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Recovery of Structural System Properties by Minimization of Finite Element Residuals Error Function

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ABSTRACT

In this study, a structural system identification technique using least square minimization of the finite element residuals error function is developed. In this technique, the structural stiffness of a two dimensional plane stress, plane strain and plate element are recovered. The recovery of element stiffnesses is considered as an inverse analysis problem and can be achieved by application of known static loads and measure the response of the structure which represents the structure associated displacements. The developed technique is based on formulation of the errors Jacobian matrix and minimizing the error of residuals function. Once the stiffness matrix of a structure is determined, the internal design forces due to any loading condition can be easily obtained. Two system identification problems, as numerical examples, were used to test the validity of the proposed technique. In these problems, the mechanical properties of plane stress and plate structures were retrieved. The displacements due to an applied loading are supposed to be measured for each structure, but in this work as a verification of the proposed approach it can be computed using any of the standard FE computer programs. The results show that this technique can accurately retrieve the structure stiffness matrix by using one or two loading cases.

Key words: System identification, inverse analysis, structural analysis, damage assessment, error minimization, Jacobian matrix

INTRODUCTION

Structures identification is very useful for the monitoring and rehabilitation process of the structures to ensure the structural safety. Damage detection technique is critical for decision making of repair, replacement and maintenance of structures (Helou, 1993). Deterioration and partial damage of an existing old structure usually caused a local reduction in the stiffness matrix (Jahn and Mehlhorn, 1998). The member properties do not conform to its original designed value. The evaluation of the existing system properties such as stiffness matrix may be affected by certain physical limitations such as material deterioration resulting from corrosive environment, cracks in reinforced concrete structures and also partial local structural damages (Yang *et al*., 2003). This is frequently encountered in industrial buildings and reinforced concrete structures as a result of loads, temperature or shrinkage where cracks begin to develop. The formulation of cracks is unavoidable in reinforced concrete structures, but it is necessary to limit the cracks' size according to several codes of practice. Cracks cause reduction in the stiffness matrix (Ngo and Scordelis, 1967). This stiffness reduction changes the behavior of the structure and its response due to different loading conditions. Using standard structural analysis, the response of the structural systems to any set of loading conditions depends on the availability of the stiffness matrix, (Hinton and Owen, 1980; Zienkiewicz and Taylor, 2000).

The stiffness matrix of the structural element depends mainly on the element's modulus of elasticity. In case of old structures that have been in use for long periods of time or when the construction materials have greatly deteriorated, the stiffness matrix cannot be accurately computed due to the change in elements properties (Arafa and Helou, 1994). Thus, the structure analysis leads to approximation which undoubtedly causes erroneous results or inaccurate ones at best.

Identifications of system parameters such as the stiffness matrix usually require a mathematical model of the structure in combination with experimental data. The identification approaches are mainly based on the change in the natural frequencies, mode shapes or measured modal flexibility. The natural frequencies are the most common dynamic parameters for system identification (Yang and Huang, 2007; Lu and Law, 2007; Furuta *et al.*, 1991; Gounaris and Papadopoulos, 1997; Lee and Jinho, 2002; Nikolakopoulos *et al.*, 1997; Wu and Li, 2006; Per and Thomas, 2008; Narkis, 1994).

In the studies of Jahn and Mehlhorn (1998) and Krex and Mehlhorn (1998), a modal test data and the structural dynamic analysis (natural frequencies) was used to identify the area and crack locations in reinforced concrete beam and bending plate.

A Combined numerical experimental model was developed by Araujo *et al.* (2000) for identification of damage locations and mechanical properties of structural plate.

The estimated starting parameter is corrected by minimization of differences between measured and corresponding analytical data. The stiffness matrices of truss and frame elements were retrieved by and Arafa and Helou (1994) using system identification based on least square minimization.

In this study, a technique to solve the inverse problem mentioned above is developed. The identification of a two dimensional plane stress, plane strain and plate structure stiffness matrix is achieved through measurement of the response of the structure due to applied known load. The element stiffnesses are recovered through application of known static forces at the elements nodes together with the measurement of the associated displacements. Then, using the least square minimization the stiffness matrix can be retrieved using Jacobian matrix of the Error function J_{Error}. The structural system is to be modeled using a finite element method which can easily model two dimensional structures (Zienkiewicz and Taylor, 2000).

