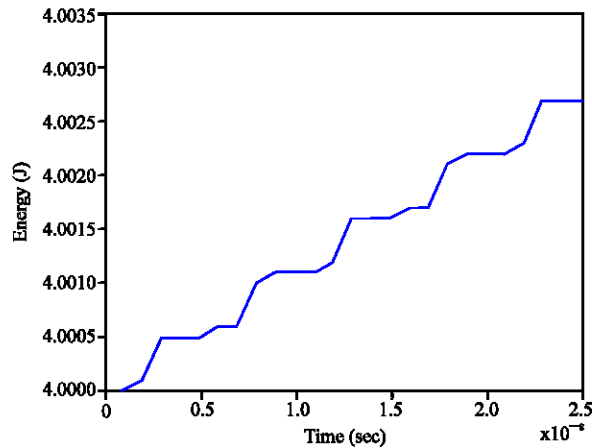


Fig. 10: Graph of  $g(t)$  versus time

The plot of  $g(t)$  (Fig. 10) was created in MATLAB by using the values of  $g(t)$  computed in Table 2.

Similarly, the energy stored in the capacitor was calculated as  $1.0003 \times 10^{-7}$  Joules.

The difference in energy storage between inductor and capacitor was calculated as follows:

$$\begin{aligned} &= (1.0003 \times 10^{-7}) - (2.8111 \times 10^{-11}) \\ &= 1.00001889 \times 10^{-7} \text{ Joules} \end{aligned}$$

## DISCUSSION

The nature of the plot presented in Fig. 7 reflects the dominance of the trigonometric sine term as observed in Eq. 36. Similarly the nature of the plot presented in Fig. 8 reflects the dominance of the linear term as observed in Eq. 37, this results in a graph having a straight line characteristic.

The results of the present analysis indicate an interesting aspect of the controller that the state plane trajectory cannot be developed for such system. This is due to the fact that state variables  $x_1(t)$  and  $x_2(t)$  are not functions of trigonometric terms alone. As  $x_1(t)$  and  $x_2(t)$  consist of terms like  $t$ ,  $t^2$ ,  $t^3$  etc., they are unable to maintain an interrelationship between themselves which is generally reflected as a state plane trajectory. As a contrast, the state variables associated with a linear state variable model automatically lead to the formation of a state plane trajectory.

The results presented in the paper (Balbis *et al.*, 2006) are in context to the state variables in a non-linear state variable model of a biological wastewater treatment process. The similarity of these results with the same presented in our paper lies in the fact that initial values have been chosen suitably. The authors of the paper (Balbis *et al.*, 2006) have presented time response of the state variables. These responses show both fluctuating and steady-state part. Similarly the results presented in our paper indicate the time response of the state variable and the fluctuations along with steady-state part. Another important similarity lies in the fact that state plane trajectory could not be developed by the authors of the paper (Balbis *et al.*, 2006). The authors of the present paper are also of the opinion that in a non-linear state plane model, state plane trajectory cannot be developed due to the reasons already explained in second paragraph of the current section.

The companion research paper (Kutkut *et al.*, 1998) basically deals with the state variable solution of a linear state variable model in the area of resonant converter. Their results are similar to the results presented in this paper in the line of selection of initial values.

The results presented in the literature (Alsharqawi and Batarseh, 2002; Qinghong and Nelms, 2002; Hey and Stein, 1998; Vuksic and Beros, 2003) are related to power electronic applications but the similarity with the present paper lies in the fact that state variable model has been developed. Again it is observed that the authors of these literatures were able to develop the state plane trajectory due to the linearity of the state variable model.

Theoretically, the energy stored in an inductor should be equal to energy stored in the capacitor. However in our results, there is a difference in the energy storage in these two elements. From the research paper (Bae *et al.*, 2008), we can correlate the fact that this difference is attributed to the natural resistance value associated with the inductor coil and the fictitious resistance (responsible for power loss) associated with the capacitor and any incremental resistance in the controller circuit. This energy difference between the inductor and capacitor is dissipated in the form of heat.

## CONCLUSIONS

The state variable model of a solar array power system has been developed using an iterative mathematical technique. Using this technique, the time response expressions for the state variables have been obtained in explicit form. With the help of these time response expressions, simulation



has been performed in MATLAB to assess the energy storage in the controller elements. From the results it appears that there is a difference between the energy stored by the inductor and capacitor. The physical significance of such difference lies in the fact that resistances associated with inductor coil, capacitor plates and also the incremental resistances in the controller circuit cannot be avoided in practice. The time response expressions developed can also be used to analyze the stability of the system. In general, the method used in this paper can be used for the analysis of any non-linear state model.

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