Formulation of element stiffness matrix: In the isoparametric element concept used in the finite elements method, the same shape functions are used to describe the elements geometry as well as the displacement within the element. The interpolation functions must satisfy at least the C^0 continuity, where the C^0 continuity means that the function itself is continuous and C^m continuity means that functions derivative up to the order of (m) are continuous (Zienkiewicz and Taylor, 2000). The shape function is defined as follows:

$$N_i(\xi_j, \eta_j) = \delta_{ij}$$

where:

$$\delta_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases} \quad \text{for } i, j \in 1, \cdots, n \tag{1}$$

$$x=\sum_{i=1}^{n} N_i(\xi_i,\eta_i)\cdot x_i$$
$$y=\sum_{i=1}^{n} N_i(\xi_i,\eta_i)\cdot y_i \tag{2}$$

The global displacement is calculated based on the nodal displacement:

$$u=\sum_{i=1}^{n} N_i(\xi_i,\eta_i)\cdot u_i$$
$$v=\sum_{i=1}^{n} N_i(\xi_i,\eta_i)\cdot v_i \tag{3}$$

Where:
n: No. of nodes per element
x_i and y_i: The x and y coordinates of the nodal joints
u_i and v_i: The x and y displacements at the nodal joints

The strain vector can be formulated as:

$$\varepsilon = B.u^T$$
$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \tag{4}$$

Using isoparametric element, the B matrix (Hinton and Owen, 1980) is defined as:

$$\varepsilon = \sum_{i=1}^{n} B_j . d_i^T \tag{5}$$

$$B = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \end{bmatrix} \tag{6}$$

The stress matrix can be determined using the following equation:

$$\sigma = De \tag{7}$$

where, D is a matrix relating stress to strain and depends on the type of the element:

$$D=\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu/2 \end{bmatrix} \tag{8}$$

where, k represents a material constant.

For plane stress element:

$$k=\frac{E}{1-\nu^2}$$

while for plane strain element:

$$k=\frac{E}{(1+\nu)(1-2\nu)}$$

and for kirchhoff plate bending element:

$$k=\frac{Et^3}{12(1-\nu^2)}$$

The stiffness matrix can be defined by the following expression:

$$K^e=\iiint_V B^T DBdV \tag{9}$$

For two dimensional element with constant thickness t:

$$K^e=t\iint_A B^T DBdxdy \tag{10}$$

Using isoparametric formulation:

$$K^e=\int_{-1}^{1}\int_{-1}^{1} B^T DBJd\xi d\eta \tag{11}$$

where, J is defined as the Jacobian matrix and can be written as:

$$J=\begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}=\begin{bmatrix} \sum \frac{\partial N_i}{\partial \xi} x_i & \sum \frac{\partial N_i}{\partial \xi} y_i \\ \sum \frac{\partial N_i}{\partial \eta} x_i & \sum \frac{\partial N_i}{\partial \eta} y_i \end{bmatrix} \tag{12}$$

Finally, the stiffness of an individual element is expressed as:

$$K^e = k_e \begin{bmatrix} a^e_{11} & a^e_{12} & \cdots & a^e_{1m'} \\ \vdots & a^e_{22} & \cdots & a^e_{2m'} \\ \vdots & \cdots & \cdots & \vdots \\ \text{Sys} & \cdots & \cdots & a^e_{m'm'} \end{bmatrix} \tag{13}$$

Where:

m': The total number of degrees of freedom per element

a^e_{ij}: The coefficient of the element stiffness matrix of element *e* at degrees of freedom i and j in the global coordinate system

The total structure stiffness matrix over the total degrees of freedom can be formulated from individual elements stiffness matrices:

$$K = \begin{bmatrix} \sum_{i=1}^{n} a^i_{11}k_i & \sum_{i=1}^{n} a^i_{12}k_i & \cdots & \sum_{i=1}^{n} a^i_{1m}k_i \\ \vdots & \sum_{i=1}^{n} a^i_{22}k_i & \cdots & \sum_{i=1}^{n} a^i_{2m}k_i \\ \vdots & \vdots & \cdots & \vdots \\ \text{Sys} & \cdots & \cdots & \sum_{i=1}^{n} a^i_{mm}k_i \end{bmatrix} \tag{14}$$

Where:

n: The total number of elements in the discretized finite element mesh.

m: Total number of degrees of freedom.

a^e_{ij}: The coefficient of the element stiffness matrix of ith element in the global degree of freedom k and j

The Jacobian matrix of the error function: The structure system equation is written as:

$$F = Kd \tag{15}$$

Where:

F: The applied force

$$\{F_1\ F_2\ ..\ F_m\} \tag{16}$$

K: Total stiffness matrix

d: The structure displacements

$$d = \{d_1\ d_2\ ...\ d_m\} \tag{17}$$

For inexact solution, the following error vector of residual can be implemented as follows:

$$e = F\text{-}Kd$$

$$\begin{Bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_m \end{Bmatrix} - \begin{bmatrix} \sum_{i=1}^{n} a_{11}^{i} k_i & \sum_{i=1}^{n} a_{12}^{i} k_i & \cdots & \sum_{i=1}^{n} a_{1m}^{i} k_i \\ \vdots & \sum_{i=1}^{n} a_{22}^{i} k_i & \cdots & \sum_{i=1}^{n} a_{2m}^{i} k_i \\ \vdots & \vdots & \cdots & \vdots \\ \text{Sys} & \cdots & \cdots & \sum_{i=1}^{n} a_{mm}^{i} k_i \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{Bmatrix} \tag{18}$$

The error function Error is defined as the sum of the square of the errors over the number degree of freedom:

$$\text{Error} = \sum_{i=1}^{m} e_i^2 \tag{19}$$

To minimize the error function, the derivative with respect to each unknown element's stiffness will be set equal to zero, this is so called least square minimization (Hinton and Campbell, 1974). Taking the first derivative of the Eq. 19 with respect to k_1, k_2, ... k_n yields the following set of equation:

$$\begin{Bmatrix} \sum_{i=1}^{m} a_{1i}^{1} d_1 & \sum_{i=1}^{m} a_{2i}^{1} d_2 & \cdots & \sum_{i=1}^{m} a_{1i}^{1} d_m \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{m} a_{1i}^{n} d_1 & \sum_{i=1}^{m} a_{ni}^{n} d_2 & \cdots & \sum_{i=1}^{m} a_{1i}^{n} d_m \end{Bmatrix} \begin{Bmatrix} F_1 - \sum_{j=1}^{n} \sum_{i=1}^{m} a_{1i}^{j} k_j d_1 \\ \vdots \\ F_m - \sum_{j=1}^{n} \sum_{i=1}^{m} a_{1i}^{j} k_j d_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{20}$$

By defining the Jacobian Matrix of the error function as follow:

$$J_{\text{Error}} = \begin{Bmatrix} \frac{\partial e_1}{\partial k_1} & \frac{\partial e_1}{\partial k_2} & \cdots & \frac{\partial e_1}{\partial k_n} \\ \frac{\partial e_2}{\partial k_1} & \frac{\partial e_2}{\partial k_2} & \cdots & \frac{\partial e_2}{\partial k_n} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial e_m}{\partial k_1} & \frac{\partial e_n}{\partial k_2} & \cdots & \frac{\partial e_m}{\partial k_n} \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{m} a_{1i}^{1} d_1 & \cdots & \sum_{i=1}^{m} a_{1i}^{n} d_1 \\ \sum_{i=1}^{m} a_{2i}^{1} d_2 & \cdots & \sum_{i=1}^{m} a_{ni}^{n} d_2 \\ \vdots & \cdots & \vdots \\ \sum_{i=1}^{m} a_{1i}^{1} d_m & \cdots & \sum_{i=1}^{m} a_{1i}^{n} d_m \end{Bmatrix} \tag{21}$$

$$\begin{Bmatrix} \sum_{i=1}^{m} a_{1i}^{1} d_1 & \sum_{i=1}^{m} a_{2i}^{1} d_2 & \cdots & \sum_{i=1}^{m} a_{1i}^{1} d_m \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{m} a_{1i}^{n} d_1 & \sum_{i=1}^{m} a_{ni}^{n} d_2 & \cdots & \sum_{i=1}^{m} a_{1i}^{n} d_m \end{Bmatrix} \begin{Bmatrix} \sum_{i=1}^{m} a_{1i}^{1} d_1 & \cdots & \sum_{i=1}^{m} a_{1i}^{n} d_1 \\ \sum_{i=1}^{m} a_{2i}^{1} d_2 & \cdots & \sum_{i=1}^{m} a_{ni}^{n} d_2 \\ \vdots & \cdots & \vdots \\ \sum_{i=1}^{m} a_{1i}^{1} d_m & \cdots & \sum_{i=1}^{m} a_{1i}^{n} d_m \end{Bmatrix} \begin{Bmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{Bmatrix}$$
$$= \begin{Bmatrix} \sum_{i=1}^{m} a_{1i}^{1} d_1 & \sum_{i=1}^{m} a_{2i}^{1} d_2 & \cdots & \sum_{i=1}^{m} a_{1i}^{1} d_m \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{m} a_{1i}^{n} d_1 & \sum_{i=1}^{m} a_{ni}^{n} d_2 & \cdots & \sum_{i=1}^{m} a_{1i}^{n} d_m \end{Bmatrix} \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{Bmatrix} \tag{22}$$

The previous equation can be written in a reduced form:

$$J_{Error}^{T} J_{Error} k = J_{Error}^{T} F \tag{23}$$

This yield:

$$k = \left(J_{Error}^{T} J_{Error}\right)^{-1} \left(J_{Error}^{T} F\right) \tag{24}$$

In some cases, $J^{T}{}_{Erroe} J_{Error}$ may not be invertible. Two or more loading cases must be used to obtain the elements stiffnesses as follows:

$$k = \left(\sum_{i=1}^{NL} \left[J_{Error}^{T}\right]_i \left[J_{Error}\right]_i\right)^{-1} \left(\sum_{i=1}^{NL} \left[J_{Error}^{T}\right]_i \{F\}_i\right) \tag{25}$$

Displacements measurement: The proposed method requires a complete set of displacement measurements at every degree of freedom of the finite element model. This can be avoided by using of interpolation techniques (Hinton and Campbell, 1974). Many computer applications like MATLAB (2009) offer one, two and three dimensional interpolation routines, which can be used to interpolate the displacements at all nodes by measurement of the displacements at sufficient defined nodes. The rotations at a certain point of a structural plate represent the derivatives of its deformed shape at that point with respect to the plate plane axis. These derivatives can be computed using numerical differentiation with the help of the measured vertical displacements at different elements nodes in the neighbourhood of that point.

Computer implementation of the proposed technique: The elements material constants to be identified are; the Modulus of elasticity of each element of the FE model is set to be as unknown variable. Figure 1 shows the methodology flow chart of the proposed method for system identification. The proposed method was implemented in a computer program using MATLAB (2009) environment with symbolic toolbox.

Numerical example: The following examples demonstrate the validity of the proposed technique for retrieval of the structure's unknown stiffnesses for plane stress and plate element. These examples are considered as numerical experiments in which displacements associated to an applied load were actually computed using standard direct stiffness methods.

Example 1 (plan stress elements): The thin plate shown in the Fig. 2 is used to test the procedure. The plate is subjected to the loading shown, the plate thickness t = 1 cm and $\nu = 0.30$. This example was also solved with the example given by Logan (2002).

The plate will be discretized into two Constant Strain Triangular elements strain (CST). Each element consists of three nodes with two degree of freedom for each node.

The global reduced stiffness matrix was obtained from standard finite element analysis for the degree of freedom at joint 3 and 4. Element No. 1 is connected to joint 1, 2 and 3, as shown in Fig. 3.

The stiffness matrix of element No. 1 corresponding to degree of freedom of node 3 and 4:

$$
\begin{array}{cc}
 & \begin{array}{cccc} \text{DO}\ 5 & 6 & 7 & 8 \end{array} \\
K^{1} = \dfrac{5}{4A(1-\nu^{2})} & \begin{bmatrix} 20E_{1} & 0 & 0 & 0 \\ 0 & 7E_{1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\end{array}
$$

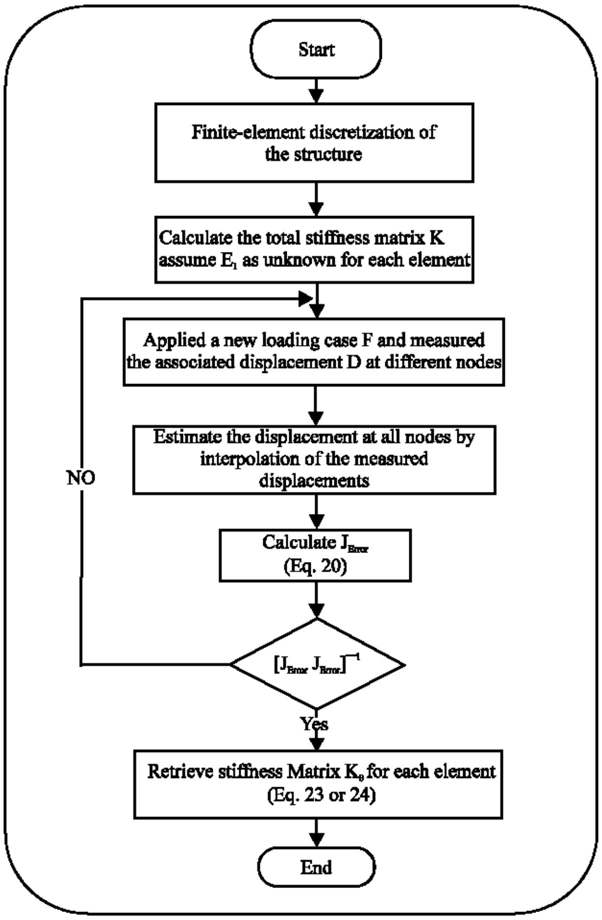

Fig. 1: Flow chart for the proposed method

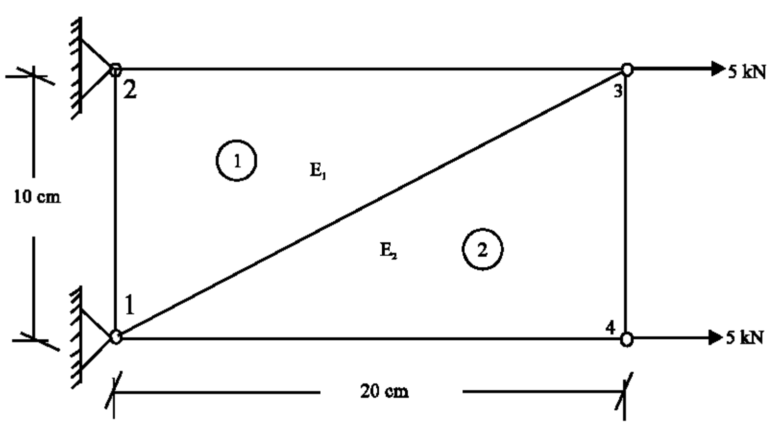

Fig. 2: Axially loaded plate

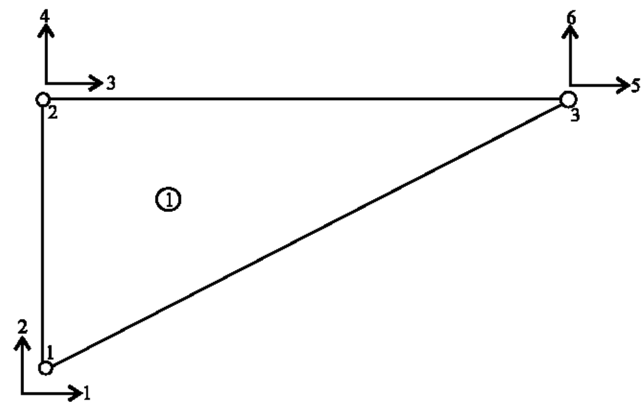

Fig. 3: Element No. 1

Similarly, the stiffness matrix of element No. 2, which connected to joint 1, 3 and 4, corresponding to degree of freedom of node 3 and 4 is:

$$K^2 = \frac{5}{4A(1-\nu^2)} \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 28E_2 & 0 & -28E_2 & 14E_2 \\ 0 & 80E_2 & 12E_2 & -80E_2 \\ -28E_2 & 12E_2 & 48E_2 & -26E_2 \\ 14E_2 & -80E_2 & -26E_2 & 87E_2 \end{bmatrix} \end{array}$$

(DOF: 5, 6, 7, 8)

The total stiffness matrix of the reduced system is:

$$K = K^1 + K^2 = \frac{1}{80\times 0.91} \times \begin{bmatrix} 20E_1+28E_2 & 0 & -28E_2 & 14E_2 \\ 0 & 7E1+80E_2 & 12E_2 & -80E_2 \\ -28E_2 & 12E_2 & 48E_2 & -26E_2 \\ 14E_2 & -80E_2 & -26E_2 & 87E_2 \end{bmatrix}$$

The error vector can be written as:

$$e = F - Kd$$

$$\begin{Bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} - \frac{1}{80\times 0.91} \times \begin{bmatrix} 20E_1+28E_2 & 0 & -28E_2 & 14E_2 \\ 0 & 7E1+80E_2 & 12E_2 & -80E_2 \\ -28E_2 & 12E_2 & 48E_2 & -26E_2 \\ 14E_2 & -80E_2 & -26E_2 & 87E_2 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix}$$

The Jacobian Matrix of the error function is:

$$J_{Error} = \frac{1}{80\times 0.91} \begin{bmatrix} 20d_1 & 28d_1 - 28d_3 + 14d_4 \\ 7d_2 & 80d_2 + 12d_3 - 80d_4 \\ 0 & -28d_1 + 12d_2 + 48d_3 - 26d_4 \\ 0 & 14d_1 - 80d_2 - 26d_3 + 87d_4 \end{bmatrix}$$

The following is the equivalent joint load action for the loading shown, together with the associated displacement used in the present numerical experiments. It was assumed that:

$$E_1 = 200 \text{ Gpa}, \quad E_2 = 180 \text{ GPa}$$

$$F = \begin{Bmatrix} 5 \\ 0 \\ 5 \\ 0 \end{Bmatrix} \text{kN} \quad \text{and} \quad D = \begin{Bmatrix} 12.95 \\ 0.638 \\ 14.758 \\ 2.914 \end{Bmatrix} 10^{-6} \text{ cm}$$

Upon performing the operation described in Eq. 23 the elements stiffnesses can be obtained. They are the same as the assumed value in the numerical experiment:

$$k = \left(J_{\text{Error}}^T J_{\text{Error}}\right)^{-1} \left(J_{\text{Error}}^T F\right) = \begin{Bmatrix} 200 \\ 180 \end{Bmatrix} \text{GPa}$$

It is noted that the result match the assumed input values for elements stiffnesses

Example 2 (plate elements): The stiffnesses of a steel plate with 4×4 m width and 10 cm thickness is to be identified. The plate is fixed along all four edges as shown in the Fig. 4.

The plate will be discretized into four rectangular bending plate elements. Each element consists of four nodes with three degree of freedoms for each node; namely vertical displacement and two rotations. Each element will be assumed to have independent modulus of elasticity as shown in Fig. 4.

The reduced stiffness matrix of element No.1 can be formulated as follows:

$$K^1 = E_1 \begin{bmatrix} \frac{11}{35} & -\frac{61}{420} & \frac{61}{420} \\ -\frac{61}{420} & \frac{19}{105} & -\frac{1}{28} \\ \frac{61}{420} & -\frac{1}{28} & \frac{19}{105} \end{bmatrix}$$

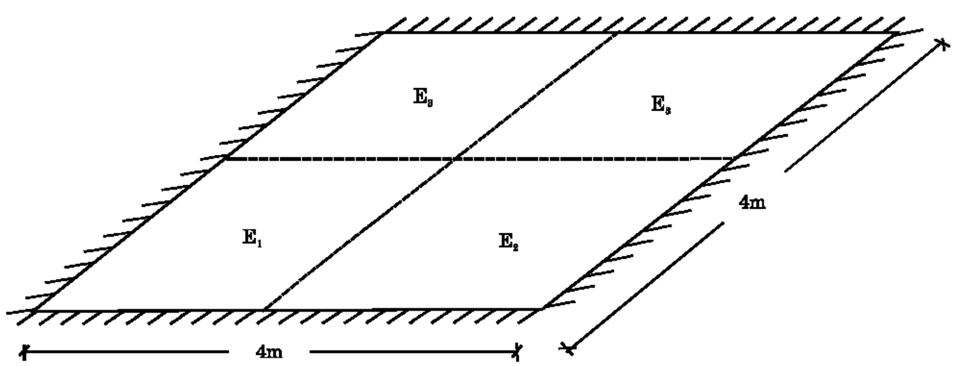

Fig. 4: Plate geometry

The total stiffness matrix of the reduced system is:

$$K = K^1 + K^2 + K^3 + K^4 =$$

$$\begin{array}{cccc} \text{DOF} & 13 & 14 & 15 \\ & \left[\begin{array}{ccc} \dfrac{11(E_1+E_2+E_3+E_4)}{35} & \dfrac{61(-E_1-E_2+E_3+E_4)}{420} & \dfrac{61(E_1-E_2-E_3+E_4)}{420} \\ \vdots & \dfrac{19(E_1+E_2+E_3+E_4)}{105} & \dfrac{(-E_1+E_2-E_3+E_4)}{28} \\ \text{sym} & \cdots & \dfrac{19(E_1+E_2+E_3+E_4)}{105} \end{array}\right] \end{array}$$

The error vector can be written as:

$$e = F - Kd$$

$$\begin{Bmatrix} e_1 \\ e_2 \\ e_3 \end{Bmatrix} = \begin{Bmatrix} F_7 \\ F_8 \\ F_9 \end{Bmatrix} - \begin{bmatrix} \dfrac{11(E_1+E_2+E_3+E_4)}{35} & \dfrac{61(-E_1-E_2+E_3+E_4)}{420} & \dfrac{61(E_1-E_2-E_3+E_4)}{420} \\ \vdots & \dfrac{19(E_1+E_2+E_3+E_4)}{105} & \dfrac{(-E_1+E_2-E_3+E_4)}{28} \\ \text{sym} & \cdots & \dfrac{19(E_1+E_2+E_3+E_4)}{105} \end{bmatrix}$$

where:

$$\begin{Bmatrix} F_7 \\ F_8 \\ F_9 \end{Bmatrix} = \begin{Bmatrix} F_{w_5} \\ F_{\theta_{x5}} \\ F_{\theta_{y5}} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} d_7 \\ d_8 \\ d_9 \end{Bmatrix} = \begin{Bmatrix} w_5 \\ \theta_{x5} \\ \theta_{y5} \end{Bmatrix}$$

The Jacobian Matrix of the error function is:

$$e = F - Kd$$

$$J_{Error}^{T} = \begin{bmatrix} \dfrac{11}{35}d_7 - \dfrac{61}{420}d_8 + \dfrac{61}{420}d_9 & -\dfrac{61}{420}d_7 + \dfrac{19}{105}d_8 - \dfrac{1}{28}d_9 & \dfrac{61}{420}d_7 - \dfrac{1}{28}d_8 + \dfrac{19}{105}d_9 \\ \dfrac{11}{35}d_7 - \dfrac{61}{420}d_8 - \dfrac{61}{420}d_9 & -\dfrac{61}{420}d_7 + \dfrac{19}{105}d_8 + \dfrac{1}{28}d_9 & -\dfrac{61}{420}d_7 + \dfrac{1}{28}d_8 + \dfrac{19}{105}d_9 \\ \dfrac{11}{35}d_7 + \dfrac{61}{420}d_8 + \dfrac{61}{420}d_9 & \dfrac{61}{420}d_7 + \dfrac{19}{105}d_8 - \dfrac{1}{28}d_9 & -\dfrac{61}{420}d_7 - \dfrac{1}{28}d_8 + \dfrac{19}{105}d_9 \\ \dfrac{11}{35}d_7 + \dfrac{61}{420}d_8 + \dfrac{61}{420}d_9 & \dfrac{61}{420}d_7 + \dfrac{19}{105}d_8 + \dfrac{1}{28}d_9 & \dfrac{61}{420}d_7 + \dfrac{1}{28}d_8 + \dfrac{19}{105}d_9 \end{bmatrix}$$

The displacements at the plate midpoint d7, d8 and d9 are supposed to be measured, were in this numerical experiment it can be computed using any of the standard FE computer program. It was assumed that E_1=180 Gpa, E_2=190 Gpa, E_3=200 Gpa and E_4=210 Gpa

Loading case No. 1: The plate is loaded with one central concentrated load of 100kN, as shown in Fig. 5.

$$\begin{Bmatrix} F_7 \\ F_8 \\ F_9 \end{Bmatrix} = \begin{Bmatrix} F_{w_5} \\ F_{\theta_{x5}} \\ F_{\theta_{y5}} \end{Bmatrix} = \begin{Bmatrix} -100 \\ 0 \\ 0 \end{Bmatrix}$$

and the resulted displacements are:

$$\begin{Bmatrix} d_7 \\ d_8 \\ d_9 \end{Bmatrix} = \begin{Bmatrix} w_5 \\ \theta_{x5} \\ \theta_{y5} \end{Bmatrix} = \begin{Bmatrix} -408.32 \\ 16.807 \\ -0.08506 \end{Bmatrix} \times 10^{-9}$$

The Jacobian Matrix of this loading case is:

$$J_{Error} = \begin{bmatrix} -13.078 & -13.076 & -12.588 & -12.590 \\ 6.2349 & 6.2342 & -5.6260 & -5.6266 \\ -5.9920 & 5.9889 & 5.8689 & -5.8719 \end{bmatrix} \times 10^{-8}$$

The matrix J^T_{Errorj} is ill conditioned and almost a singular matrix. Thus it is invertible for this load condition. Therefore another independent loading condition is needed.

Loading case No. 2: The plate is now loaded with a new independent loading case shown in Fig. 6.

$$\begin{Bmatrix} F_7 \\ F_8 \\ F_9 \end{Bmatrix} = \begin{Bmatrix} F_{w_5} \\ F_{\theta_{x5}} \\ F_{\theta_{y5}} \end{Bmatrix} = \begin{Bmatrix} -3 \\ 1 \\ -1 \end{Bmatrix}$$

the resulted displacements are:

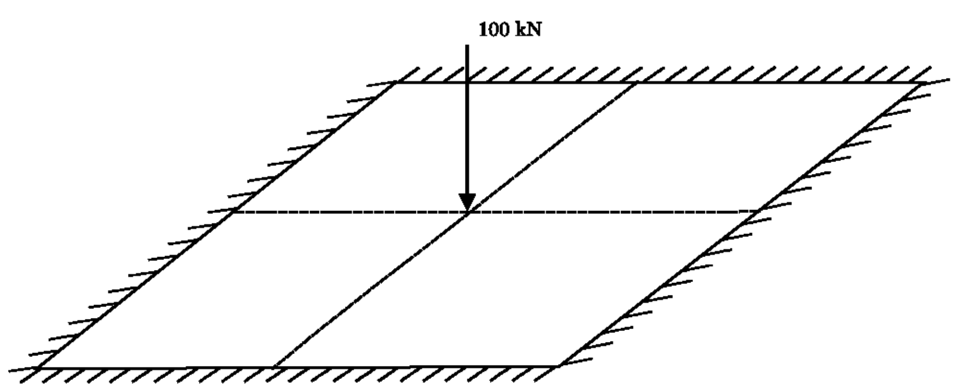

Fig. 5: Loading case No. 1

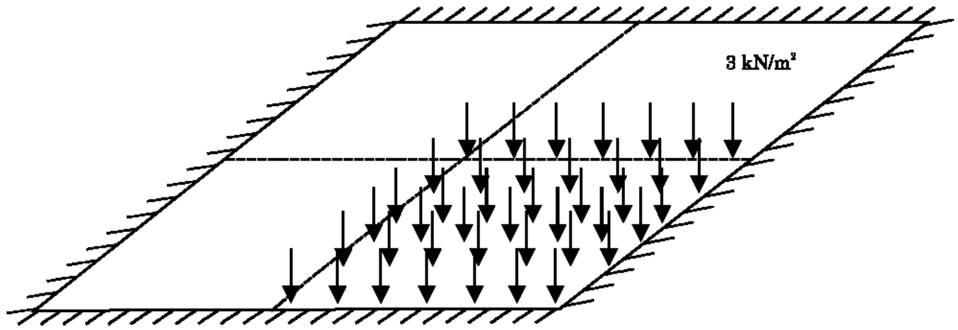

Fig. 6: Loading case No.

$$\begin{Bmatrix} d_7 \\ d_8 \\ d_9 \end{Bmatrix} = \begin{Bmatrix} w_5 \\ \theta_{x5} \\ \theta_{y5} \end{Bmatrix} = \begin{Bmatrix} -124.19 \\ 76.322 \\ -71.236 \end{Bmatrix} \times 10^{-6}$$

$$J_{Error} = \begin{bmatrix} -6.046 & -3.977 & -1.760 & -3.829 \\ 3.439 & 2.930 & -0.168 & -0.677 \\ -3.365 & 7.872 & 0.242 & -2.820 \end{bmatrix} \times 10^{-8}$$

The matrix J^T_{Error} J of this loading condition is ill conditioned and almost a singular matrix. Now apply Eq. 25 to evaluate the value of unknown element stiffnesses:

$$k = \left(\sum_{i=1}^{NL} \left[J^T_{Error} \right]_i \left[J_{Error} \right]_i \right)^{-1} \left(\sum_{i=1}^{NL} \left[J^T_{Error} \right]_i \{F\}_i \right)$$

$$k = \{180 \;\; 190 \;\; 200 \;\; 210\}^T \text{ GPa}$$

This is exactly as the assumed values in the numerical experiment.

CONCLUSION

A system identification technique based on the least square minimization was developed. The retrieval of plane stress, plane strain, axisymmetric and plate element stiffness matrix was achieved by measurement of the structure response due to an applied load. Two numerical experimental examples were used to test the validity of this technique. In these examples, the displacements due to an applied loading are supposed to be measured, but in these numerical experiments it can be computed using any of the standard FE computer programs. The results show that this technique can accurately retrieve the structure stiffness matrix by using one or two loading cases. A complete set of readings must be available at every degree of freedom of the structure using the proposed method. This shortcoming can be avoided through using data interpolation and further research in this area is still required.

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Zienkiewicz, O.C. and R.L. Taylor, 2000. The Finite Element Analysis. Vol. 2. 5th Edn., Butterworth-Heinemann, UK., ISBN: 978-0-7506-5055-7.Fig. 6: Loading case No.